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Visibility Estimation and Calibration

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Outline

- Visibility
- Visibility estimation via fringe scanning
- Signal-to-noise ratio
- Incoherent and coherent averaging
- Detection biases
- Atmospheric and instrumental biases
- Single-mode fibers
- Interspersed data from PTI

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Visibility

- Visibility is the fundamental observable for interferometric imaging
- Visibility is related to the object irradiance distribution via the van Cittert–Zernike theorem
- Visibility is generally complex, viz. $\Gamma = Ve^{-j\phi}$
- In optical/IR interferometry, "visibility" frequently refers to the visibility amplitude: $V = |\Gamma|$
- To get the true object phase requires either phase referencing or closure phase

Measuring Visibility

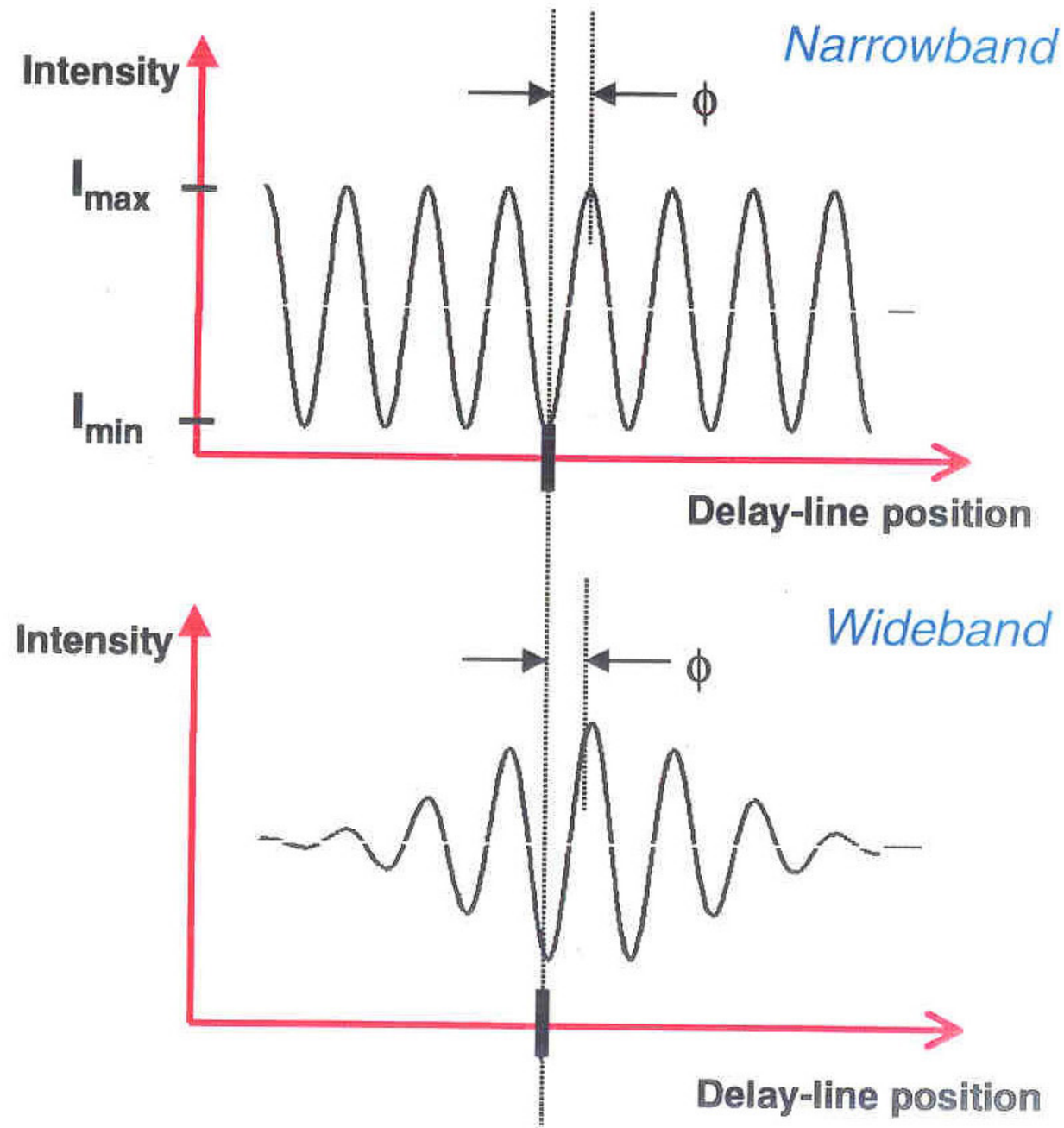
Visibility Amplitude
(or just visibility)

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$0 < V < 1$$

- In general, fringe visibility is a complex number

$$\Gamma = V e^{-j\phi}$$



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Measuring visibility

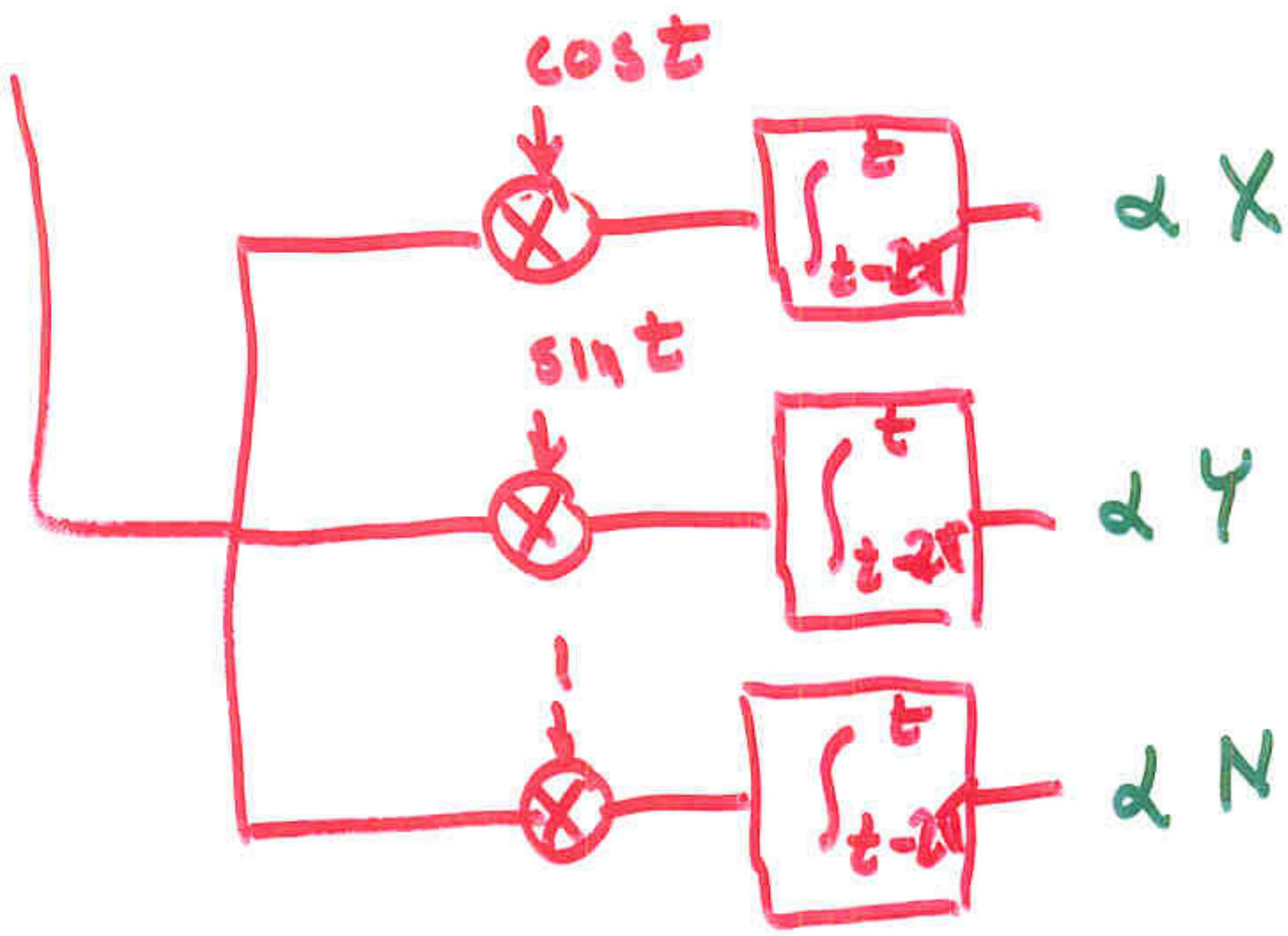
- Visibility is just the contrast of the spatial fringe pattern
- Most measurement schemes involve converting the spatial pattern to a temporal pattern
  - We know how to measure the contrast of an electrical sinusoid
  - These are all variants of schemes used for phase shifting interferometry for optical testing.
    - » Options
      - Step or continuous scanning (blurring, settling)
      - 4, 6, or 8 bins (match sin wave, #reads)
      - Triangle or sawtooth waveform (#reads, retrace time, overlap)

