An Introduction to Closure Phases

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Interferometry Summer School
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Outline of Tutorial

A. Introduction
B. Closure Phases and Amplitudes
C. What to do with Φ_{CP}?
D. Mapping with Φ_{CP} i.e. self-calibration
E. New Developments

DETECTION OF ELECTRIC FIELD AT EACH TELESCOPE INTRODUCES A COMPLEX GAIN $\tilde{G}$:

$$\tilde{E}_i^{\text{MEASURED}} = \tilde{G}_i \cdot \tilde{E}_i^{\text{TRUE}}$$

$$= |G_i| e^{i \Delta\phi_i} \tilde{E}_i^{\text{TRUE}}$$

THIS AFFECTS MEASUREMENT OF THE COMPLEX VISIBILITY $\tilde{V}$:

$$\tilde{V}_{ij} \propto \tilde{E}_i \cdot \tilde{E}_j^* \quad \text{HENCE}$$

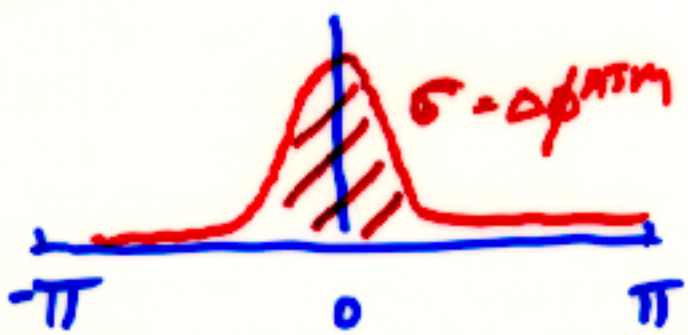
$$\tilde{V}_{ij}^{\text{MEASURED}} = \tilde{G}_i \tilde{G}_j^* \tilde{V}_{ij}^{\text{TRUE}}$$

$$= |G_i||G_j| e^{i(\phi_i - \phi_j)} \tilde{V}_{ij}^{\text{TRUE}}$$

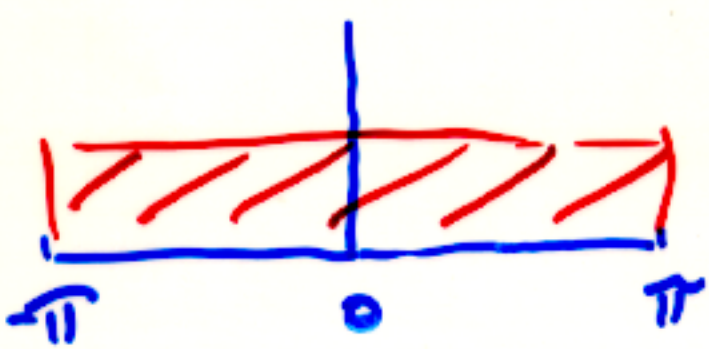
WHY NOT JUST AVERAGE THE SEPARATE PHASES?

$$\phi^{measured} = \phi^{true} + \Delta\phi^{ATM}$$

<u>CASE 1</u>: $\Delta\phi^{ATM} < \pi$



<u>CASE 2</u>: $\Delta\phi^{ATM} > \pi$ <u>THEN</u>



NO PHASE INFORMATION AT ALL

Fourier Phases are crucial for image reconstructions of all but the simplest sources. What to do?

