Interferometry with

Two Telescopes

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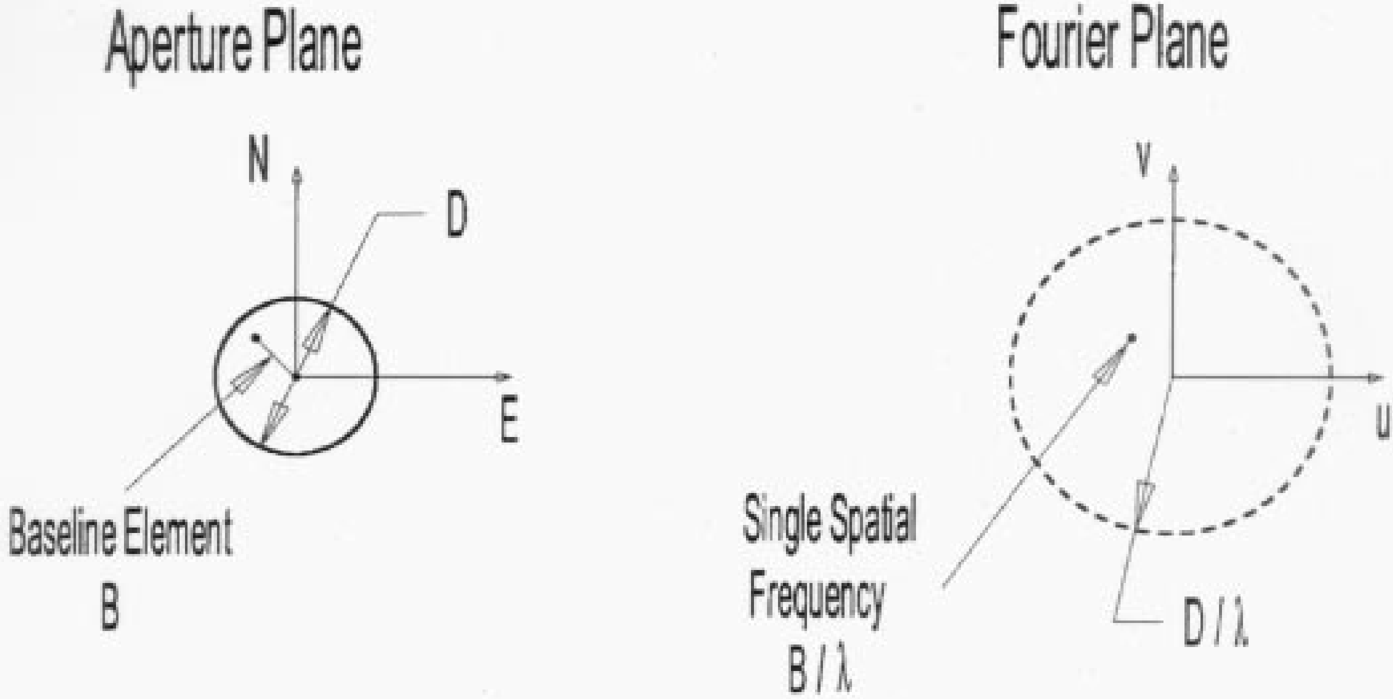
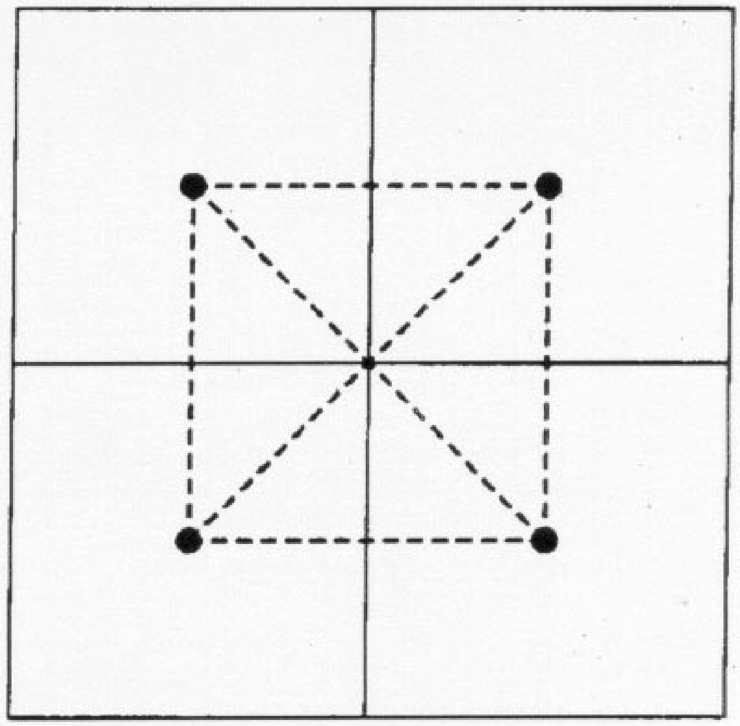
Aperture Synthesis Concepts



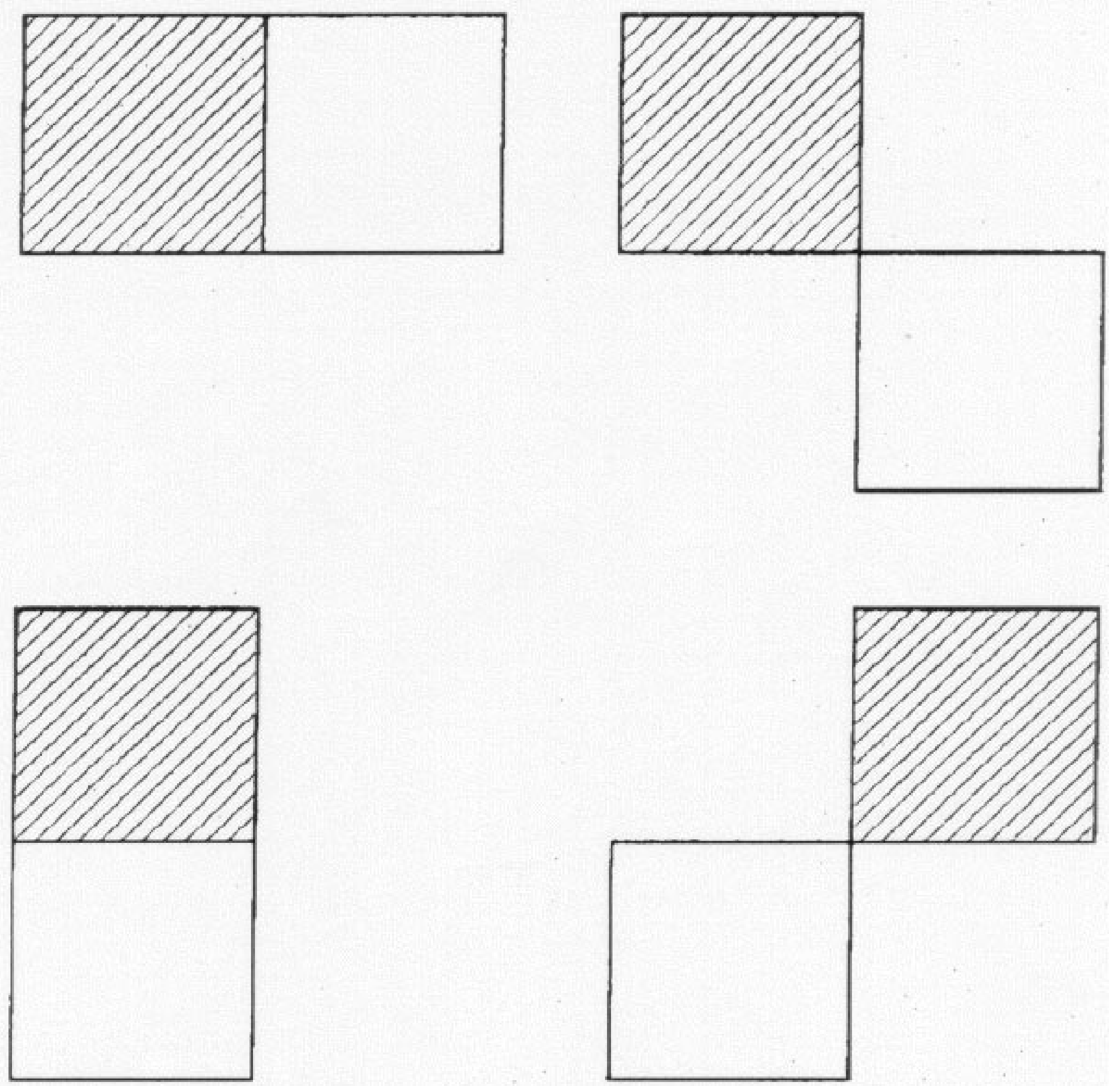
Sparse Aperture Baselines



4 apertures => 6 baselines

(2 are redundant)