Visibility Estimation as

- coherent demodulation
- quadrature demodulation
- matched filtering

- Use fringe scanning to convert spatial pattern to temporal pattern

$$I = N(1 + V\cos(t+\phi))$$
$$= N + X\cos t + Y\sin t$$



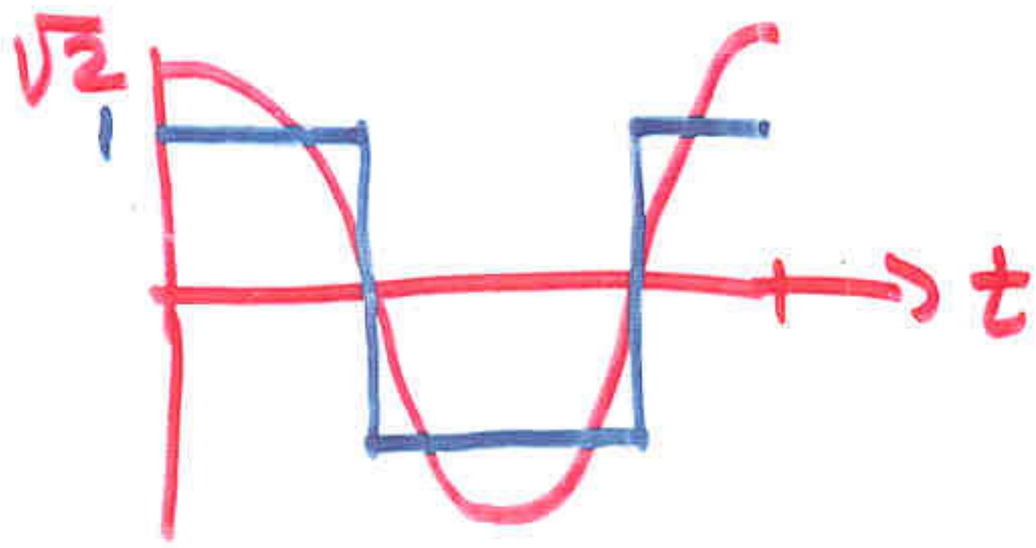
$$\hat{\phi} = \tan^{-1}\frac{\hat{Y}}{\hat{X}}$$

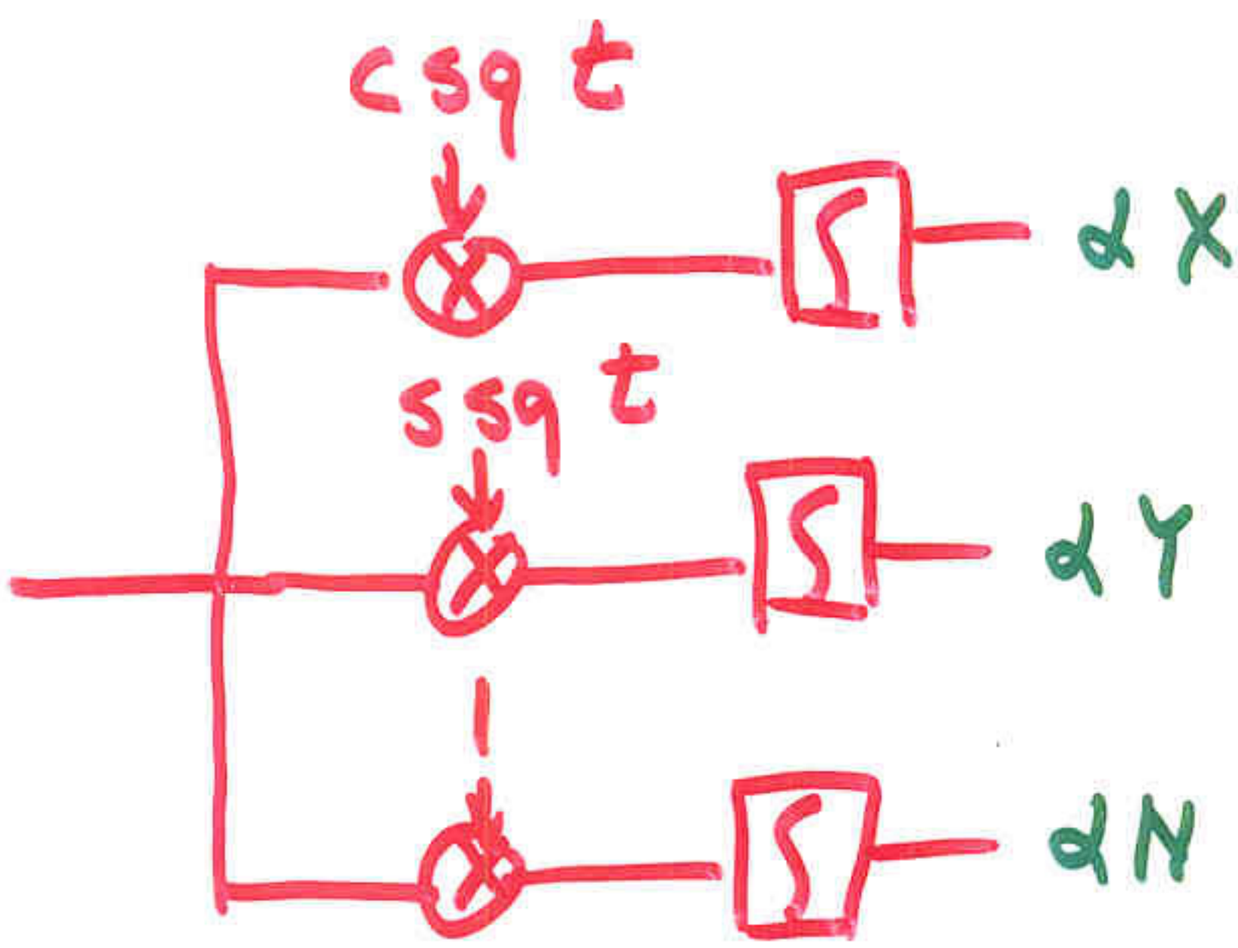
$$\widehat{NV} \propto \sqrt{\hat{X}^2 + \hat{Y}^2}$$

$$\hat{V} \propto \frac{\sqrt{\hat{X}^2 + \hat{Y}^2}}{N}$$

4-bin algorithm

Approximate sines, cosines with square waves





slightly non optimal, as it's a mismatch to the proper waveform (0.9 dB)