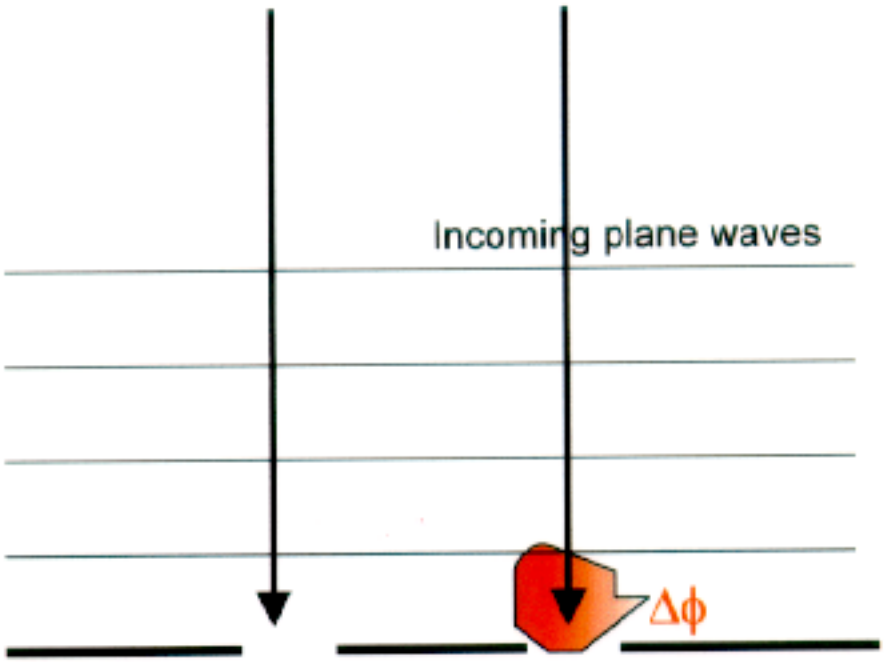
Phase Referencing

- **nearby sources**
- **measuring $\Delta\phi_{atm}$**
- **spectral lines**

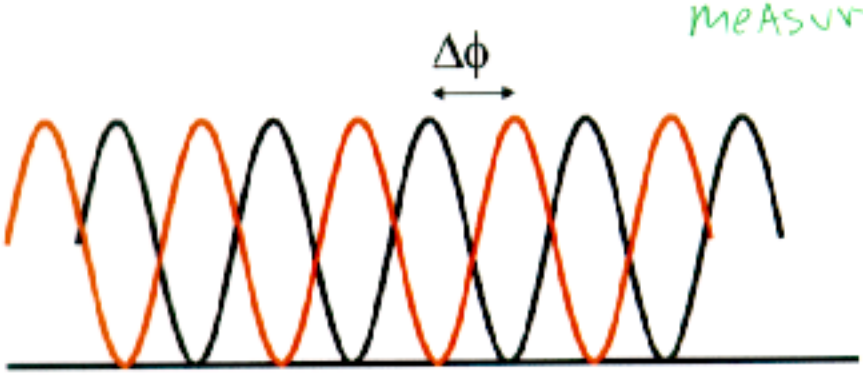
or

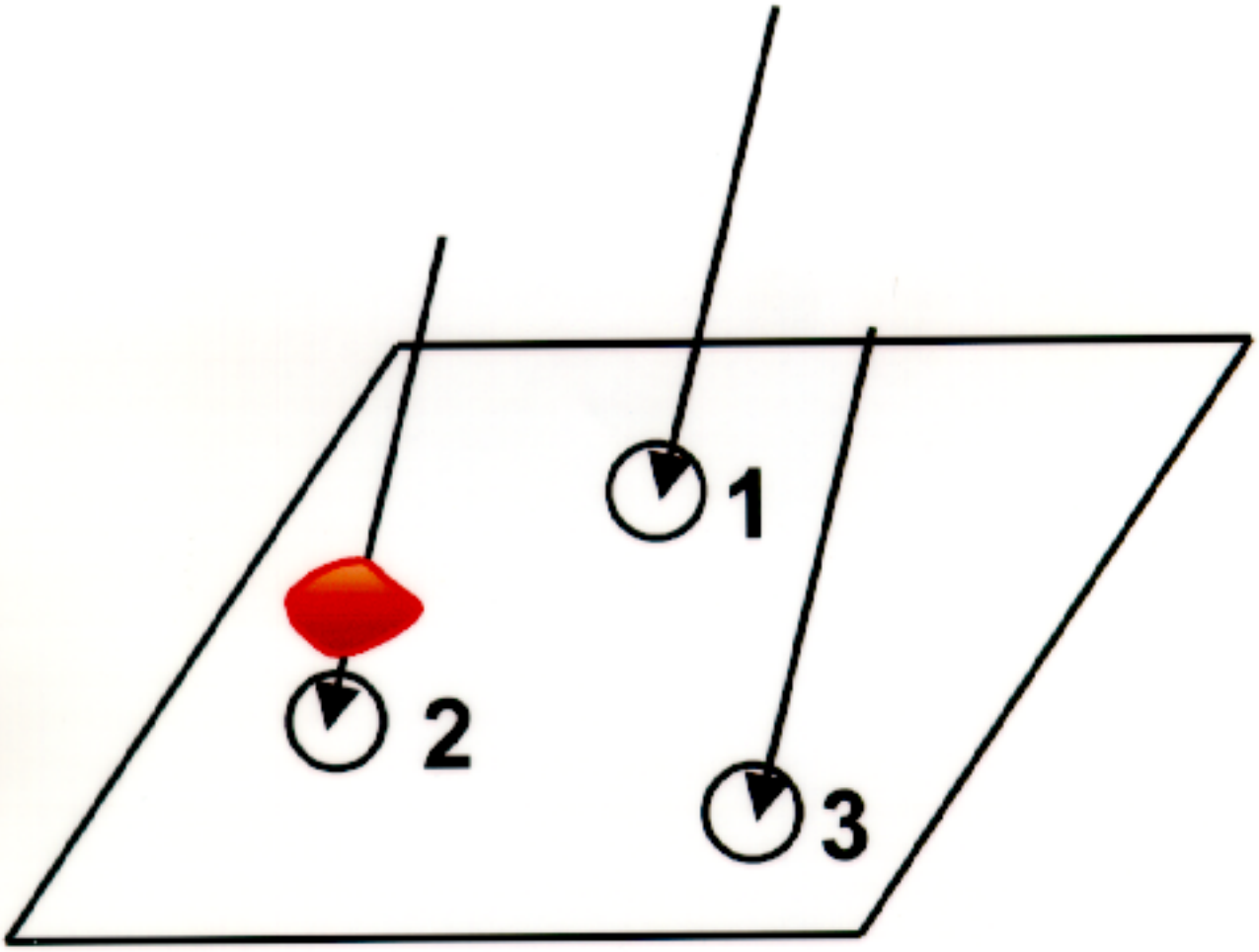
Closure Phases

Point source
at infinity



$$\tilde{V}_{measured} = \tilde{V}_0 \cdot e^{i\Delta\phi}$$

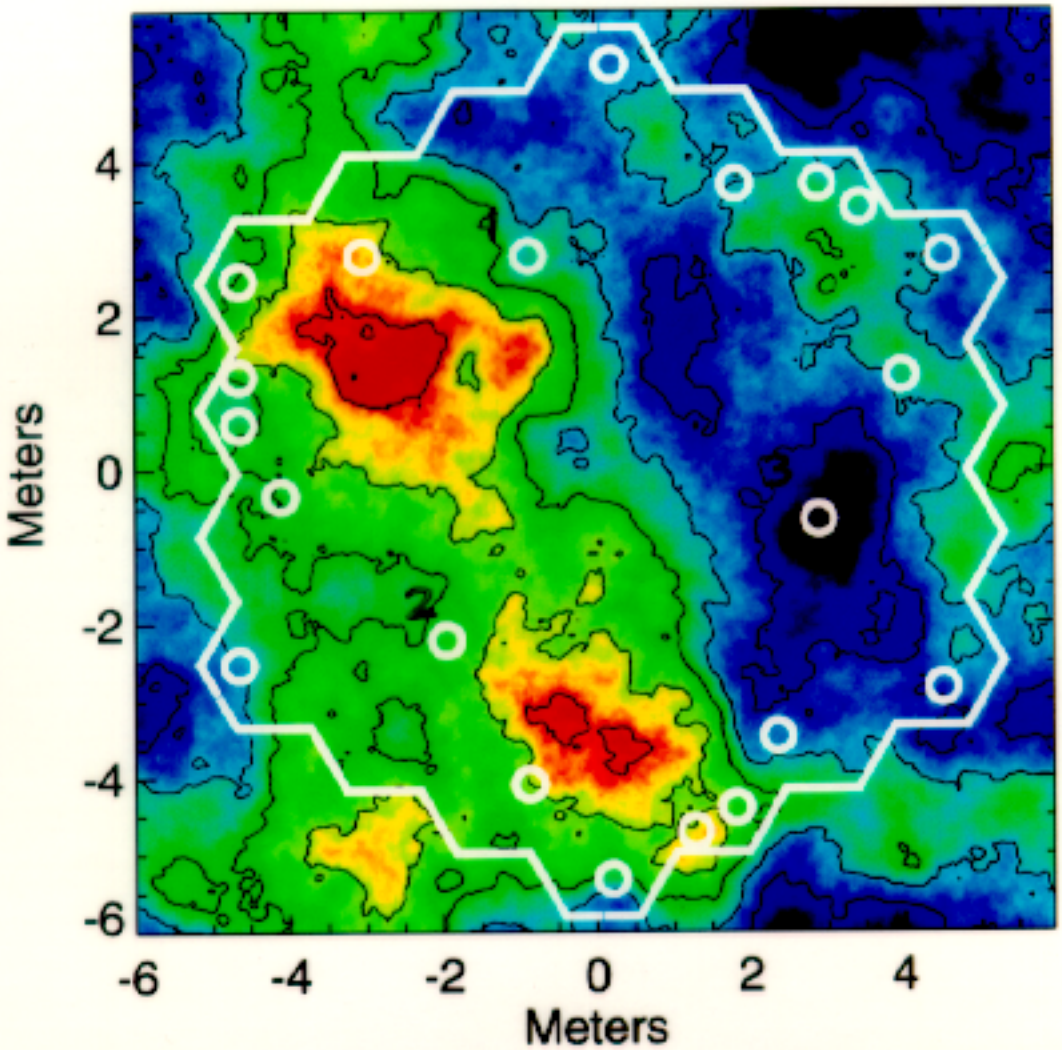




Observed		Intrinsic		Atmosphere
$\Phi(1\text{-}2)$	=	$\Phi_o(1\text{-}2)$	+	$[\phi(2)-\phi(1)]$
$\Phi(2\text{-}3)$	=	$\Phi_o(2\text{-}3)$	+	$[\phi(3)-\phi(2)]$
$\Phi(3\text{-}1)$	=	$\Phi_o(3\text{-}1)$	+	$[\phi(1)-\phi(3)]$

$$\text{Closure Phase } (1\text{-}2\text{-}3) = \Phi_o(1\text{-}2)+\Phi_o(2\text{-}3)+\Phi_o(3\text{-}1)$$

Phasescreen



Observed Intrinsic Atmosphere

$$\Phi(1\text{-}2) = \Phi_o(1\text{-}2) + [\phi(2)\text{-}\phi(1)]$$
$$\Phi(2\text{-}3) = \Phi_o(2\text{-}3) + [\phi(3)\text{-}\phi(2)]$$
$$\Phi(3\text{-}1) = \Phi_o(3\text{-}1) + [\phi(1)\text{-}\phi(3)]$$

$$\text{Closure Phase } (1\text{-}2\text{-}3) = \Phi_o(1\text{-}2) + \Phi_o(2\text{-}3) + \Phi_o(3\text{-}1)$$

WHAT

THE BISPECTRUM

TRIPLE PRODUCTS OF COMPLEX VISIBILITIES AROUND CLOSED TRIANGLES

$$\tilde{B}_{ijk} = \hat{\gamma}_{ij}\,\hat{\gamma}_{jk}\,\hat{\gamma}_{ki}$$

Hence, AMPLITUDE OF BISPECTRUM

$$|\tilde{B}_{ijk}| = |\hat{\gamma}_{ij}|\,|\hat{\gamma}_{jk}|\,|\hat{\gamma}_{ki}|$$

AND PHASE OF BISPECTRUM

$$= \Phi_{ij} + \Phi_{jk} + \Phi_{ki}$$

$$= \Phi^{CP}_{ijk} \rightarrow \text{THE CLOSURE PHASE}$$

General Statements

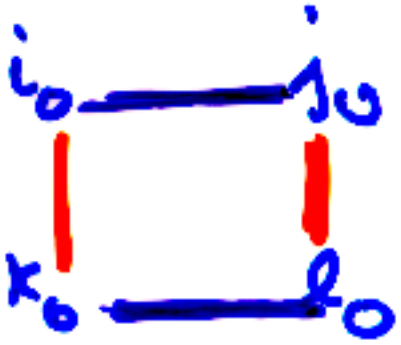
The Bispectrum is always REAL for sources with *point-symmetry* (i.e. the closure phases are 0° or 180°)

Closure Phases are not sensitive to an overall translation of image (a translation is indistinguishable from atmospheric phase delays for any given closing triangle)

CLOSURE AMPLITUDES

RECALL:

$$\tilde{V}_{ij} = |\tilde{G}_i||\tilde{G}_j| e^{i(\phi_i - \phi_j)} \times \tilde{V}_{ij}^{TRUE}$$