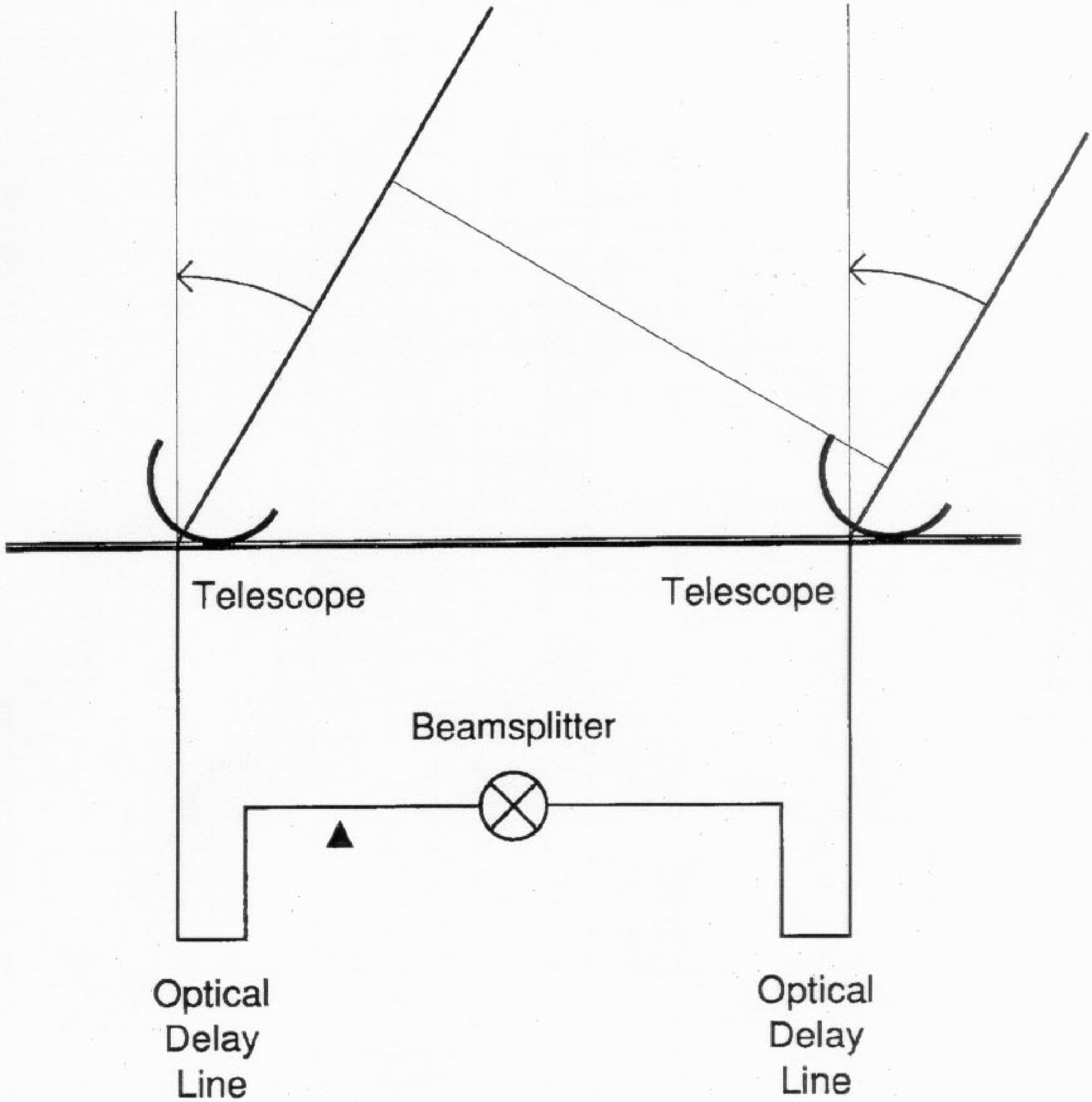
Synthesizing the 4-square Telesope by moving 2 apertures



Aperture Synthesis

- **By moving the telescopes in the array**

- **By allowing the earth to rotate and change the spacing between telescopes**


Telescope
Telescope
Beamsplitter
Optical
Delay
Line
Optical
Delay
Line

What does a two-telescope interferometer measure?

- **I(x) = source brightness in one spatial dimension, x.**

- **Interferometers measure the FOURIER TRANSFORM of I(x):**

$$\tilde{I}(s) = FT\ [I(x)]\ ,$$

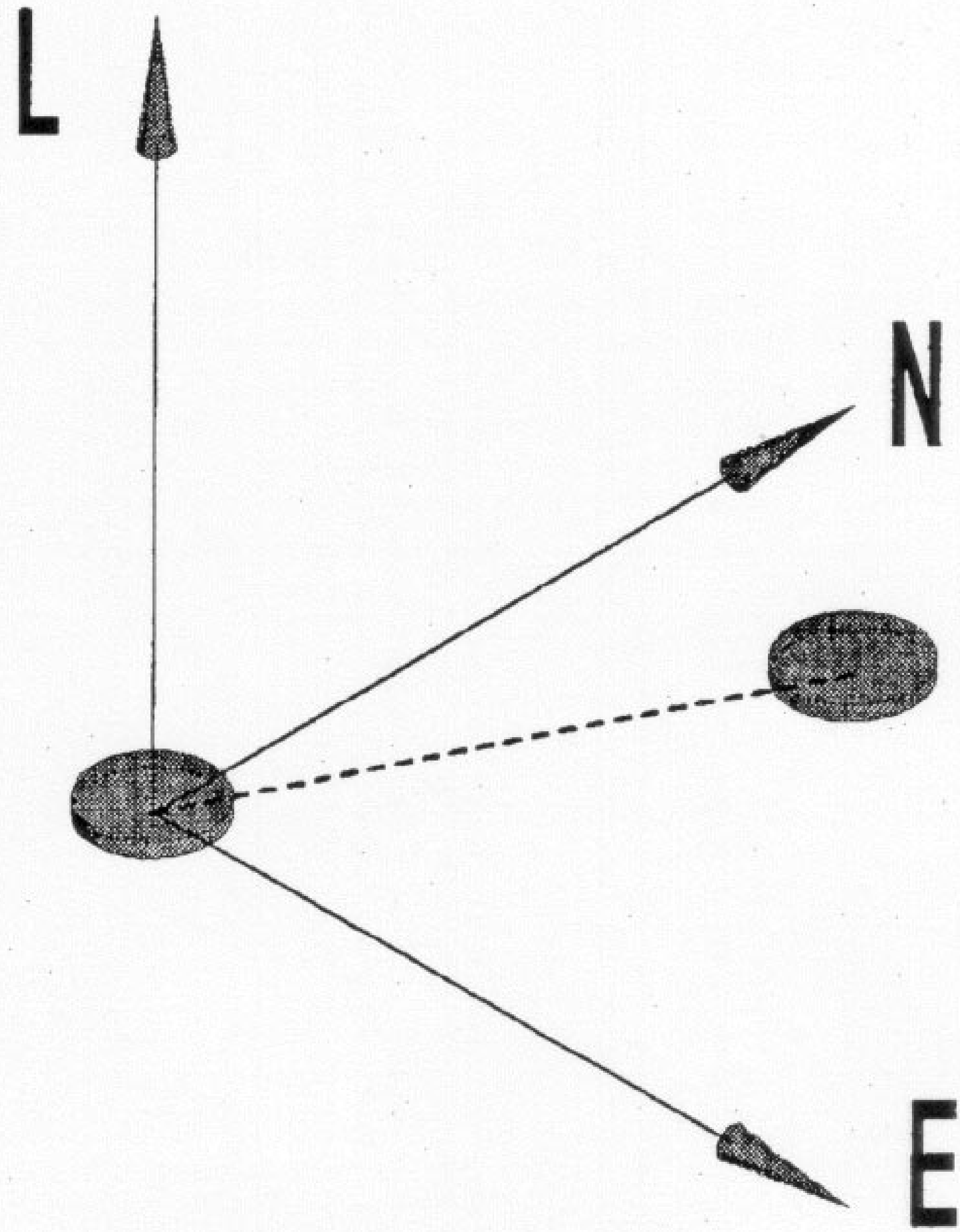
where s = B/λ = SPATIAL FREQUENCY,
B = interferometer telescope spacing and
λ = wavelength.

- **The visibility amplitude is:**

$$V(s) = |\tilde{I}(s)|$$

FOR MORE THAN TWO TELESCOPES, THE PROBLEM GENERALLY BECOMES A TWO-DIMENSIONAL ONE.

Two-telescope Coordinate System



Fundamentals

- **Adapted from Fomalont & Wright (1974)**

- **Definitions:**

b = mean latitude of the interferometer, neglecting curvature of the earth,

δ = declination of the source,

h = hour angle of the source,

B_E, B_N, B_L = east, north & elevation components of the vector describing a single baseline, and

λ = wavelength of the observation.

Fundamentals

- **U,V coordinates**

$$U = \frac{1}{206265\ \lambda}\ (B_E \cos h\ -\ B_N \sin b \sin h\ +\ B_L \cos b \sin h)$$

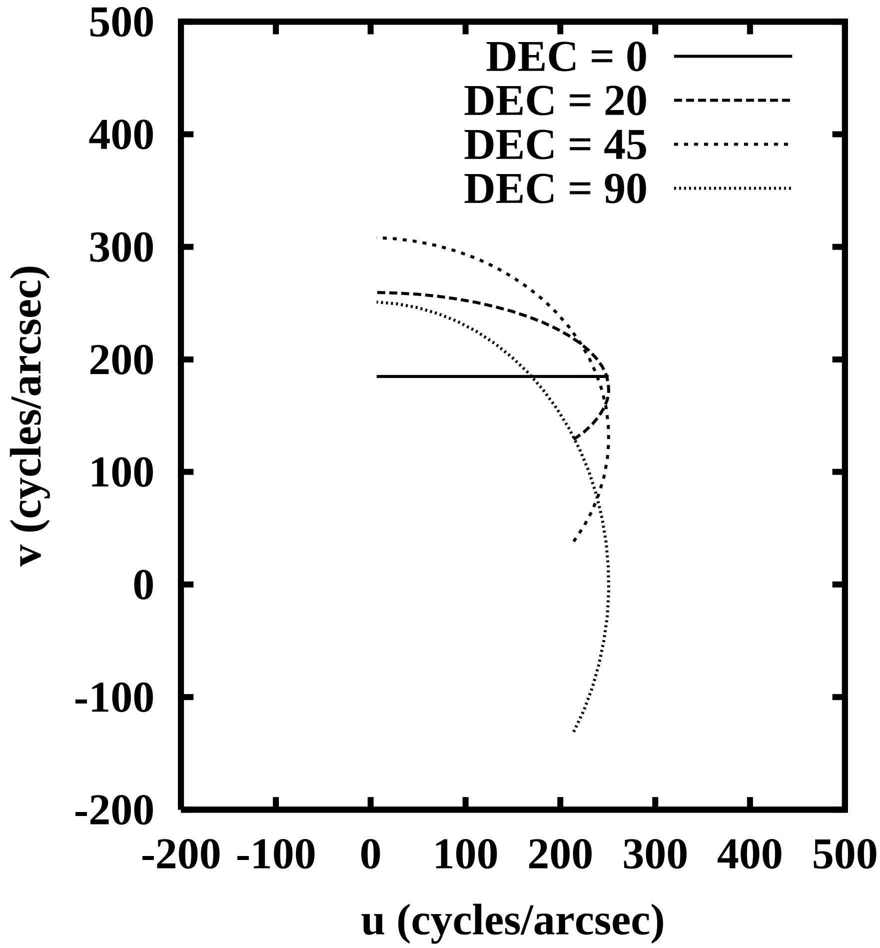
$$V = \frac{1}{206265\ \lambda}[B_E \sin\delta \sin h + B_N (\sin b \sin\delta \cos h$$
$$+\ \cos b \cos\delta)\ -\ B_L (\cos b \sin\delta \cos h\ -\ \sin b \cos\delta)]$$