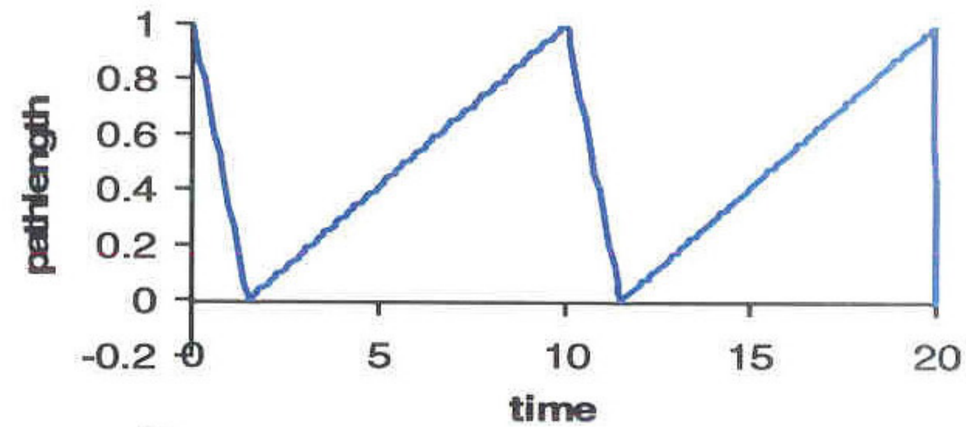
- but minimizes # reads

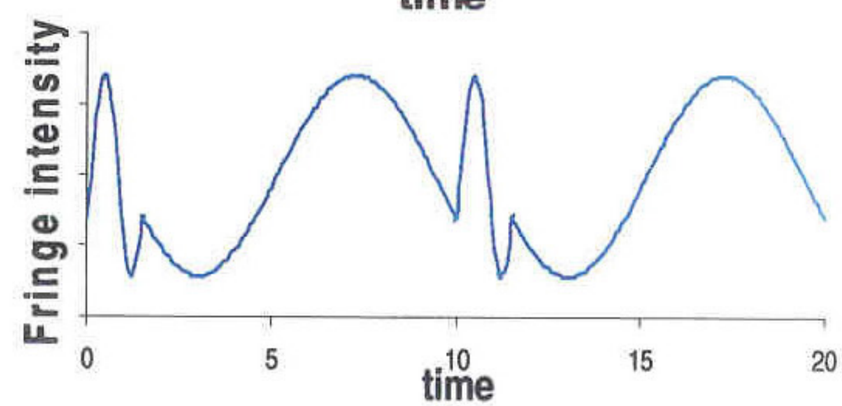
Fringe measurements

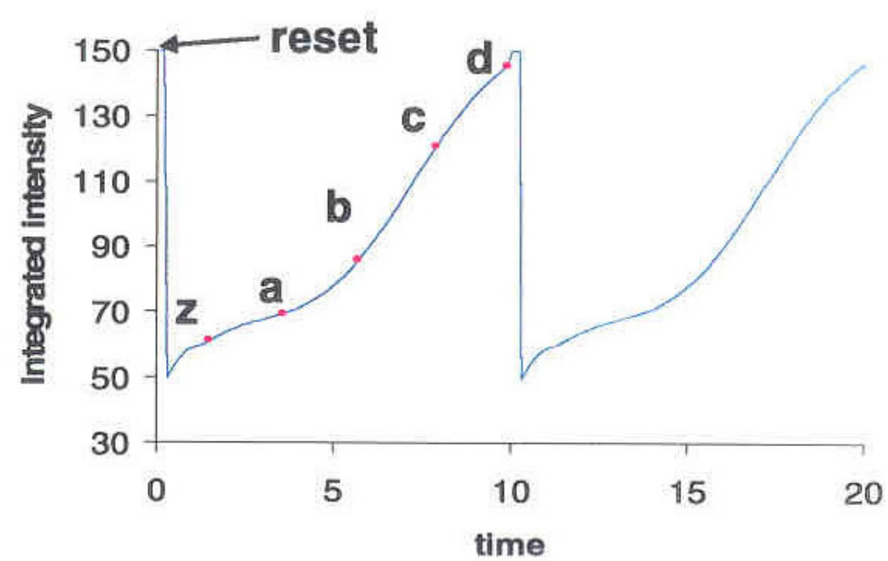
- Fringe-scanning modulation, implemented on delay line
- Sawtooth waveform to minimize number of reads per frame
- Retrace occurs during array settling time
- A, B, C, D bins computed as
  - A = a - z, etc.
- Timing varies with wavelength so that each time bin corresponds to λ/4 at the wavelength of interest
- Let X = A-C, Y = B-D, N = A+B+C+D

$$\phi = \arctan\left(\frac{Y}{X}\right)$$

$$V^2 \propto \frac{X^2 + Y^2 - \text{bias}}{N^2}$$







Forming the estimator

V^2, rather than V, to avoid the square root & associated biases

$$V^2 = \frac{\pi^2}{2} \frac{X^2 + Y^2 - \text{Bias}}{N^2}$$

typically, inadequate SNR to get good measure in one sample

- average numerator and N separately

$$\langle V^2 \rangle = \frac{\pi^2}{2} \frac{\langle X^2 + Y^2 - \text{Bias} \rangle}{\langle N \rangle^2}$$ } "Num"

Calculating SNR

- V^2 is a squared quantity of Gauss & Poiss. RVs - need 4th order stats
- typically assume all noise in numerator, N constant

special cases:

<u>Photon-noise only</u>

$$\frac{1}{\sigma_{V^2}} \propto \begin{bmatrix} \sqrt{N} & , N \gg 1 \\ N & , N \ll 1 \end{bmatrix}$$

<u>Read-noise limited</u>

$$\frac{1}{\sigma_{V^2}} \propto \left(\frac{N}{\sigma_{rn}}\right)^2 , N \ll \sigma_{rn}$$