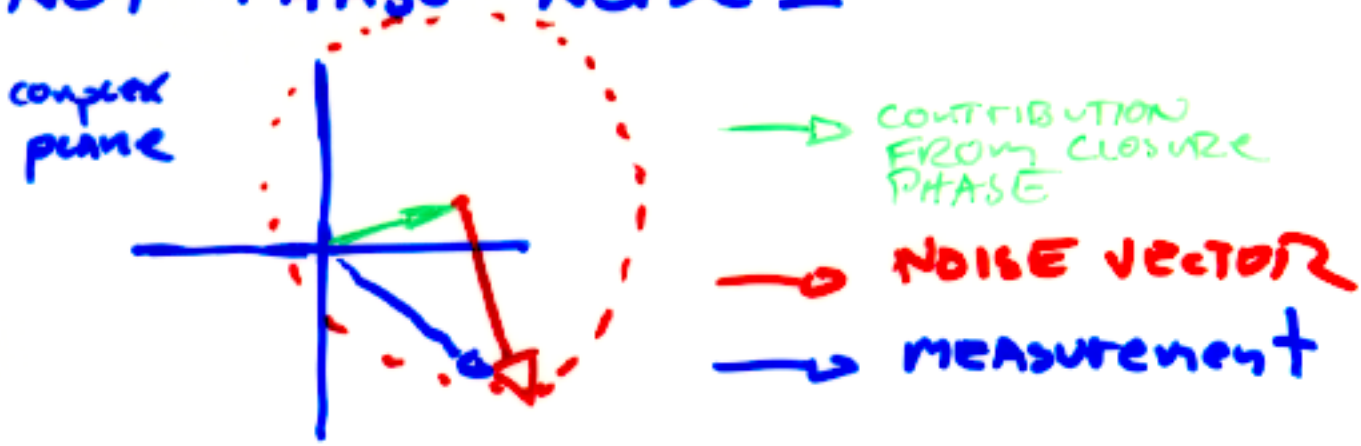
DEFINE A_{ijkl}:

$$A_{ijkl} = \frac{|V_{ij}|\cdot|V_{kl}|}{|V_{ik}|\cdot|V_{jl}|} = \frac{|G_i||G_j||\tilde{V}_{ij}^{T}||G_k||G_l||\tilde{V}_{kl}^{TRUE}|}{|G_i||G_k||\tilde{V}_{ik}^{TRUE}||G_j||G_l||\tilde{V}_{jl}^{TRUE}|}$$

$$= \frac{|\tilde{V}_{ij}^{TRUE}|\cdot|\tilde{V}_{kl}^{TRUE}|}{|\tilde{V}_{ik}^{TRUE}|\cdot|\tilde{V}_{jl}^{TRUE}|},$$

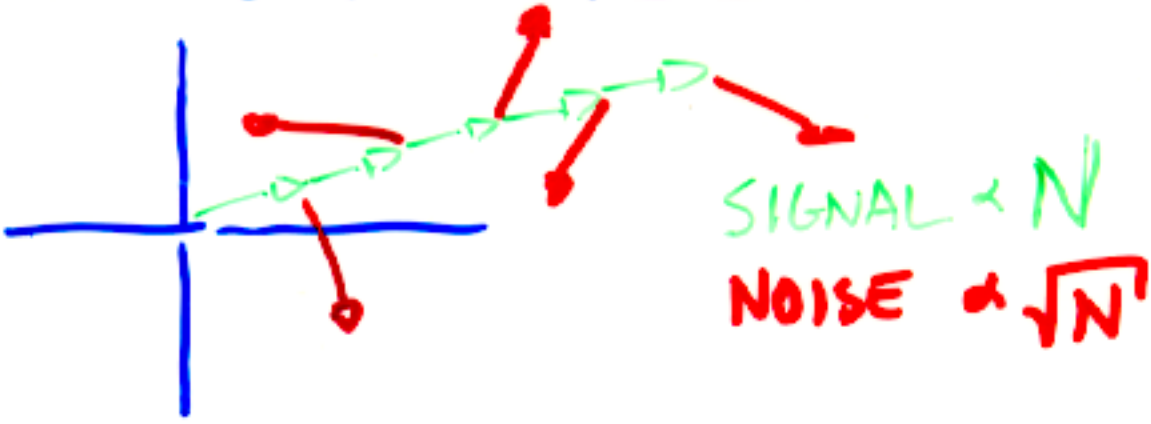
THIS QUANTITY IS INDEPENDENT OF TELESCOPE-SPECIFIC GAINS

CLOSURE PHASE MEASUREMENTS SUFFER FROM ADDITIVE NOISE, NOT "PHASE" NOISE —

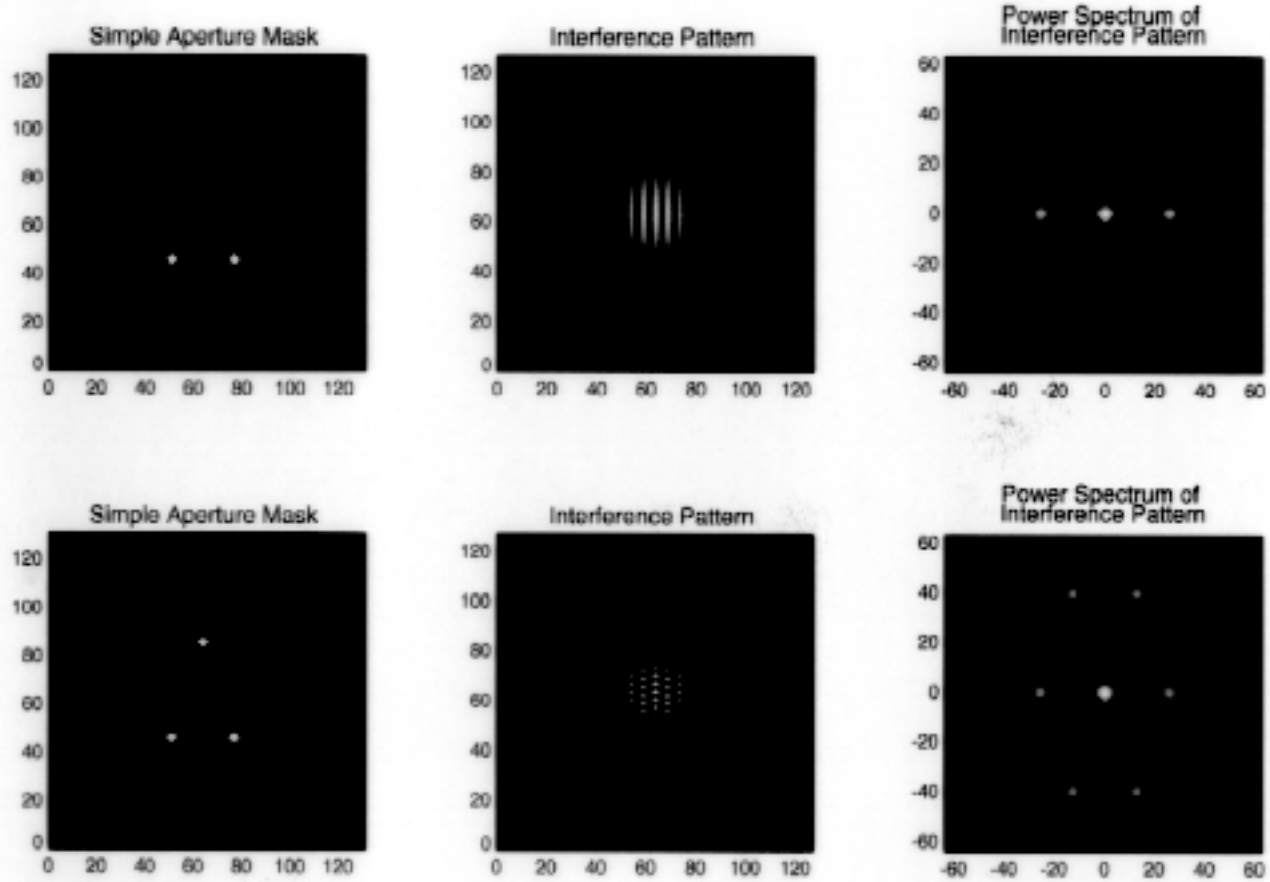


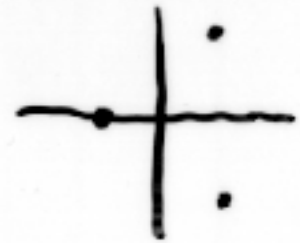
$$\tilde{B}^{\text{measured}}_{ijk} = \tilde{B}^{\text{true}}_{ijk} + \widetilde{\text{NOISE}}$$

BUT, AVERAGING WORKS —



HENCE, SNR OF BISPECTRUM $\propto \sqrt{N}$


Simple Aperture Mask
Interference Pattern
Power Spectrum of Interference Pattern
Simple Aperture Mask
Interference Pattern
Power Spectrum of Interference Pattern



THE EFFECT OF THE PHASE CENTER

$$\tilde{\gamma}' \to \tilde{\gamma} \cdot e^{i \bar{k} \cdot \overline{\Delta x}}$$