- **Optical delay**

$$\Delta = \frac{1}{206265\lambda}[-\ B_E \cos\delta\, \mathrm{sinh}\ -\ B_N (\sin b \cos\delta \cos h$$
$$-\ \cos b \sin\delta) B_L (\cos b \cos\delta \cos h\ +\ \sin b \sin\delta)]$$

U,V Tracks for Two Telescopes

- $B_N = B_E = 100$ m
- $b = 33^o$
- $\lambda = 2.2\ \mu m$
- Various source declinations
- Source hour angle range: $-4 \le h \le 4$



DEC = 0
DEC = 20
DEC = 45
DEC = 90
500
400
300
200
100
0
-100
-200
v (cycles/arcsec)
-200 -100 0 100 200 300 400 500
u (cycles/arcsec)

Summary

- **U,V plane coverage minimal with two telescopes**

- **Want to observe simplest astrophysical systems to keep from being fooled**

Simple Astrophysical Systems

- **Binary star orbits**
  - ★ **Stellar masses**
- **Single star angular diameters**
  - ★ **Linear radii**
  - ★ **Effective temperatures**
- **Limb darkening**
  - ★ **Atmospheric temperature structure**
- **Circumstellar shells**
  - ★ **Mass loss phenomena**
- **Departures from circular symmetry in stars**
  - ★ **Non-radial pulsation phenomena**
  - ★ **Rapid rotation**

Two Unresolved Points

Let

B1 = brightness of source 1

B2 = brightness of source 2

r = separation between sources (arcsec).

The visibility amplitude is given by

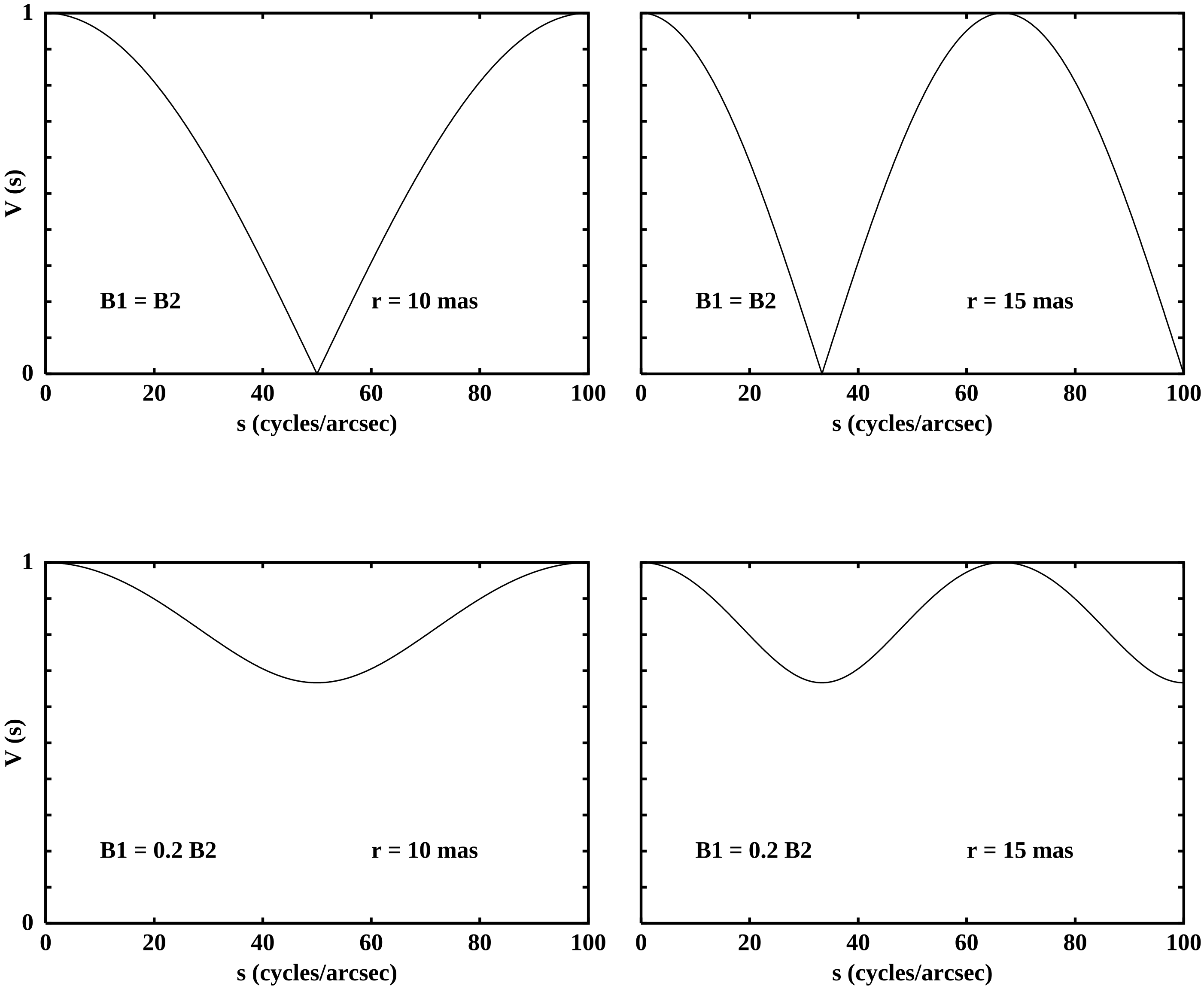
$$V(s) = \sqrt{[P_0 + (1 - P_0)\cos^2(\pi s r)]}$$

where

s = spatial frequency (cycles/arcsec)

and

$$P_0 = \frac{(B1 - B2)^2}{(B1 + B2)^2}$$



1
V (s)
0
B1 = B2
r = 10 mas
0
20
40
60
80
100
s (cycles/arcsec)
B1 = B2
r = 15 mas
0
20
40
60
80
100
s (cycles/arcsec)
1
V (s)
0
B1 = 0.2 B2
r = 10 mas
0
20
40
60
80
100
s (cycles/arcsec)
B1 = 0.2 B2
r = 15 mas
0
20
40
60
80
100
s (cycles/arcsec)

Two Unresolved Points

Let

V_{min} = first minimum in the visibility amplitude and

s_{min} = spatial frequency at which V_{min} occurs.

Then, we may derive the binary star parameters from the observations:

$$r = \frac{1}{2\ s_{min}}$$

$$\frac{B1}{B2} = \frac{1 + V_{min}}{1 - V_{min}}$$

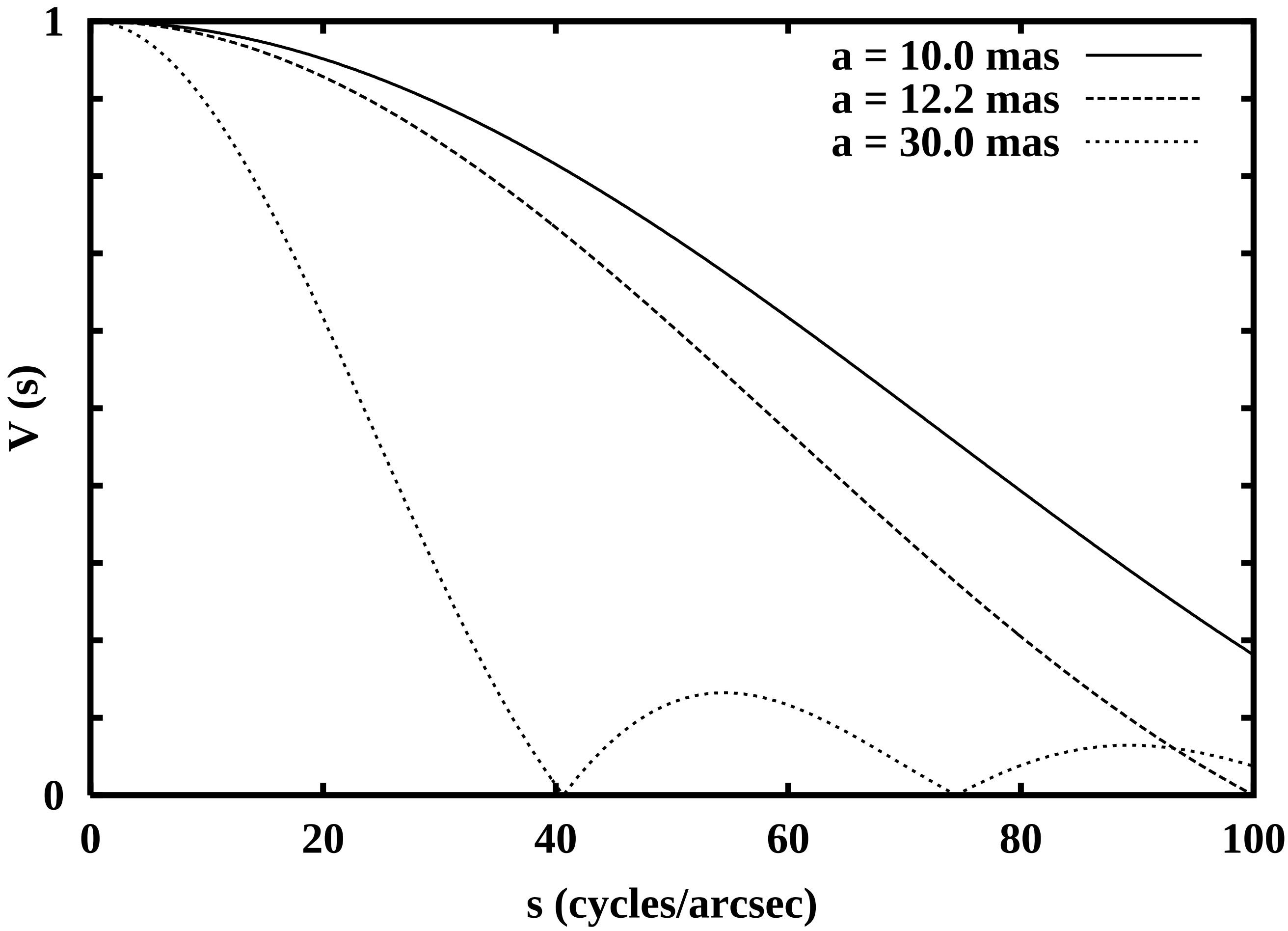
Uniformly-bright Circular Disk

Let

a = diameter of disk (arcsec).

Then, the visibility function is given by

$$V(s) = \left| \frac{2 \; J_1(\pi a s)}{\pi a s} \right| .$$


a = 10.0 mas
a = 12.2 mas
a = 30.0 mas
V (s)
s (cycles/arcsec)
1
0
0
20
40
60
80
100

Uniformly-bright Circular Disk

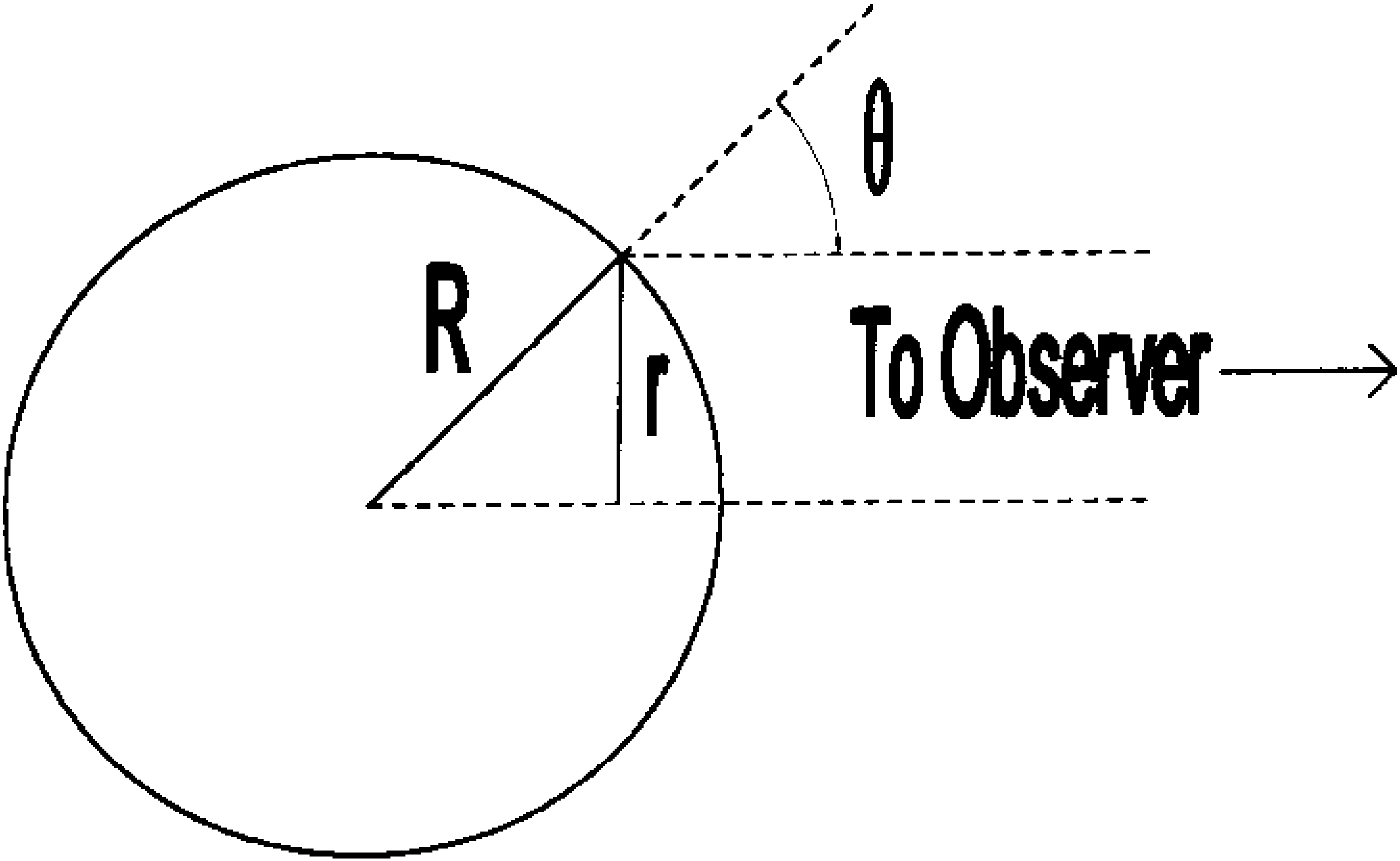
From the observed visibility amplitude we may determine the angular diameter of the source. If

s_0 = spatial frequency of the first null of V(s),

then

$$a \approx \frac{1.22}{s_0} \ .$$

Limb-darkening Geometry



Limb-Darkening Functions

SHOULD use exact model atmosphere brightness profiles to generate limb-darkening visibility curves.

CAN use convenient functional form first suggested by Michelson & Pease (1921).

Recommended by Hestroffer (1997) as a generally useful representation.

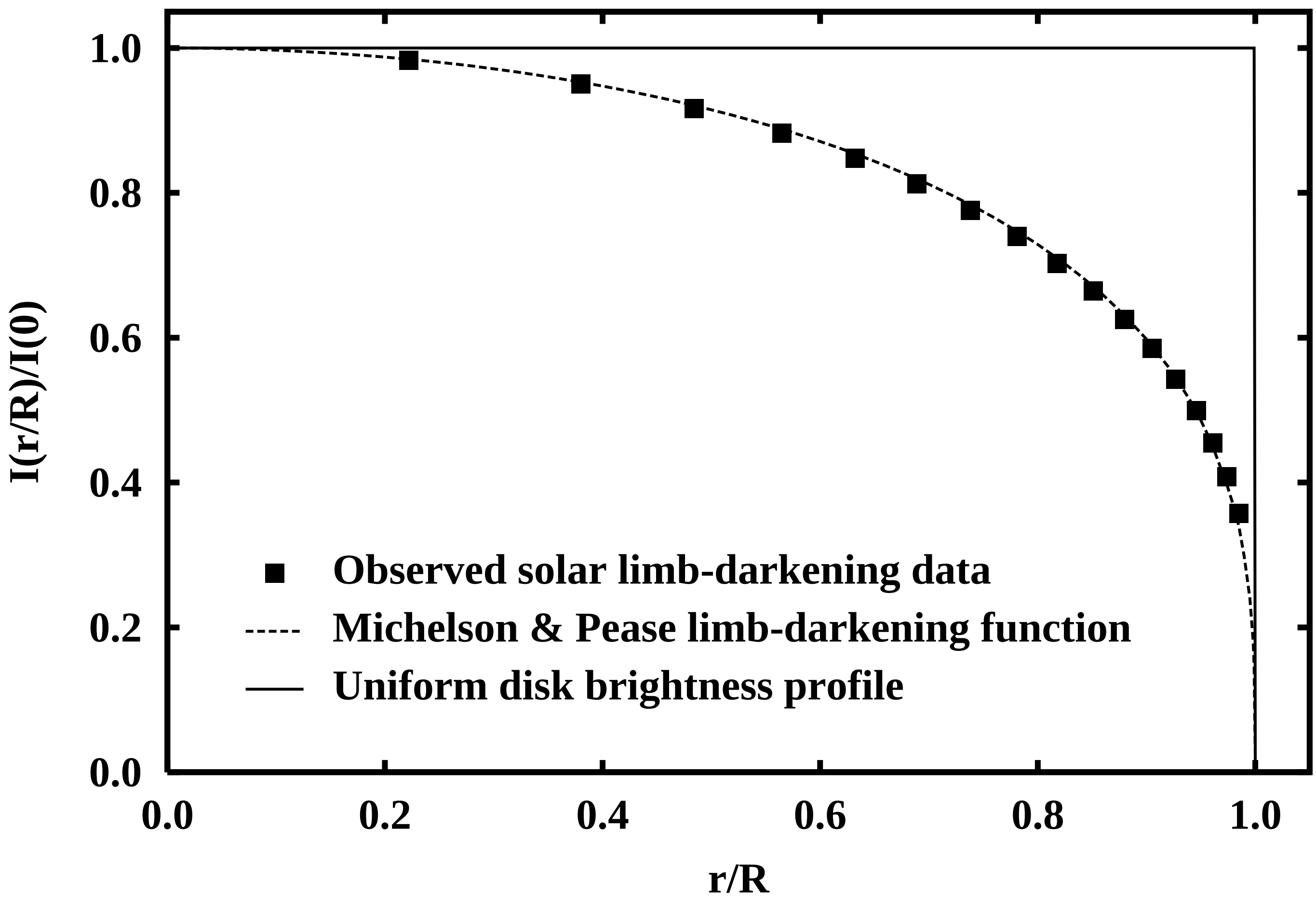
$$\frac{I(\mu)}{I(0)} = \mu^{\alpha} \ ,$$

where

$I(\mu)$ = disk brightness at μ,

$\mu = \cos\theta = [1 - (r/R)^2]^{1/2}$ and

α = an exponent best describing the model atmosphere.