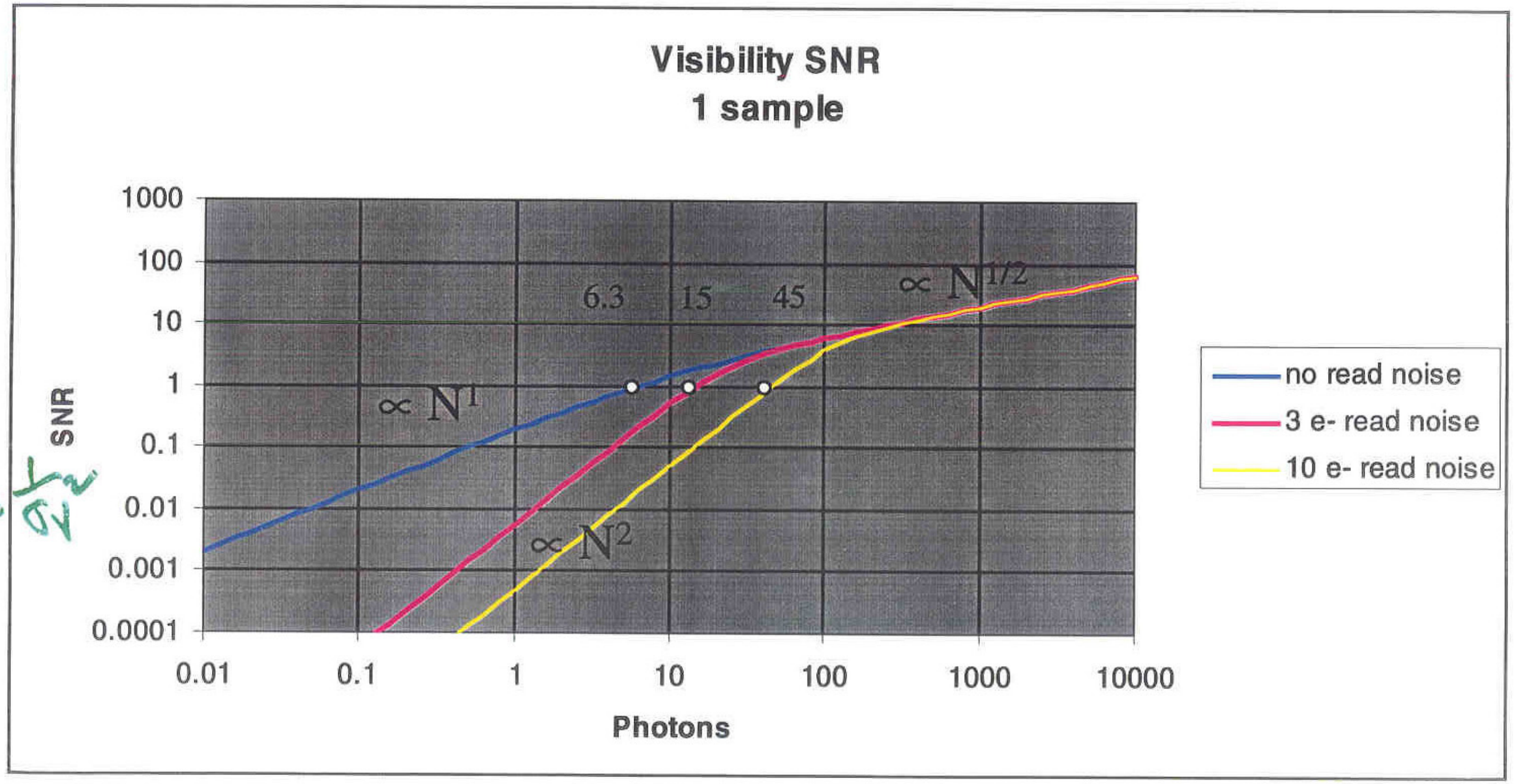
read noise

NB: when photon-starved
SNR $\neq \sqrt{N}$

in general:

$$\frac{1}{\sigma_{V^2}} \propto \left(\frac{N^4}{N^2 + aN^3V^2 + b\sigma_{rn}^4}\right)^{1/2}$$

Signal-to-noise ratio



$\equiv \frac{1}{\sigma_V^2}$

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Coherent vs. incoherent estimation

- Incoherent estimation (sum the energy)
  - Combine many independent estimates of V^2
  - At PTI, we combine 5 spectral channels for 125 sec at 50-100 samples/sec
    - » Increases final SNR by ~200
    - » Scatter on 25 sec points allow estimation of internal errors
  - Doesn't help photon starved condition

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Coherent vs. incoherent estimation

- Coherent estimation (sum the phasors)
  - Use a phase reference to measure the phasor rotation
  - Derotate the fringe phasor
  - Sum the fringe quadratures together
  - No improvement over incoherent estimator when shot noise limited
    - » Some disadvantage due to extra biases
  - Use to get out of photon starved regime

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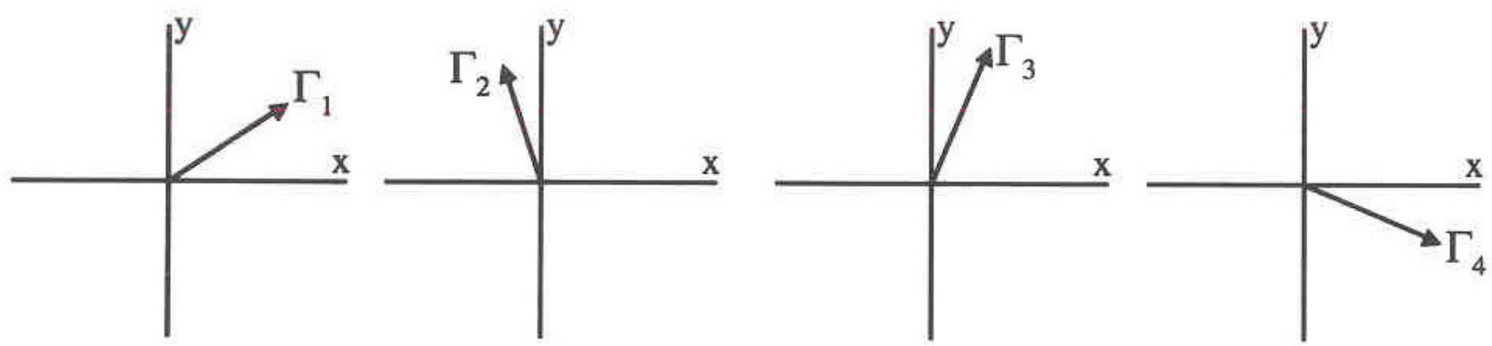
Fringe derotation and stacking (coadding)

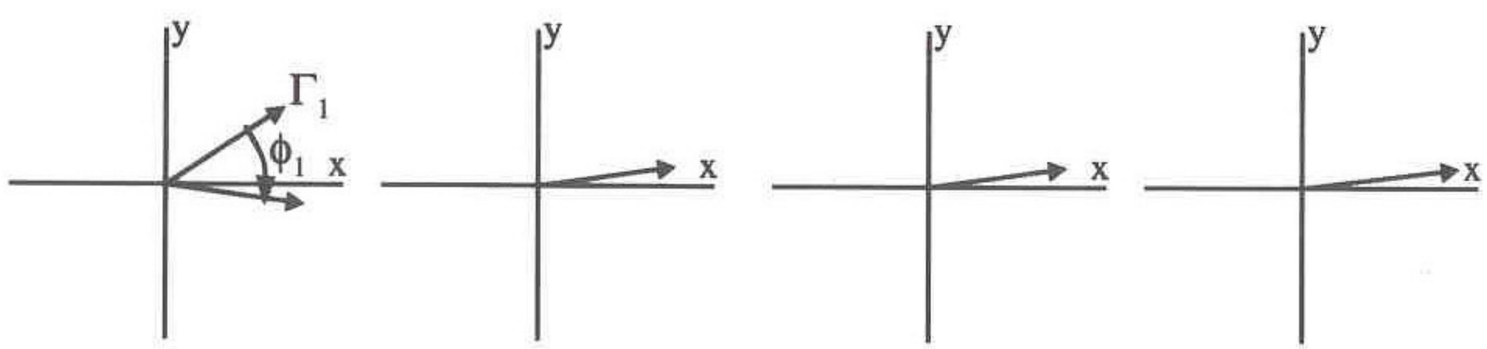
Raw phasors



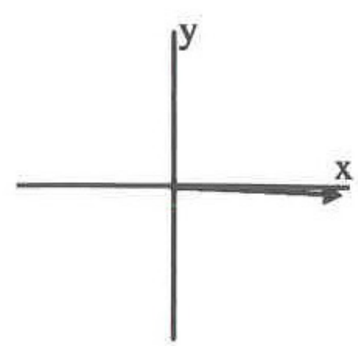
Phase reference ϕ_1 ϕ_2 ϕ_3 ϕ_4

De-rotate

(transformation matrix)