Hence,

$$\phi'_{ij} \to \phi_{ij} + \bar{k}_{ij} \cdot \overline{\Delta x}$$

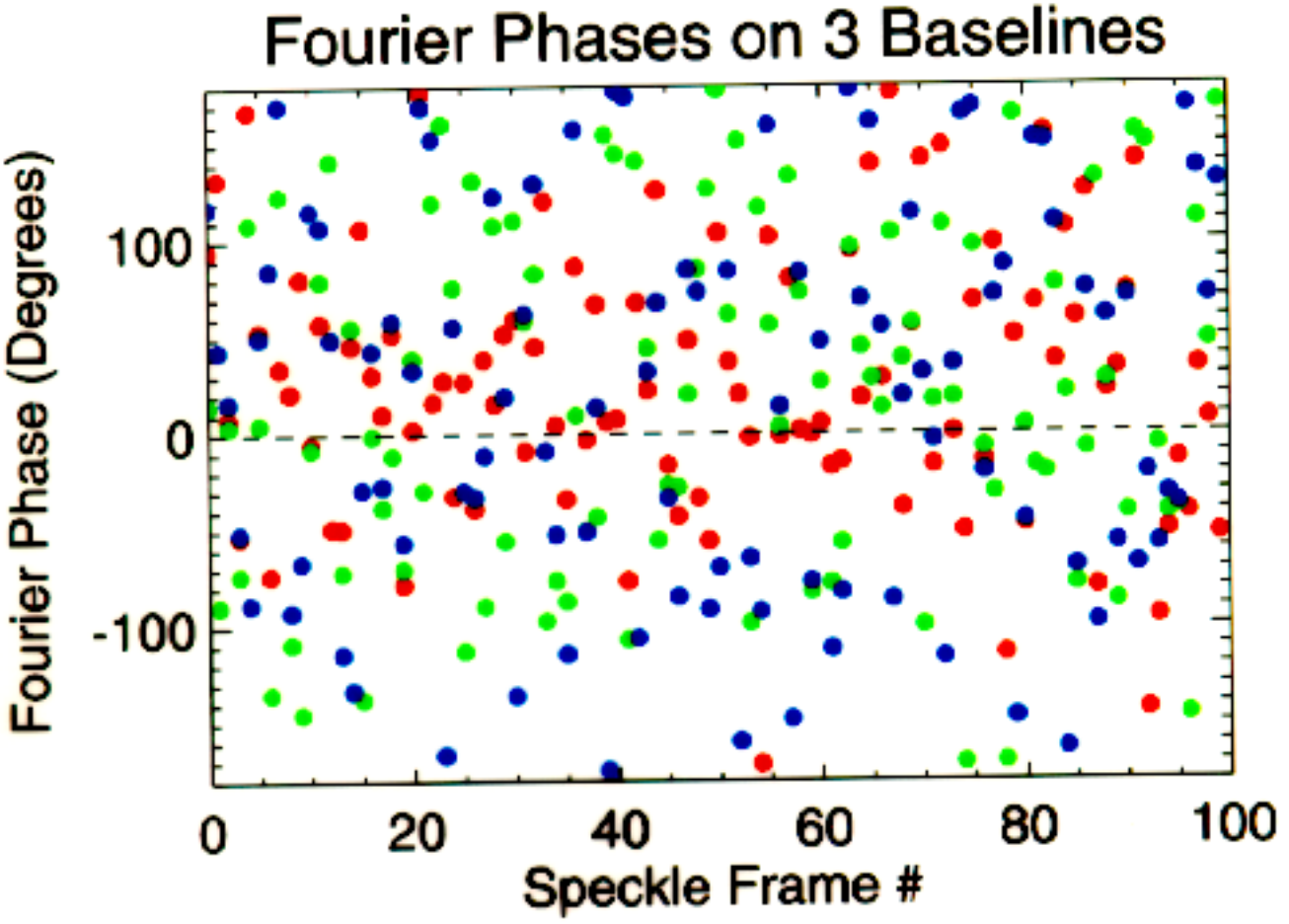
$$\phi'_{jk} \to \phi_{jk} + \bar{k}_{jk} \cdot \overline{\Delta x}$$

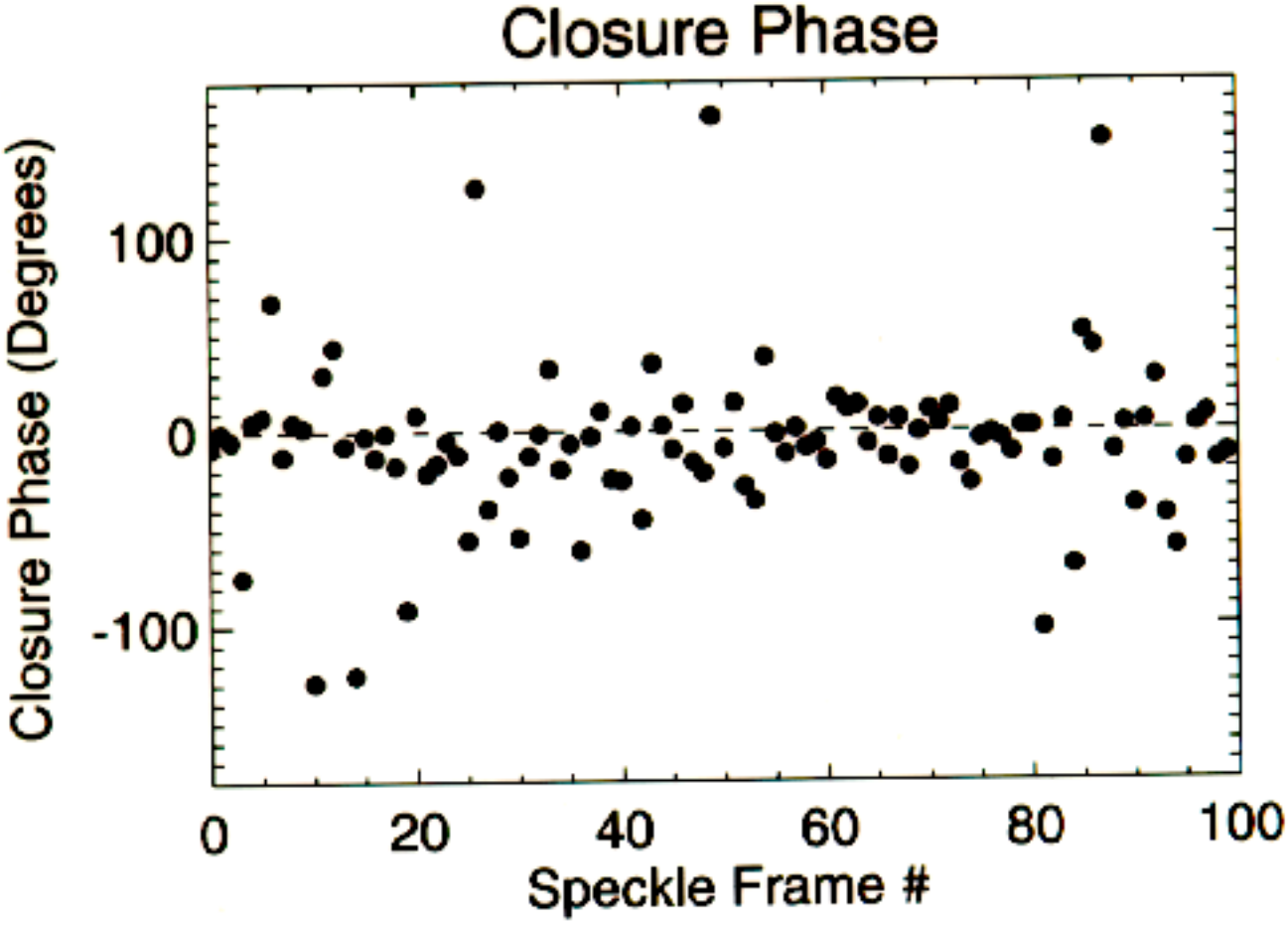
$$\phi'_{ki} \to \phi_{ki} + \bar{k}_{ki} \cdot \overline{\Delta x}$$