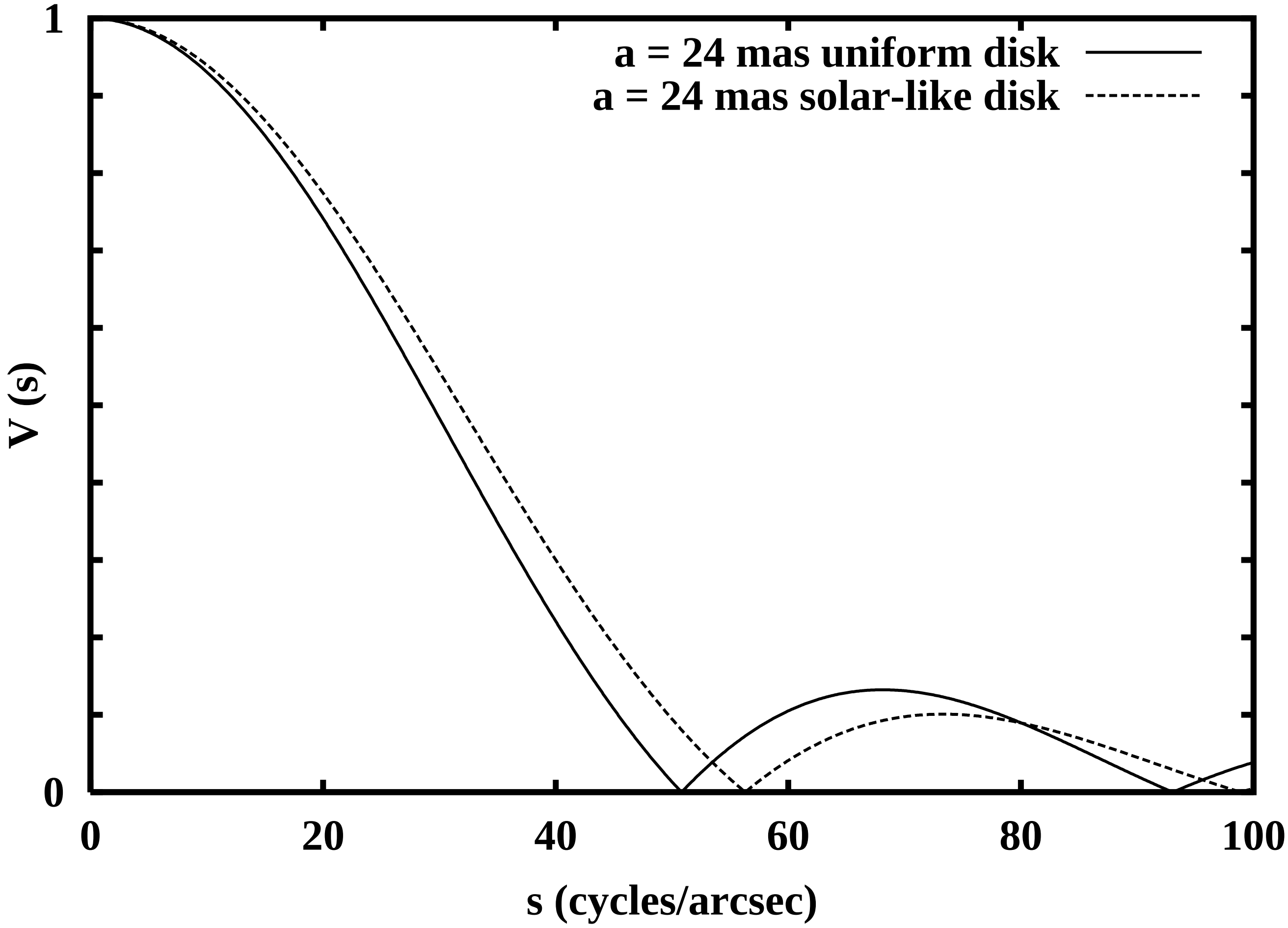
1.0
0.8
0.6
0.4
0.2
0.0
I(r/R)/I(0)
0.0
0.2
0.4
0.6
0.8
1.0
r/R
Observed solar limb-darkening data
Michelson & Pease limb-darkening function
Uniform disk brightness profile

Limb-Darkening Functions

The visibility function for the Michelson & Pease (1921) intensity profile is shown by Hestroffer (1997) to be

$$V(s) = \Gamma(\nu + 1)\frac{|J_\nu(\pi a s)|}{(\pi a s)^\nu}$$

where v = (α+2)/2.



a = 24 mas uniform disk
a = 24 mas solar-like disk
V (s)
s (cycles/arcsec)
1
0
0
20
40
60
80
100

Infinitesimally-thin Ring

Let

d = diameter of the shell.