Sum (average)



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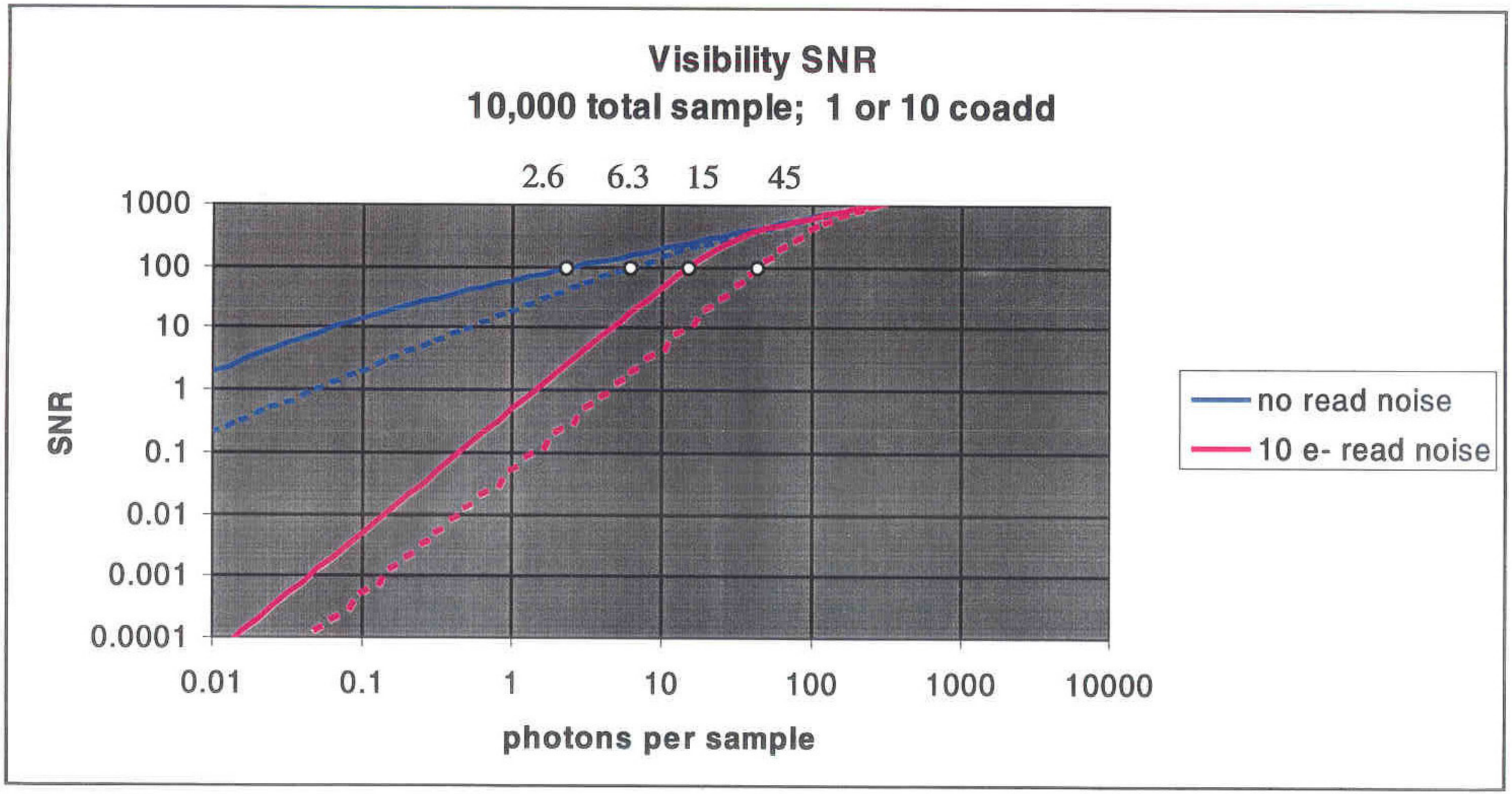
Signal-to-noise ratio with averaging and coadding