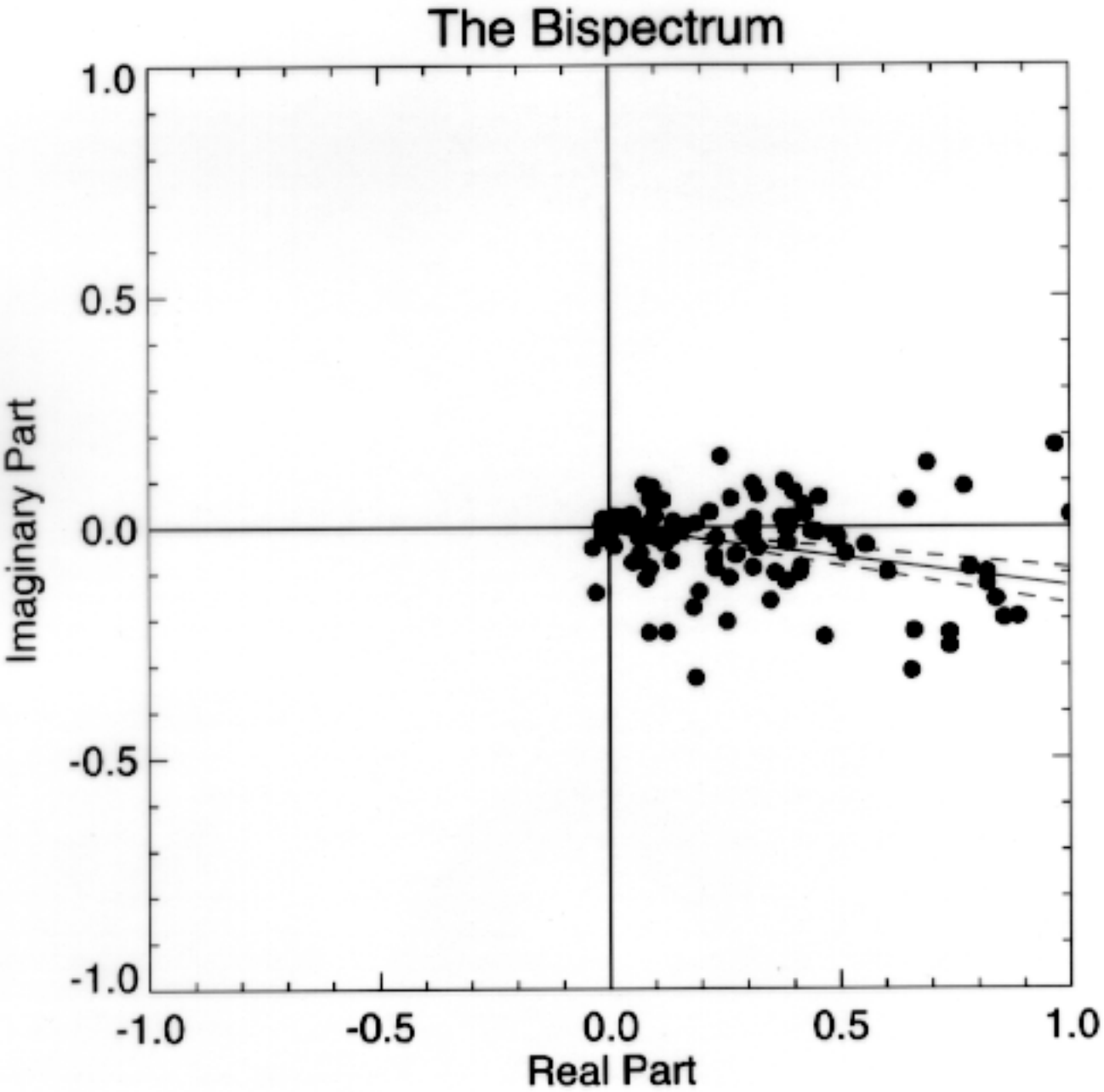
$$\Phi^{cp\,\prime}_{ijk} \to \Phi^{cp}_{ijk} + \left[\bar{k}_{ij} + \bar{k}_{jk} + \bar{k}_{ki}\right] \cdot \overline{\Delta x}$$

BUT, WHAT IS $\bar{k}_{ij}$? $\bar{k}_{ij} = 2\pi \dfrac{\bar{B}_{ij}}{\lambda}$

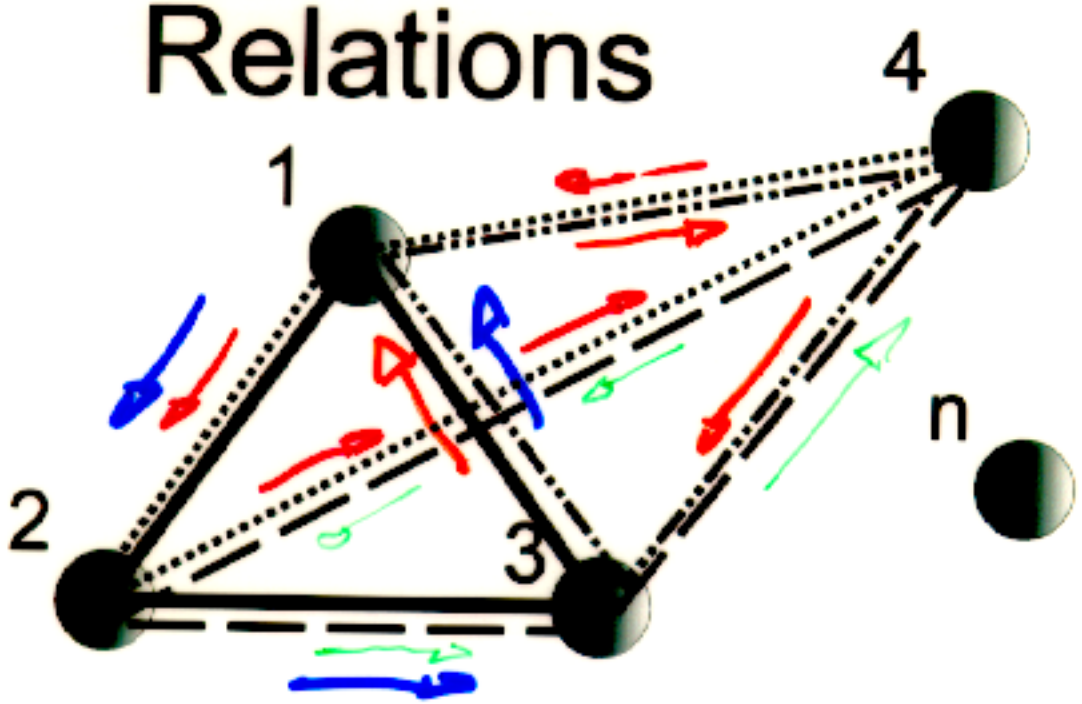
Hence, $\bar{k}_{ij} + \bar{k}_{jk} + \bar{k}_{ki} \propto \underbrace{\bar{B}_{ij} + \bar{B}_{jk} + \bar{B}_{ki}}_{\text{ZERO FOR CLOSING TRIANGLES}}$


Fourier Phases on 3 Baselines
Fourier Phase (Degrees)
100
0
-100
0
20
40
60
80
100
Speckle Frame #


Closure Phase
Closure Phase (Degrees)
100
0
-100
0
20
40
60
80
100
Speckle Frame #


The Bispectrum
1.0
0.5
0.0
-0.5
-1.0
Imaginary Part
-1.0
-0.5
0.0
0.5
1.0
Real Part

Closure Phase Relations



$$\Phi(1\text{-}2\text{-}3)=\Phi(1\text{-}2\text{-}4) + \Phi(4\text{-}2\text{-}3) + \Phi(1\text{-}4\text{-}3)$$

In General:

$$\Phi(1\text{-}2\text{-}3)=\Phi(1\text{-}2\text{-}n) + \Phi(n\text{-}2\text{-}3) + \Phi(1\text{-}n\text{-}3)$$

FOR NON-REDUNDANT ARRAY,

NUMBER OF INDEPENDENT Φ^{CP}:

$$\binom{N-1}{2} = \frac{(N-1)(N-2)}{2}$$

NUMBER OF FOURIER PHASES:

$$\binom{N}{2} = \frac{N\cdot(N-1)}{2}$$

N	NUMBER OF PHASES	NUMBER OF Φ^{CP}	% OF PHASE INFO
3	3	1	33%
7	21	15	71%
21	210	190	90%
27	351	325	93%
50	1225	1176	96%

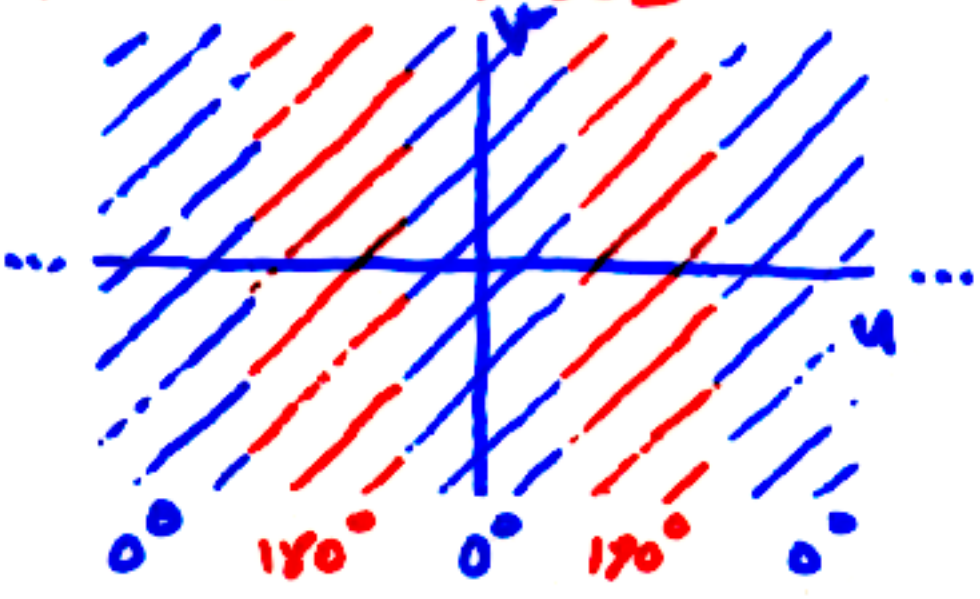
EXAMPLE: AN EQUAL BINARY

$$\tilde{\gamma} \propto e^{-i\frac{\bar{\rho}\cdot\bar{k}}{2}} + e^{+i\frac{\bar{\rho}\cdot\bar{k}}{2}}$$