The visibility amplitude is given by

$$V(s) = |\, J_0\,(\pi\, s\, d)\,|$$

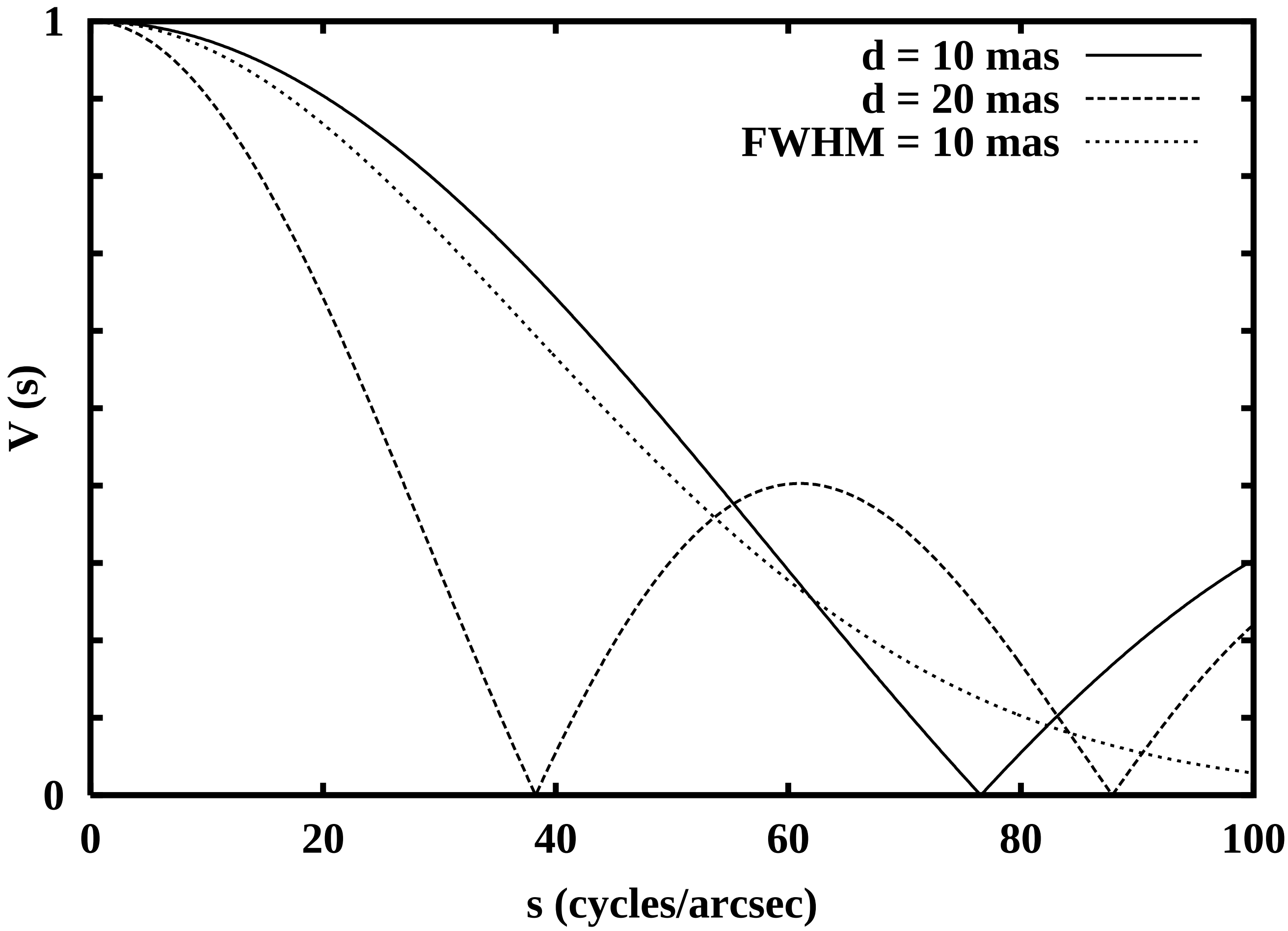
Circularly-symmetric Gaussian

If

FWHM = full width at half maximum of the distribution,

then the visibility amplitude is given by

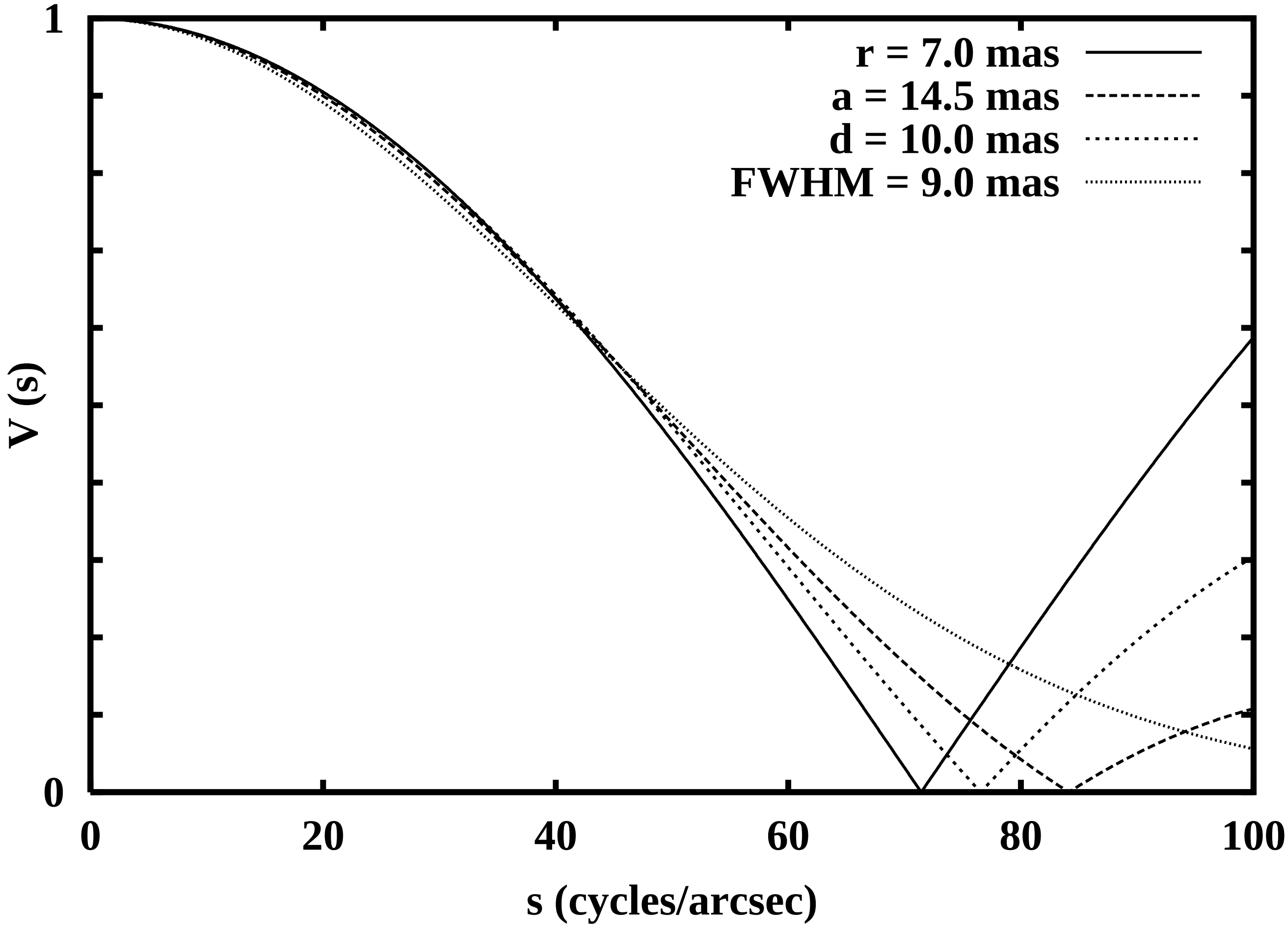
$$V(s) = e^{-\frac{2\pi^2}{8\ln 2}(\mathrm{FWHM})^2 s^2}\ .$$



d = 10 mas
d = 20 mas
FWHM = 10 mas
1
0
V (s)
0
20
40
60
80
100
s (cycles/arcsec)

Caveat

All cats are gray with limited resolution....


r = 7.0 mas
a = 14.5 mas
d = 10.0 mas
FWHM = 9.0 mas
V (s)
s (cycles/arcsec)
1
0
0
20
40
60
80
100

Uniform Disk + Unresolved Source

Let

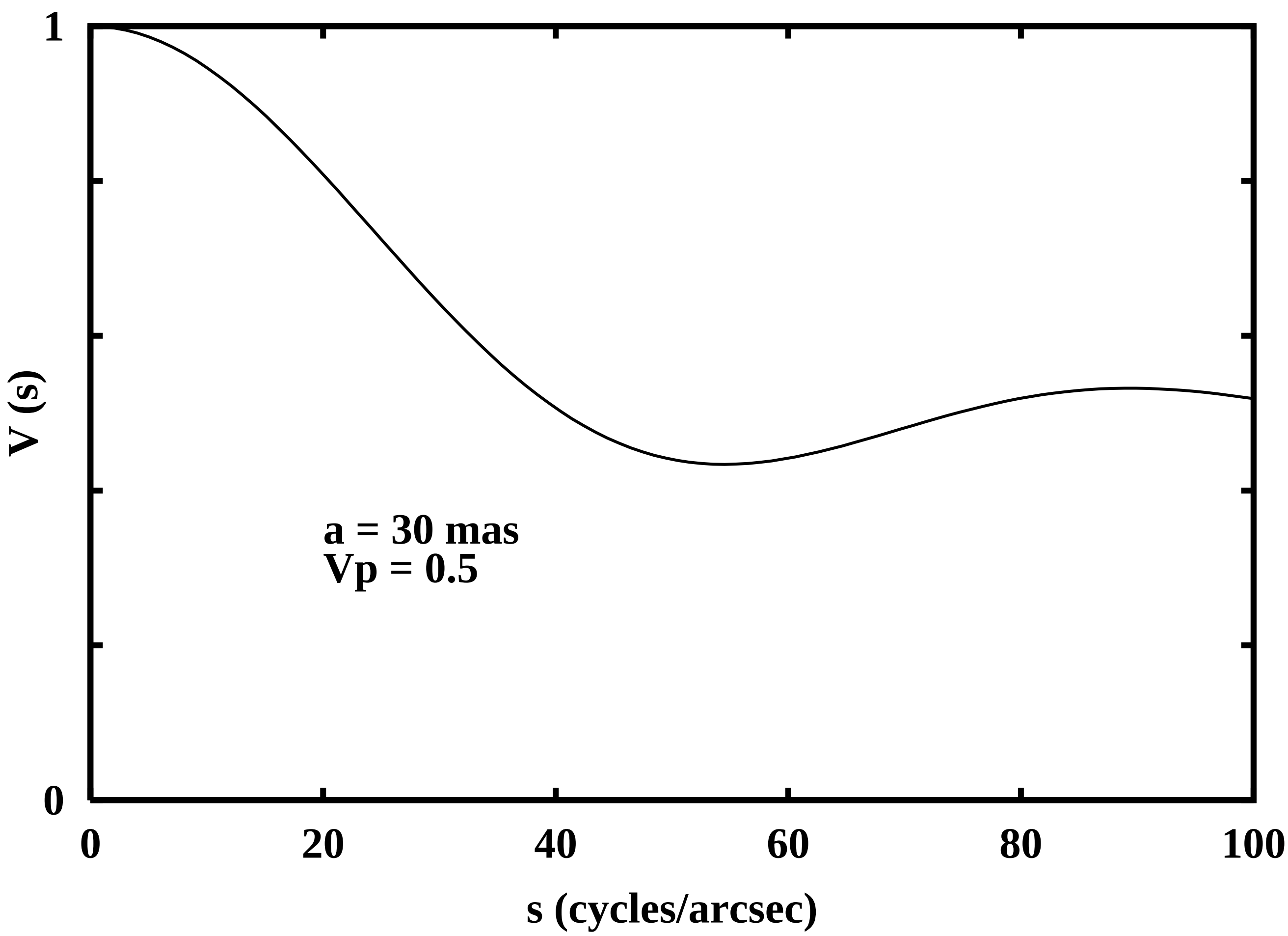
a = diameter of the uniform disk

and

$$V_P = \frac{\text{Point source radiated power}}{\text{Total radiated power}} \;.$$

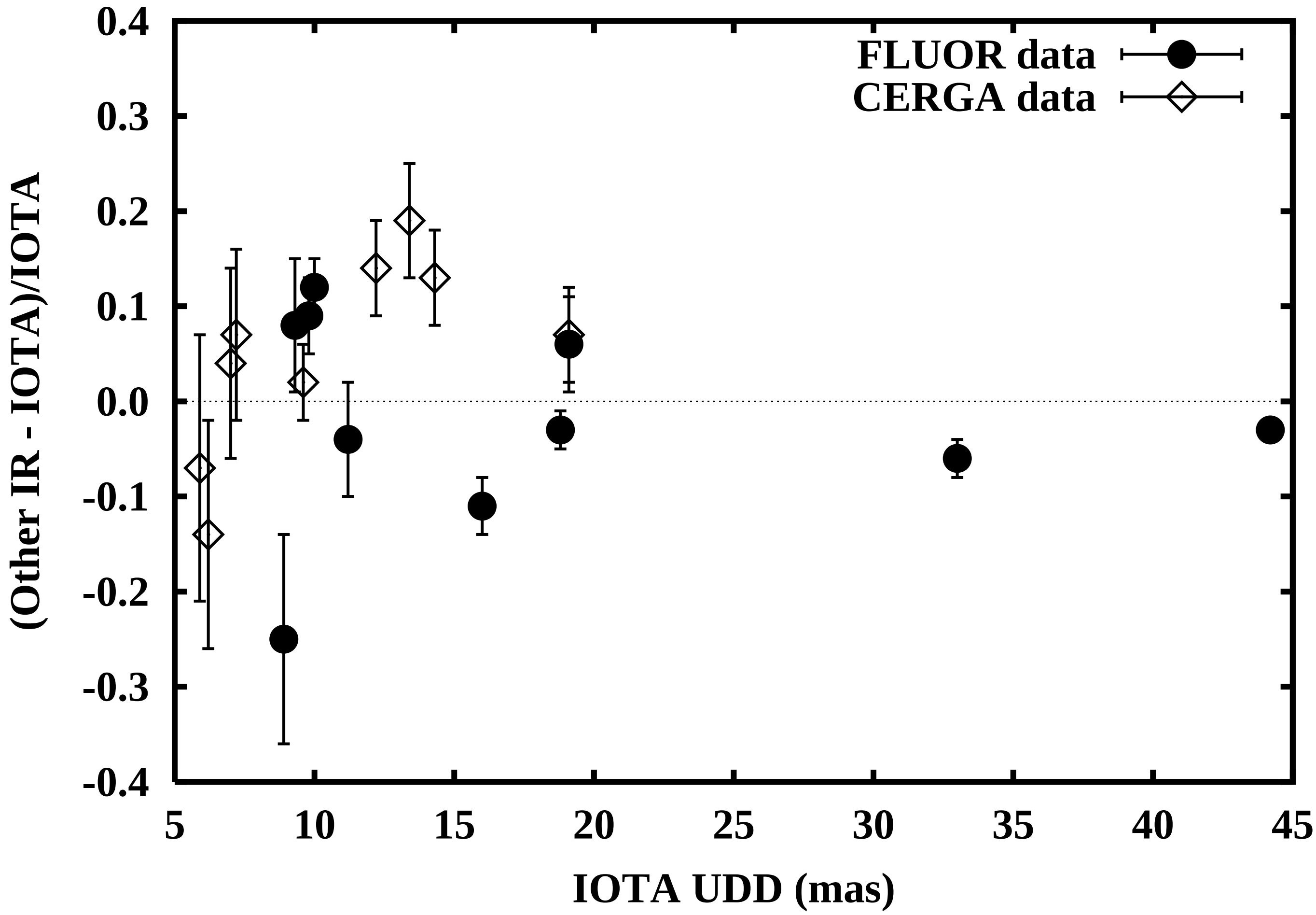
The visibility function for the combination is