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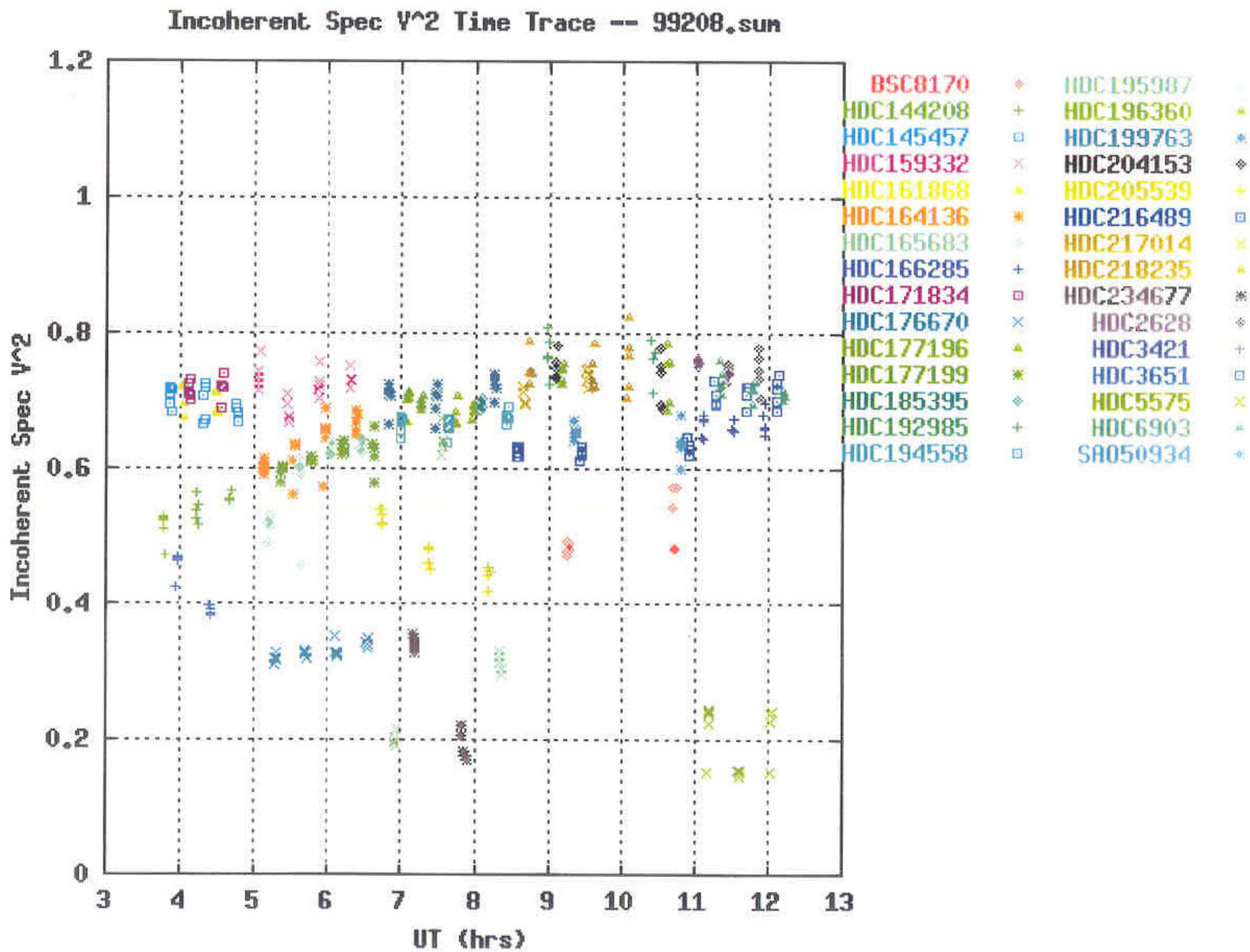
Incoherent Spec V^2 Time Trace -- 99208.sum
Incoherent Spec V^2
UT (hrs)
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0.2
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1.2
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HDC145457
HDC159332
HDC161868
HDC164136
HDC165683
HDC166285
HDC171834
HDC176670
HDC177196
HDC177199
HDC185395
HDC192985
HDC194558
HDC195987
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HDC199763
HDC204153
HDC205539
HDC216489
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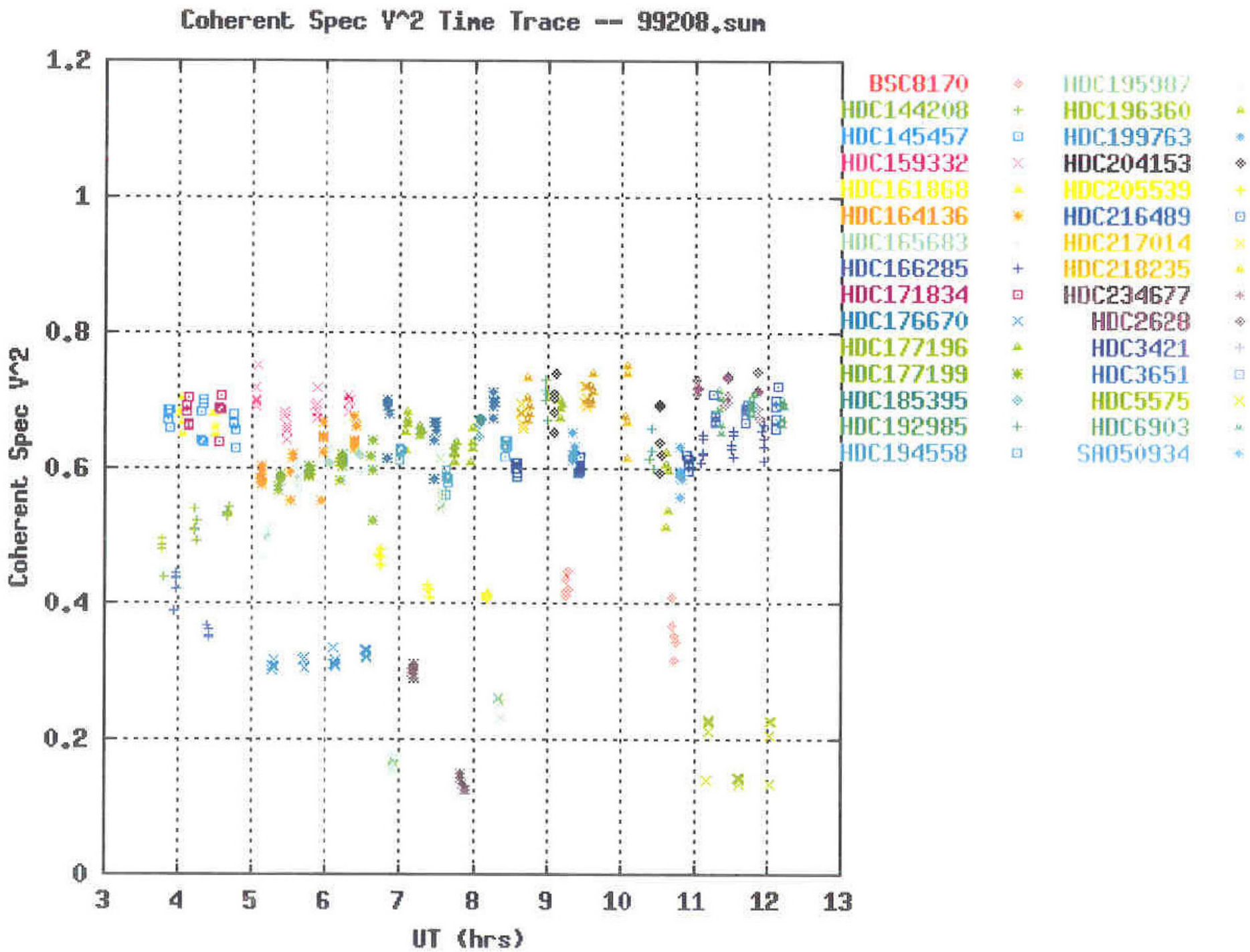
Coherent Spec V^2 Time Trace -- 99208.sum
Coherent Spec V^2
UT (hrs)
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HDC3421
HDC3651
HDC5575
HDC6903
SA050934

Estimating Bias Terms, I

- offsets

$$X \rightarrow X - B_X$$

$$Y \rightarrow Y - B_Y$$

$$N \rightarrow N - B_N$$

B_N : just dark current + background

$B_X = X(V=0)$
$B_Y = Y(V=0)$

} - should be zero
- non zero due to detector imperfections

Estimating Bias Terms, II

- Numerator biases

$$\text{NUM} \propto \langle X^2 + Y^2 - \underline{\underline{\text{bias}}} \rangle$$

- photon noise

$$\langle X^2 + Y^2 \rangle = k N$$

N: counts (adc units)

k: electronic gain (adc units/e^-)

- + Detector noise

$$\langle X^2 + Y^2 \rangle = 4 k^2 \sigma_{rn}^2$$

σ_{rn}: double correlated read noise

4: 4 reads for 4-bin algorithm

Measuring Biases

Dark Sky

$\langle X \rangle = B_X$

$\langle Y \rangle = B_Y$

$\langle N \rangle = B_N$

$\langle X^2 + Y^2 \rangle = 4k^2\sigma_{rn}^2$ (read-noise bias term)

On-source, no fringes or lamp

@ some N: $\langle X^2 + Y^2 \rangle - 4k^2\sigma_n^2 = kN$

(photon-noise bias term)

- divide by N to get gain k
- can now use k with actual observed photon rate for bias correction (PT1)
- allows averaging of read-noise bias term over multiple stars

Noise Attributable to Bias Terms

$$V^2 \propto \langle X^2 + Y^2 - kN - 4k^2\sigma_{rn}^2 \rangle$$

when read-noise limited, this term dominates the V^2 bias

- error in estimating this term is same as error in estimating V^2

$\Rightarrow$ if spend equal time measuring V^2 & bias term, SNR degrades by $\sqrt{2}$

- approaches
  - average bias over multiple star
  - coherent estimator
  - unbiased estimator