$$\propto 2\cos\left(\frac{\bar{\rho}\cdot\bar{k}}{2}\right)$$

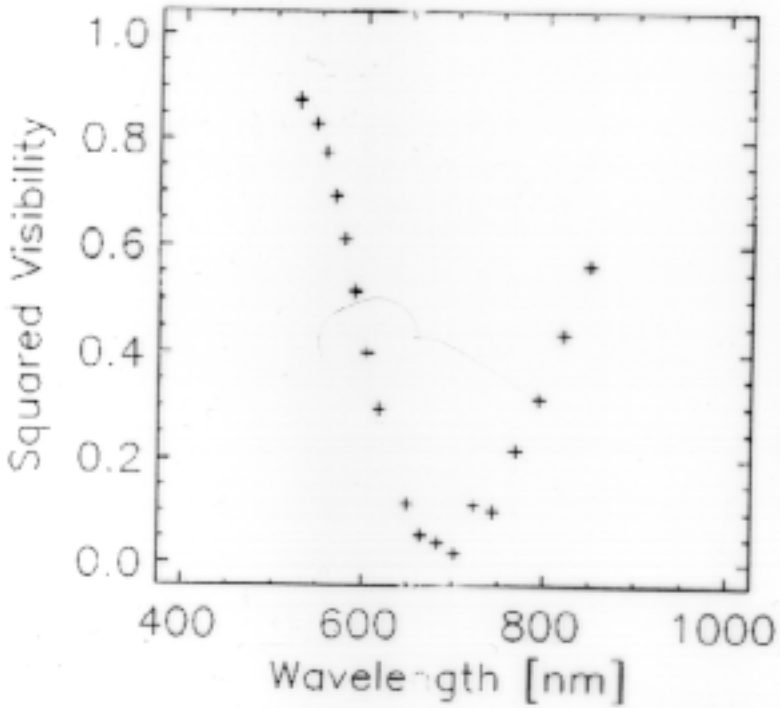
HENCE, PHASE EITHER 0°, 180°

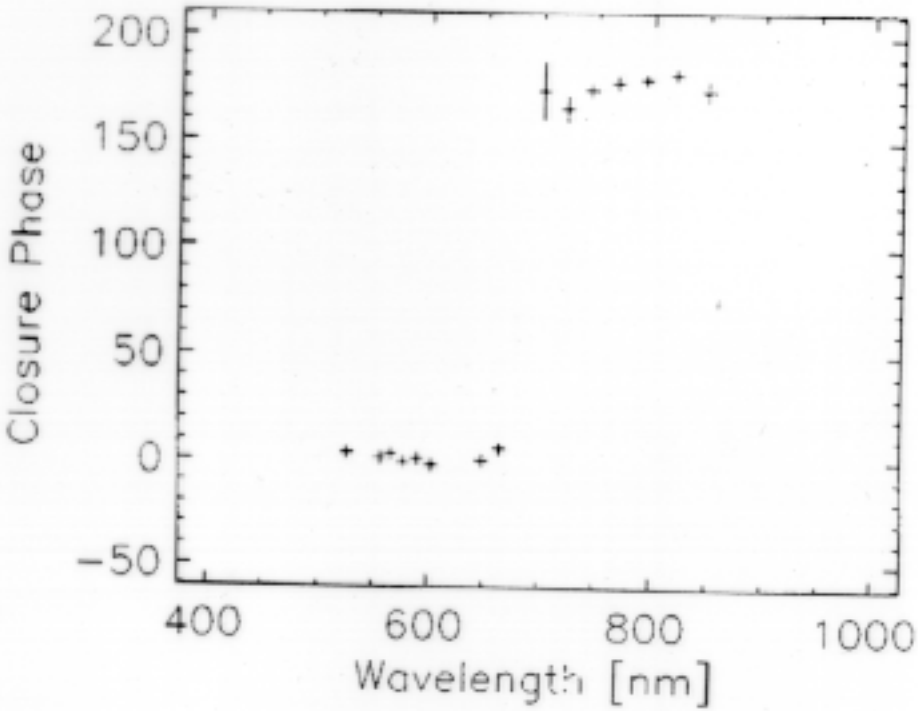
FOURIER PHASES



NOTICE ABRUPT PHASE JUMPS WHEN VISIBILITY AMPLITUDE GOES THROUGH NULLS

PHASE DISCONTINUITIES SMOOTHED OUT FOR UNEQUAL BINARIES


Squared Visibility
1.0
0.8
0.6
0.4
0.2
0.0
400
600
800
1000
Wavelength [nm]


Closure Phase
200
150
100
50
0
−50
400
600
800
1000
Wavelength [nm]

EXAMPLE: FAINT HOTSPOT ON STELLAR SURFACE

NOTE: ELLIPTICAL STARS YIELD $\Phi^{CP} = 0°, 180°$ ONLY.

AT LOW SPATIAL RESOLUTION,

STELLAR DISK IS CENTRO SYMMETRIC, i.e. $\Phi_{CP} = 0°, 180°$

AT HIGH SPATIAL RESOLUTION,

STELLAR DISK IS RESOLVED, LEAVING ONLY THE HOTSPOT, i.e. $\Phi_{CP} = 0°, 180°$

HOWEVER, CLOSING TRIANGLES OF INTERMEDIATE BASELINES WILL YIELD NON-ZERO CLOSURE PHASES

MODELS: DISK + HOTSPOTS

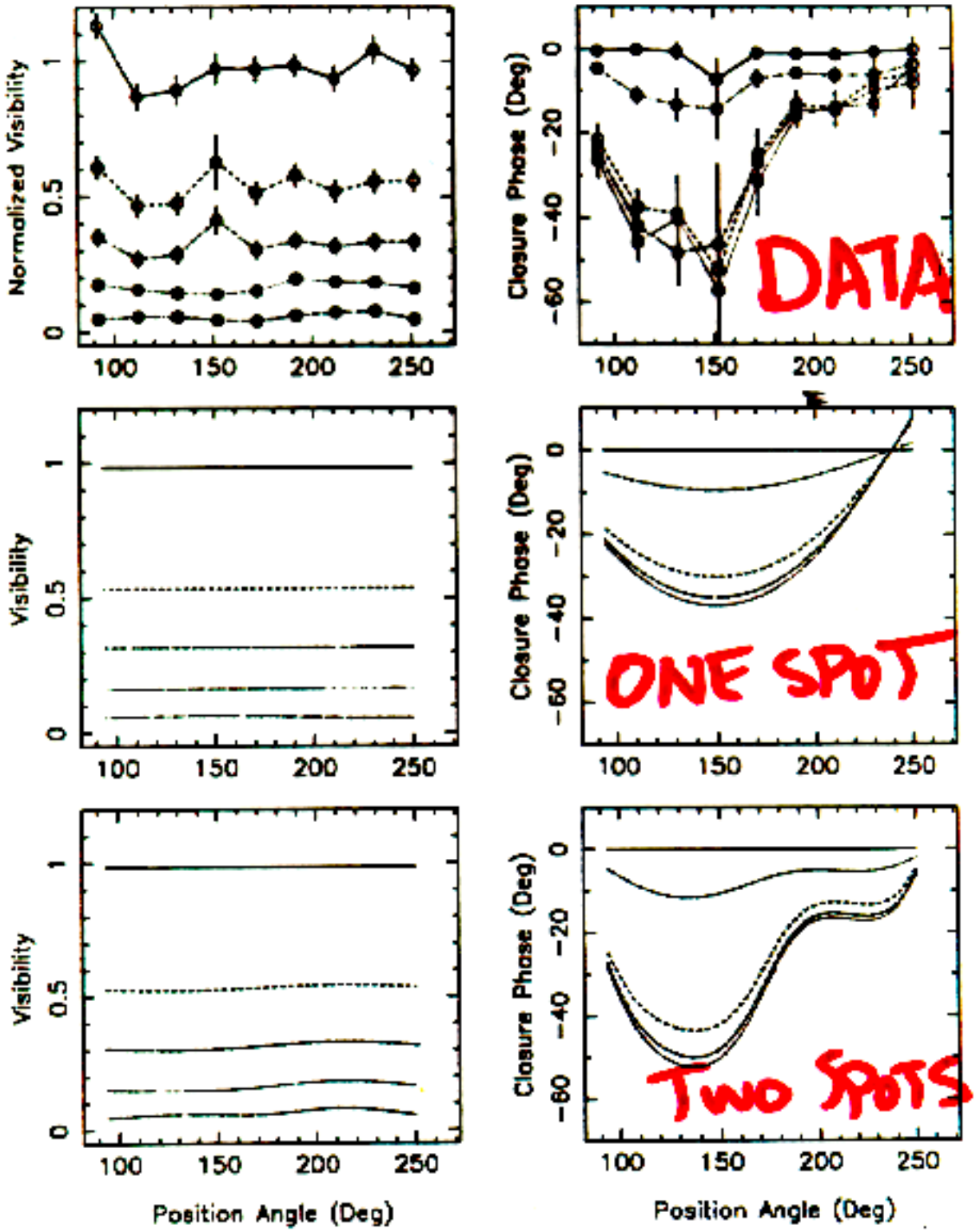
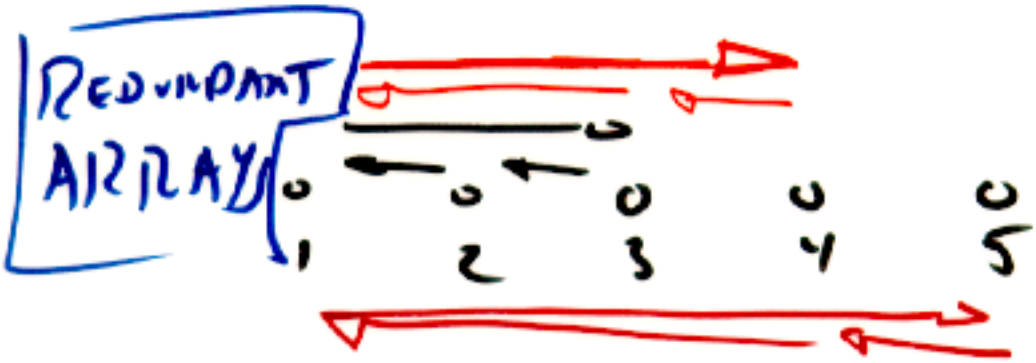