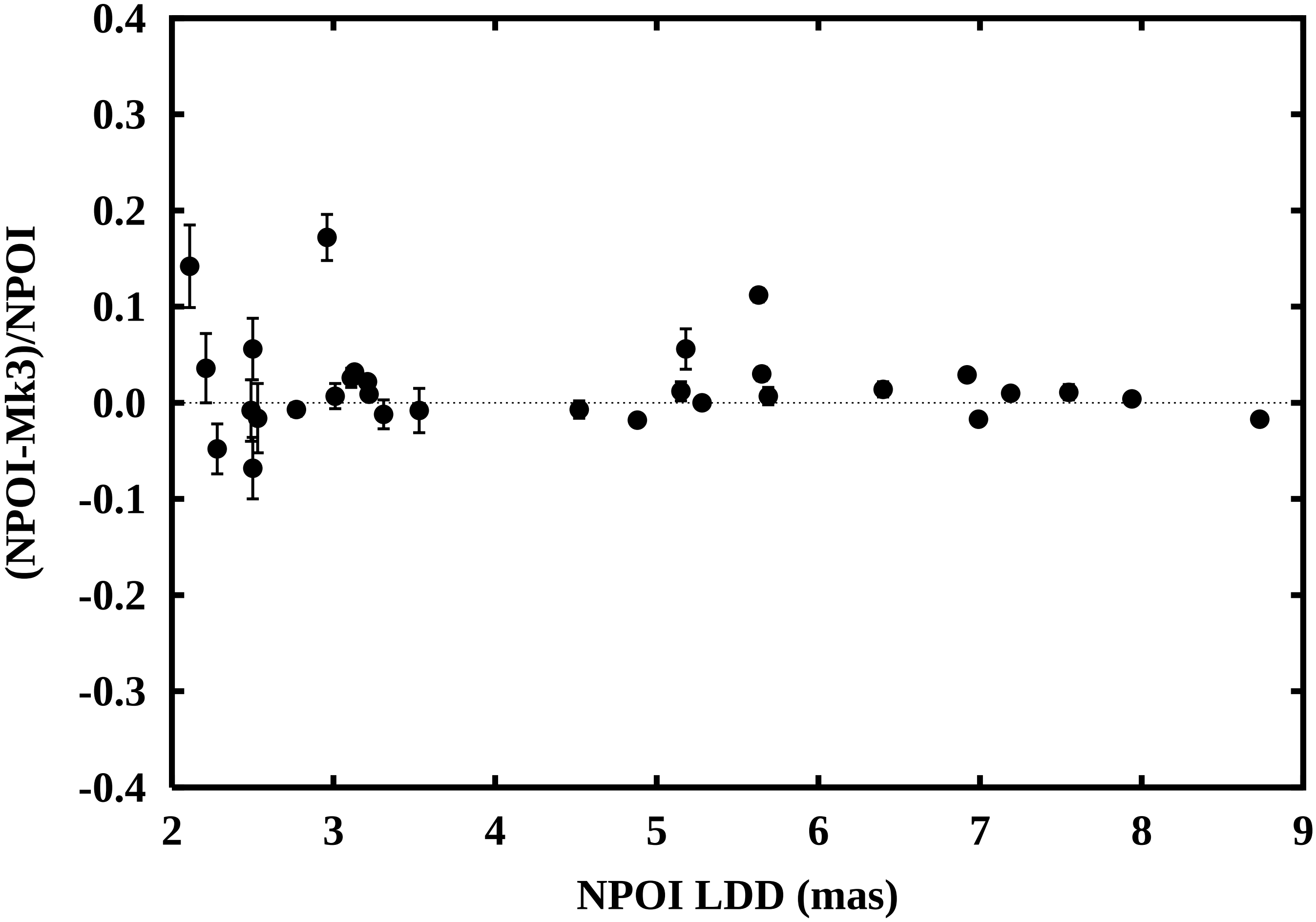
$$V(s) = V_P + (1 - V_P)\left[\frac{2\; J_1(\pi\, a\, s)}{\pi\, a\, s}\right] .$$


1
0
V (s)
a = 30 mas
Vp = 0.5
0
20
40
60
80
100
s (cycles/arcsec)

Choice of model determined by:

- **Signal-to-noise ratio of the observations**
- **Confidence in the level of systematic errors**
- **Spatial resolution of the observations**
- **Number of spacings (baselines) observed**
- **Expected level of complexity of the source**


FLUOR data
CERGA data
(Other IR - IOTA)/IOTA
IOTA UDD (mas)
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
5
10
15
20
25
30
35
40
45


0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
(NPOI-Mk3)/NPOI
2
3
4
5
6
7
8
9
NPOI LDD (mas)

Intercomparing Interferometers

- **IOTA versus other infrared**

$$\frac{\text{Other IR} - \text{IOTA}}{\text{IOTA}} = 0.017 \pm 0.107$$

- **NPOI versus the MarkIII**

$$\frac{\text{NPOI} - \text{MarkIII}}{\text{NPOI}} = 0.018 \pm 0.052$$

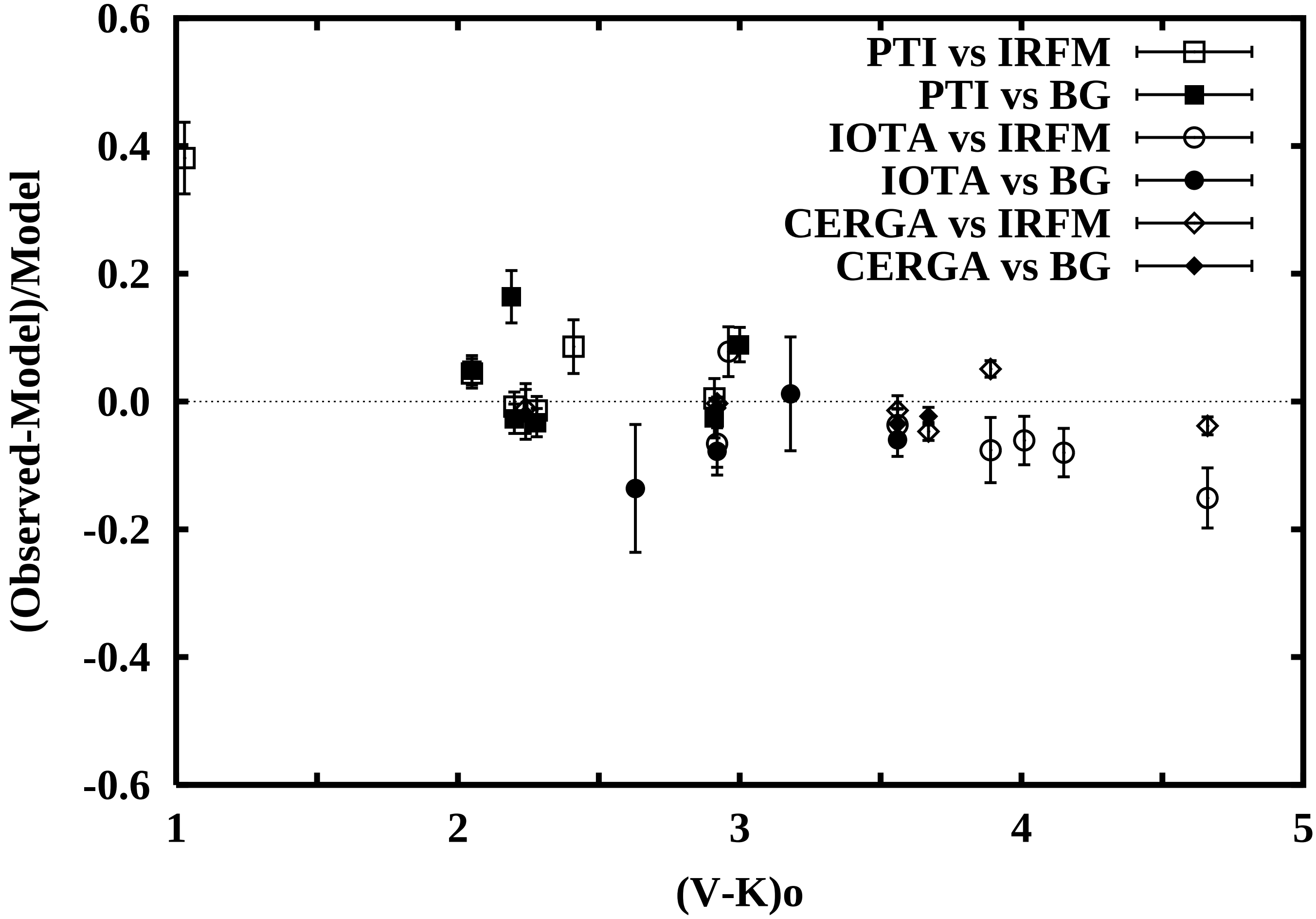
Predicting Angular Diameters

- **The angular diameter may be predicted from total flux and a model atmosphere**