Other biases

spatial wavefront error

$$\langle V^2 \rangle \simeq \exp(-2\sigma_\phi^2)$$

$$= \exp\left(-2.06\left(\frac{d}{r_0}\right)^{5/3}\right)$$ for slow guiding

- fixed biases are easy to deal with
- these biases vary
  1) statistical fluctuations
     - atmosphere is a random process
  2) seeing non stationarity
     - structure constant isn't

post combination single-mode fiber

Write combined E field as

$$E_1(\bar{r}) + E_2(\bar{r}) = A_1 e^{j\alpha_1} G'(\bar{r}) + B_1 e^{j\beta_1} G''(\bar{r}) + \ldots$$
$$+ A_2 e^{j\alpha_2} G'(\bar{r}) + B_2 e^{j\beta_2} G''(\bar{r}) + \ldots$$

[orthog. basis functions $G'(\bar{r}) \perp G''(\bar{r})$]

optical fiber selects just this mode

contains ~ (V^2_{raw}) of light

[x mode-matching efficiency]

In the fiber, we just have the one mode

$$E_{fiber} = A_1 e^{j\alpha_1} + A_2 e^{j\alpha_2}$$

$$\left[\begin{array}{l} I_1 \equiv A_1^2 \\ I_2 \equiv A_2^2 \end{array}\right]$$

if $A_1 = A_2$, $V^2 = 1$

if $A_1 \neq A_2$, $V^2 = \dfrac{4 I_1 I_2}{(I_1 + I_2)^2}$

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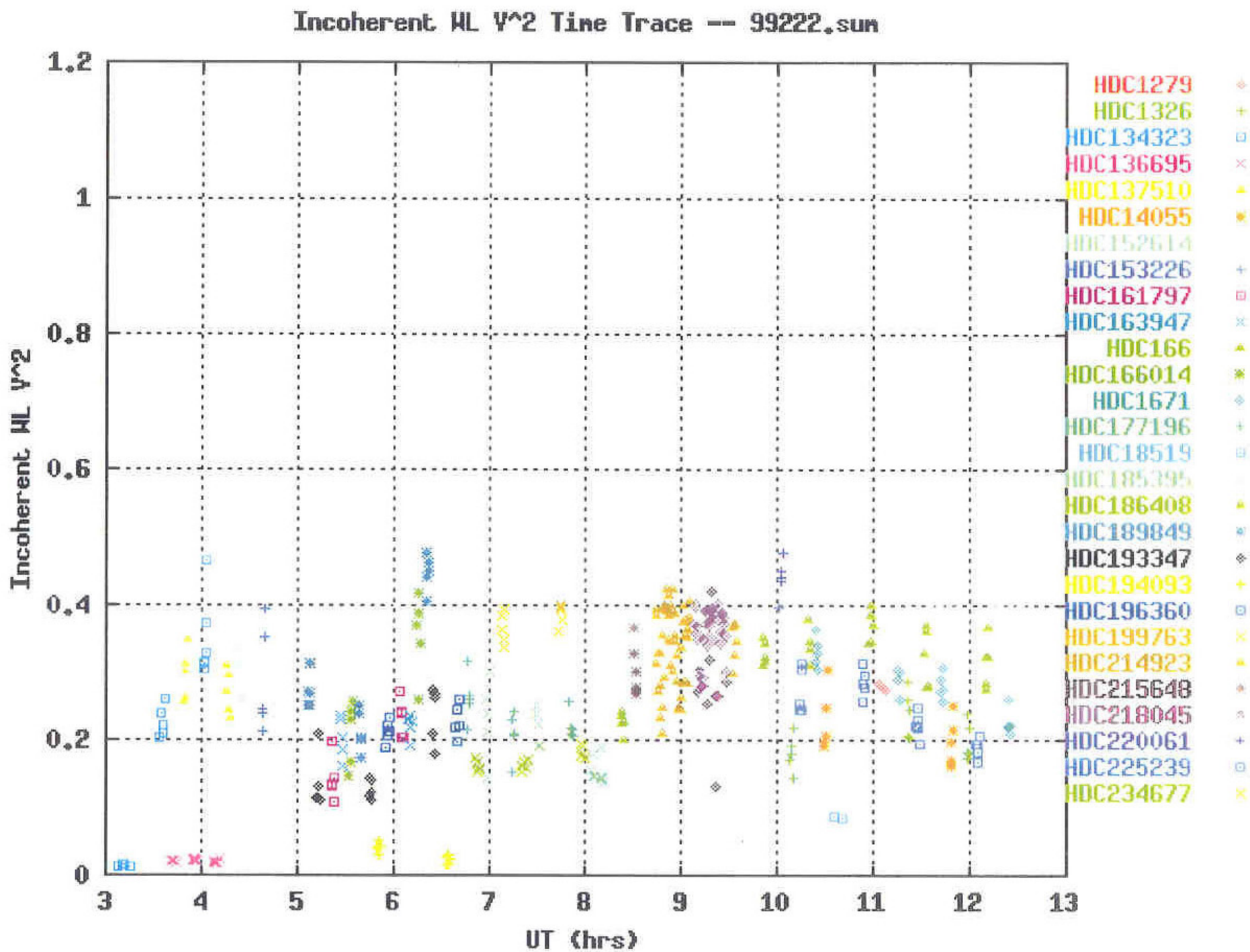
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Incoherent WL V^2
UT (hrs)
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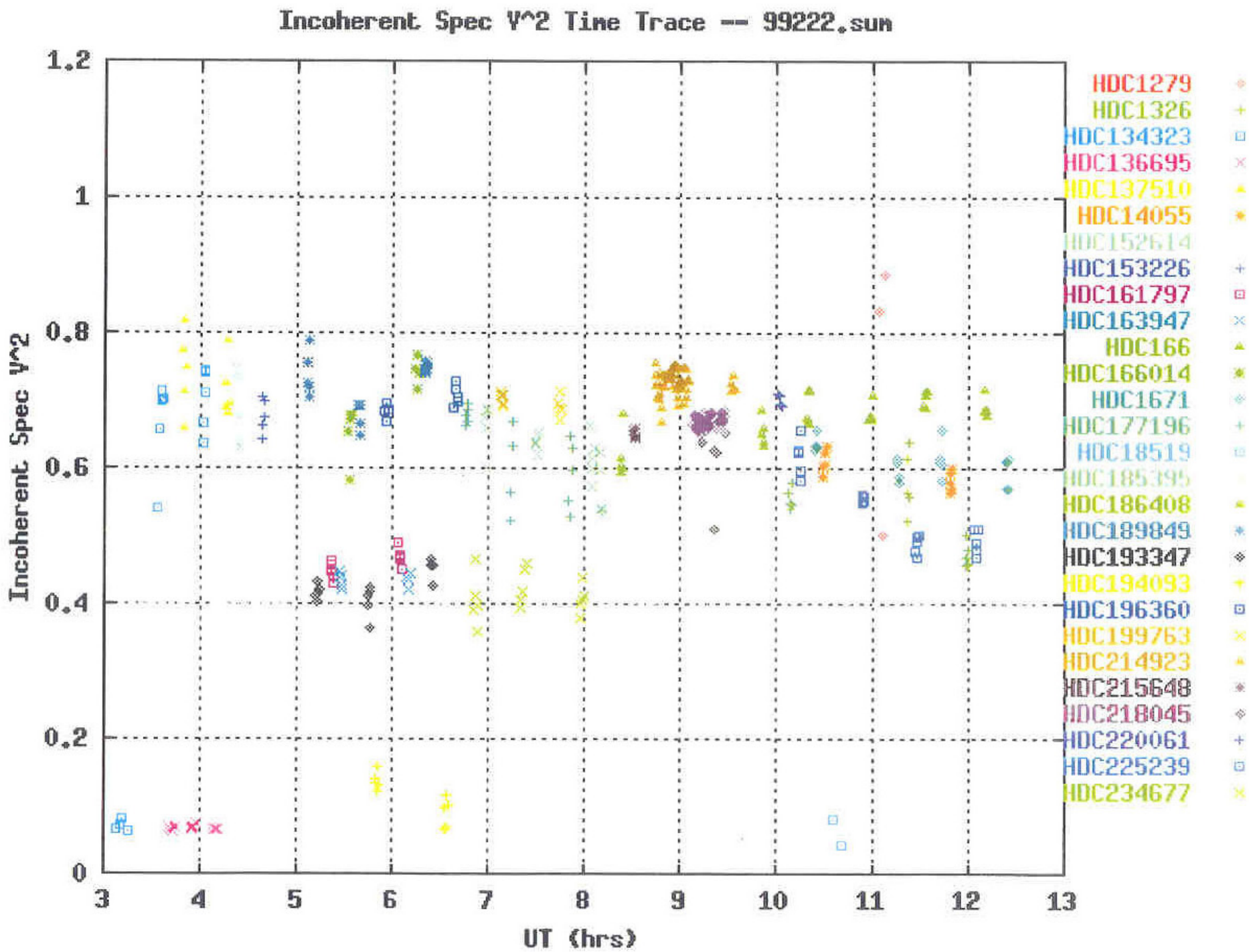
Incoherent Spec V^2 Time Trace -- 99222.sum
Incoherent Spec V^2
UT (hrs)
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HDC225239
HDC234677