Figure 4. Visibility amplitudes and closure phases as measured

TUTHILL 1994

IMAGING

CONVERTING BISPECTRUM TO FOURIER PHASES.



$$\phi_{12} = 0$$

$$\phi_{13} = \Phi_{132} - \phi_{32} - \phi_{21}$$

SOLVE FOR ALL PHASES DIRECTLY!

BUT! POOR NOISE PROPAGATION CHARACTERISTICS.

INEFFICIENT USE OF TELESCOPES
POOR (U,V) COVERAGE.

LINEAR operation $\Rightarrow$ MATRICES.
except complicated by mod 2π.

Image Reconstruction

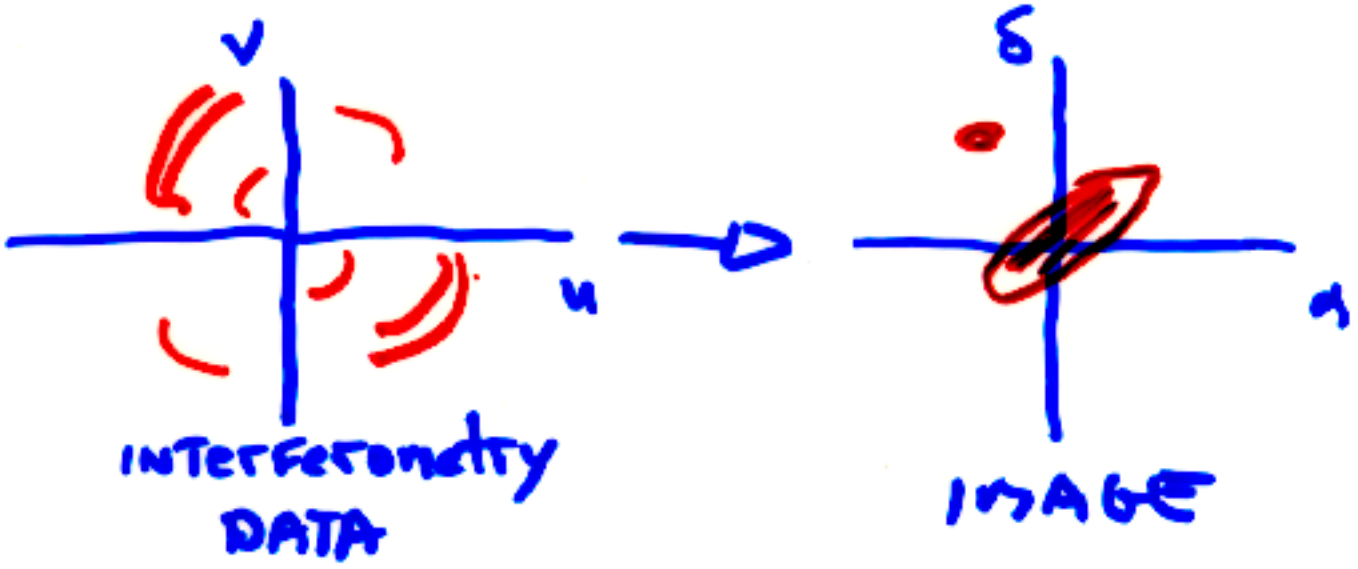
- **Find image which fits both visibility amplitudes and closure phases (to within experimental uncertainty)**

Further Constraints

- **Positive Definite**
- **Limited Field-of-View**
- **"Smooth" – e.g. MEM**
- **Use *a priori* information**

"STANDARD" APERTURE SYNTHESIS IMAGING:

converts unevenly sampled Fourier Phases and Amps (i.e. complex Visibility) into a map free of artifacts



ALGORITHMS: CLEAN
MEM

Iteration Cycle

Start with Initial Model

↓

Generate Fourier phases consistent with Closure phases, using Model phases as necessary.

↓

Using these candidate phases and visibility amplitudes, use CLEAN/MEM to generate map

↓

If not converged, use new map as starting Model and repeat

Readhead & Wilkinson 1978

Self-Calibration

Models intrinsic Fourier phases *plus* telescope errors

Generate Fourier phases consistent with Closure Phases & begin with initial trial Image

↓

Calculate complex Visibility (Amplitudes & Phases) of trial image

↓

Adjust telescope errors so "measured" Fourier phases are best fit by combination of trial image phases plus telescope errors

↓

Correct trial phases based on new estimates of telescope errors, and map using CLEAN/MEM

↓

If not converged, use new map as the next trial image

Cornwell & Wilkinson 1981

$$\Phi_{\text{"MEASURED"}} = \Phi_{\text{TRUE}} + [\phi_1 - \phi_i]$$

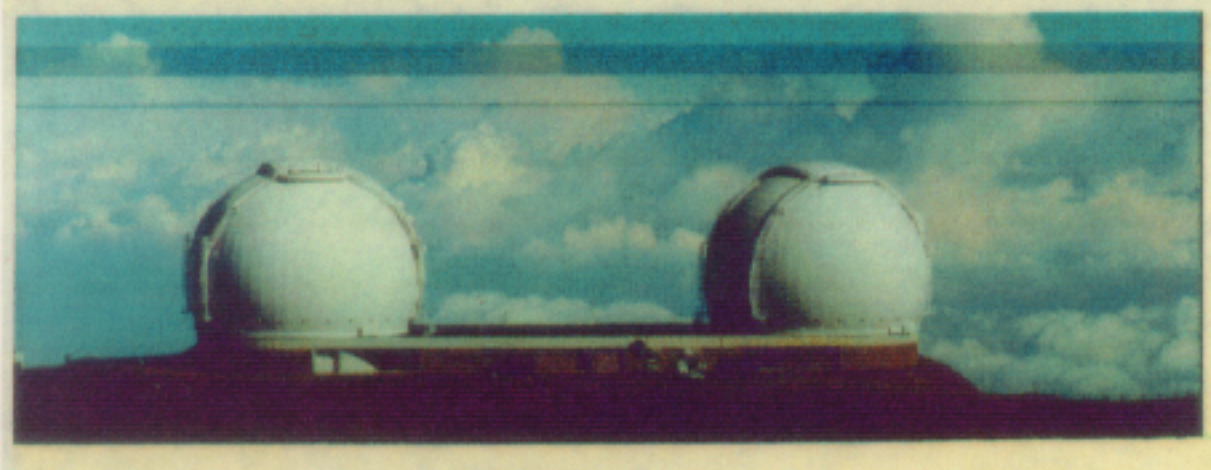
ATMOSPHERE DOES NOT CHANGE THE CLOSURE PHASES