- **The IRFM and others have done this successfully**

- **May use TABULATED estimates of angular diameter**

- **May also use the BEST FIT relation to the IRFM model data (Blackwell, D. E. & Lynas-Gray, A. E. 1994), valid in the range 4000K - 8500K**



PTI vs IRFM
PTI vs BG
IOTA vs IRFM
IOTA vs BG
CERGA vs IRFM
CERGA vs BG
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
1
2
3
4
5
(Observed-Model)/Model
(V-K)o

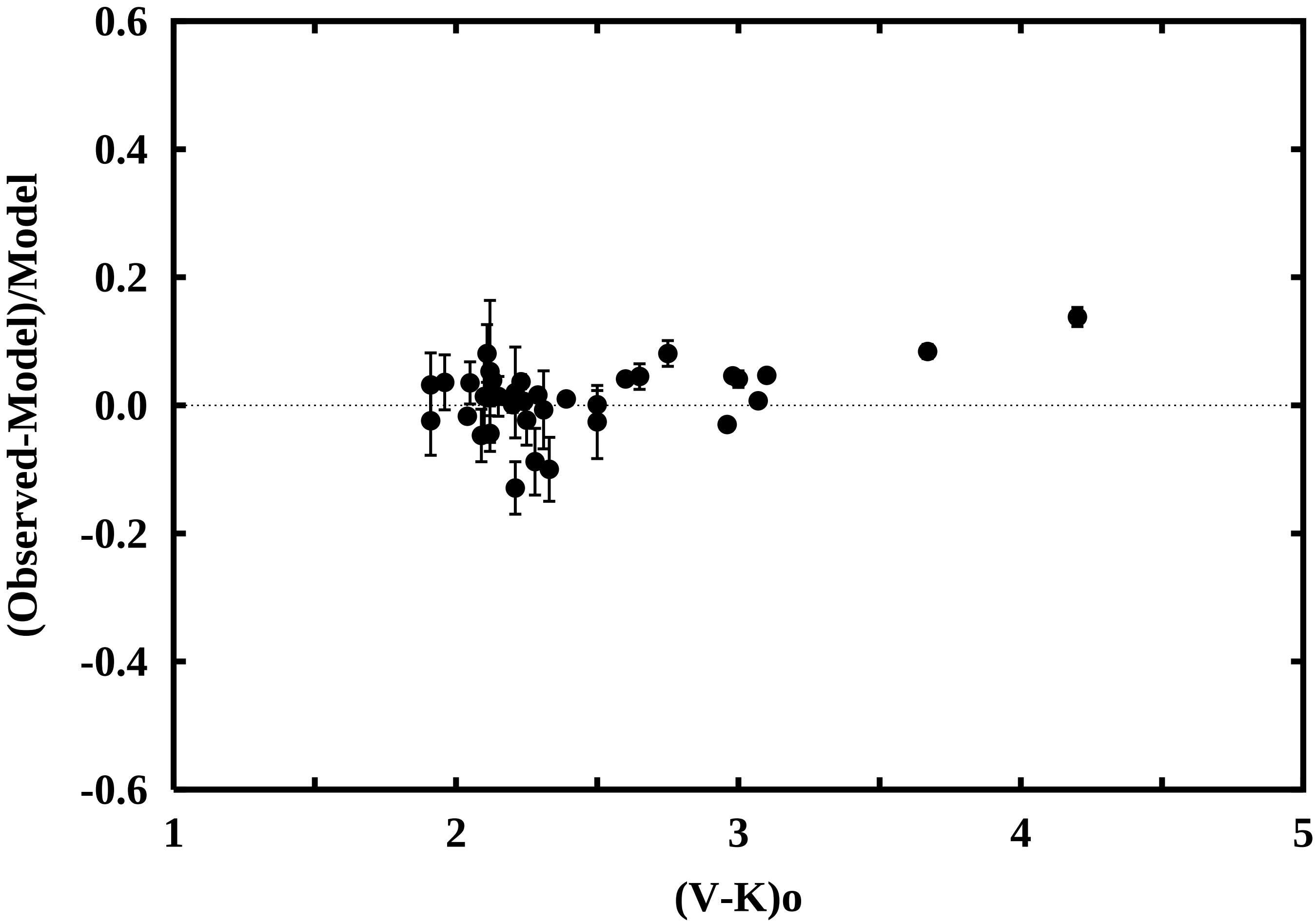
Near-IR Interferometer Diameters

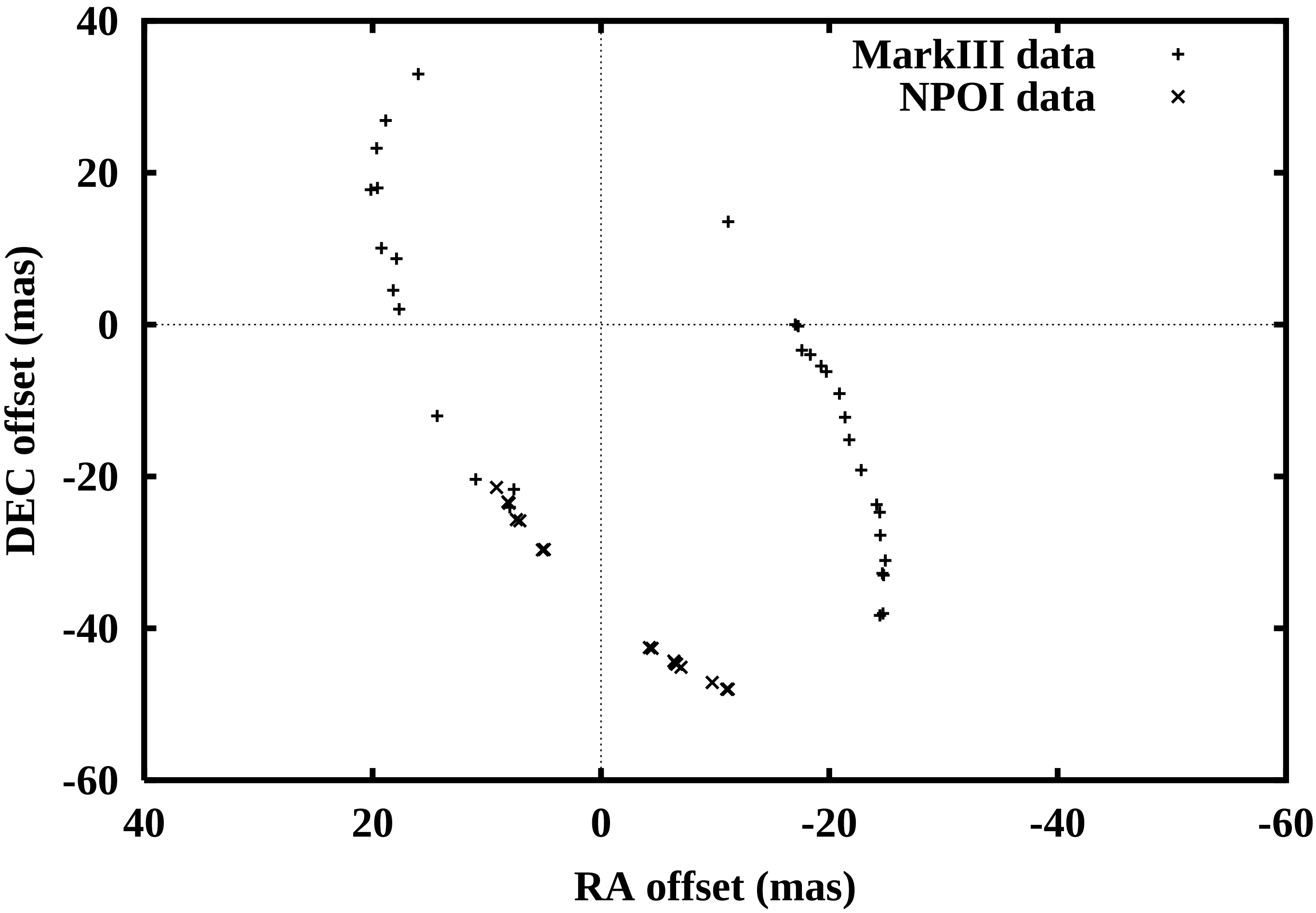
- **Comparing to the IRFM TABULATED values (excluding the 41 Cyg data point):**

$$\frac{\mathrm{IR} - \mathrm{IRFM}}{\mathrm{IRFM}} = -0.019 \pm 0.061$$

- **Comparing to the Bell & Gustafsson (1989) TABULATED values:**

$$\frac{\mathrm{IR} - \mathrm{BG}}{\mathrm{BG}} = -0.009 \pm 0.071$$


0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
(Observed-Model)/Model
1
2
3
4
5
(V-K)o


MarkIII data
NPOI data
DEC offset (mas)
RA offset (mas)
40
20
0
-20
-40
-60
40
20
0
-20
-40
-60

Results from the NPOI

- **Comparing diameters to the IRFM BEST FIT:**

$$\frac{\text{NPOI - IRFM}}{\text{IRFM}} = 0.011 \pm 0.053$$

- **Comparing Mizar observations to fitted orbit:**

$$\mathbf{(O\text{-}C)_{rms} \approx 60\ \mu as}$$

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- **Limb darkening models**

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Michelson, A. A. & Pease, F. G. 1921, ApJ, 53, 249

- **Measuring binary stars**

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