Calibrating for $I_1 \neq I_2$

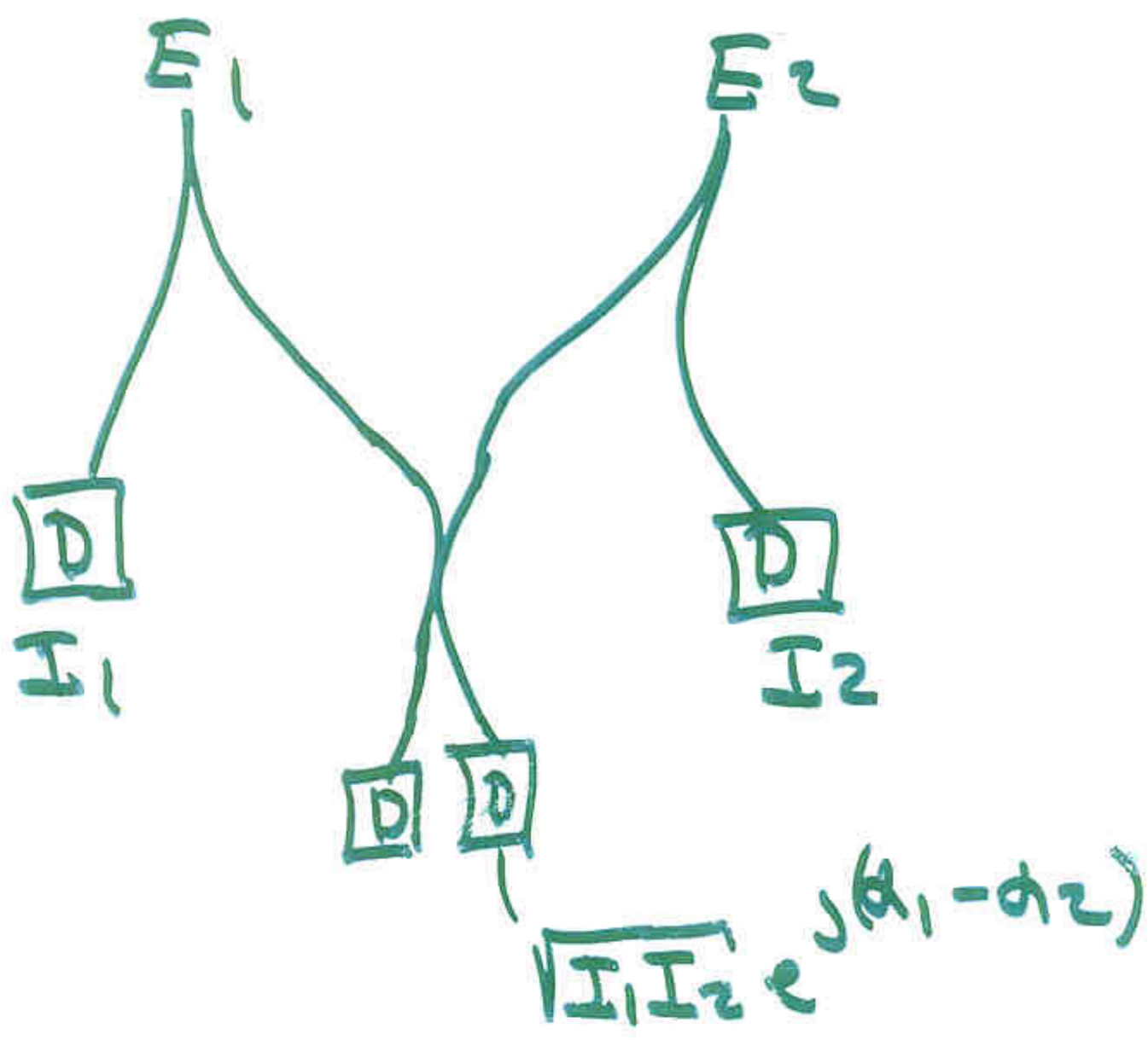
1) can show that only the average ratio $\langle I_1 \rangle / \langle I_2 \rangle$ over the integration time matters

- But it's hard to get an accurate simultaneous measurement

2) Use a nearby calibrator

- assumes integration time long enough that only systematic part of ratio remains

3) Fully single-mode combiner (FLUOR)



Temporal Biases

coherent integration time

$$\langle V^2 \rangle \propto \exp\left(-\left(\frac{T}{T_{0,2}}\right)^{5/3}\right)$$

coherence Time

2-aperture variance definition

- can estimate $T_{0,2}$ based on phase jitter

$$\sigma_{\Delta\phi}$$

- can apply correction of form

$$V^2 \rightarrow V^2 \exp\left(C_0 \sigma_{\Delta\phi}^2\right)$$

Finite envelope width

$$V^2 \propto \mathrm{sinc}^2 \frac{\pi x}{R\lambda}$$

x ← tracker offset (μm)

λ ← wavelength

R ← spectrometer resolution $\lambda/\Delta\lambda$

for accurate visibility measurement

- calibrate shape of envelope
- work in a narrow band

can incoherently average V^2 from several narrow bands to create synthetic wide-band data with long coherence length

stuff not touched on

- $\mathrm{San} \neq \lambda$

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Conclusion

- "Visibility" is the modulus of the complex visibility
- You typically measure it by converting a spatial fringe pattern to a temporal one
  - Becomes a matched-filter problem
- When photon starved, SNR behaves worse than $\sqrt{N}$
- Calibration is important
  - Background & dark current
  - Read noise and photon noise
  - Detector imperfections and offsets
  - Atmospheric and systematic effects
- Fibers are good
- Almost *everything* reduces visibility