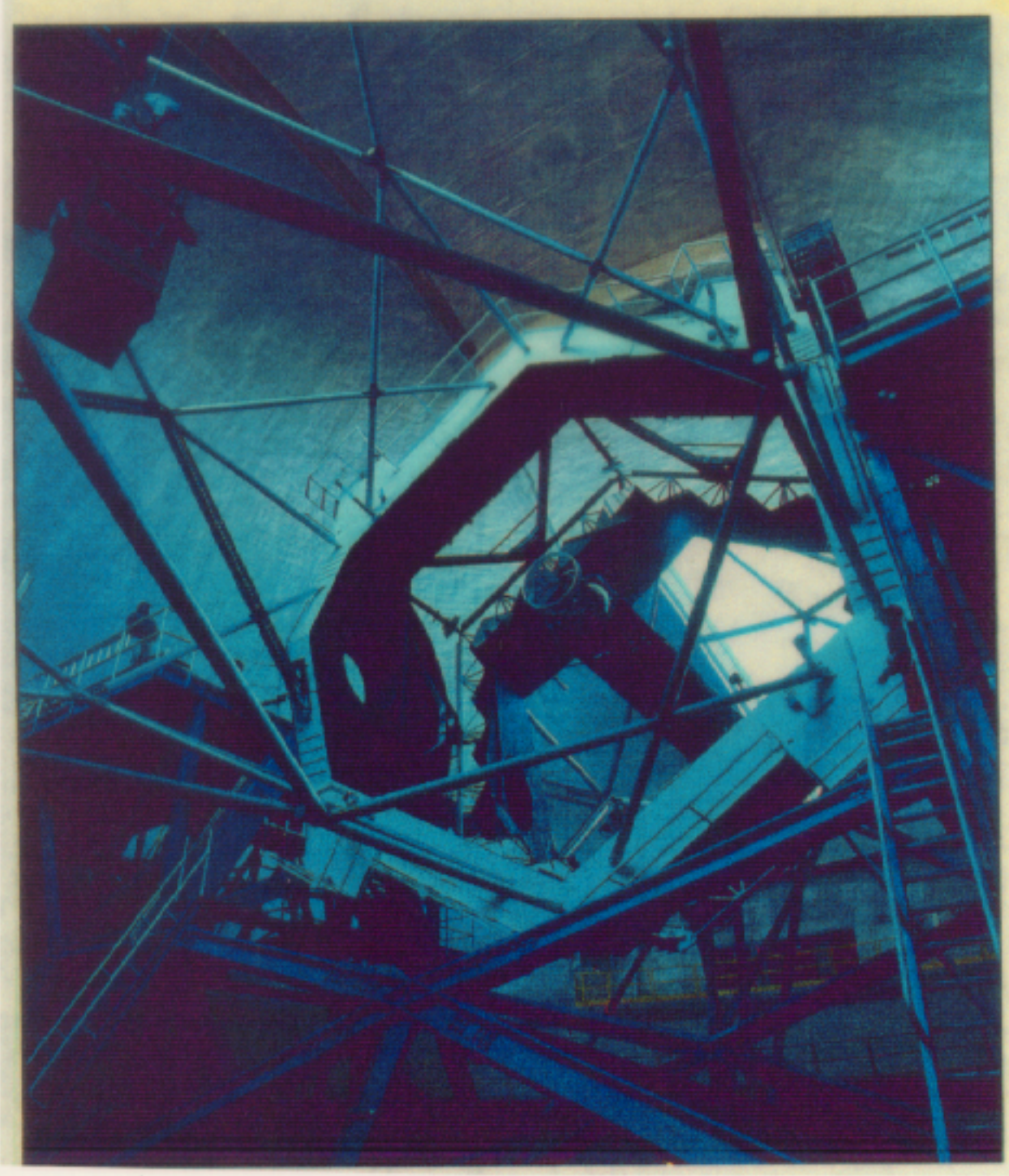
ADJUST THESE TERMS ITERATIVELY, APPLYING IMAGE CONSTRAINTS DURING EACH CYCLE.

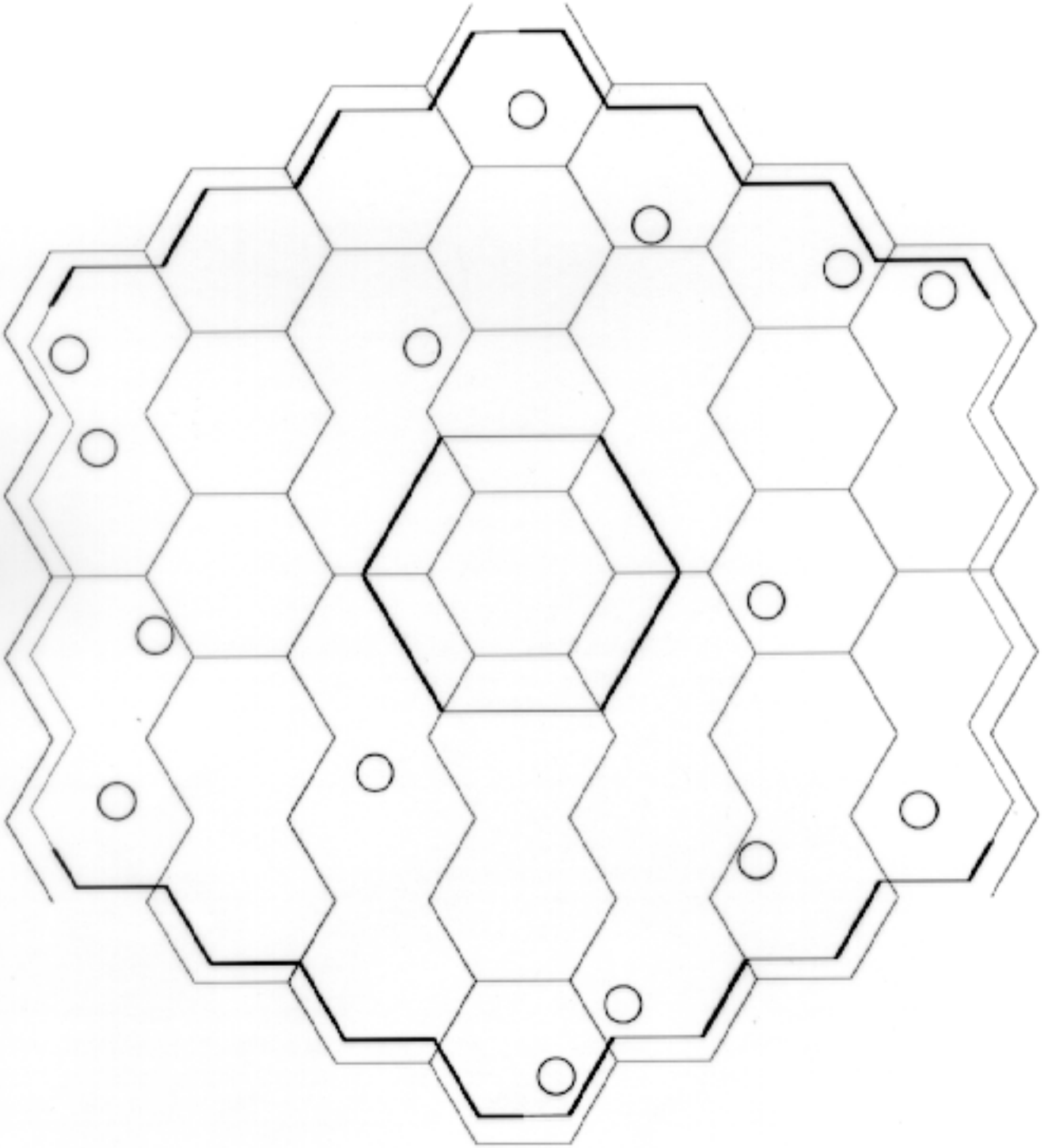
Works extremely well –

FOR HIGH SNR (~5) AND GOOD FOURIER COVERAGE.

EMERGING IMAGING INTERFEROMETERS (3-6 telescopes) PROBING NEW REGIME...

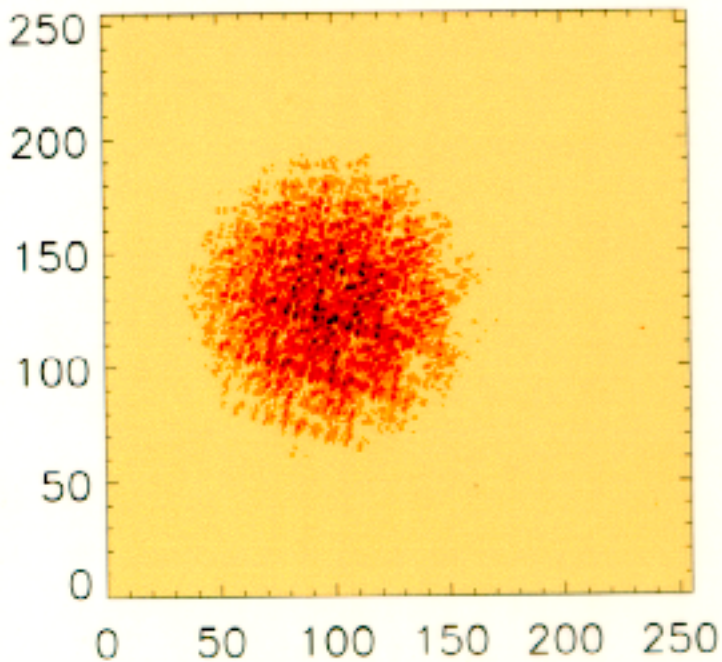




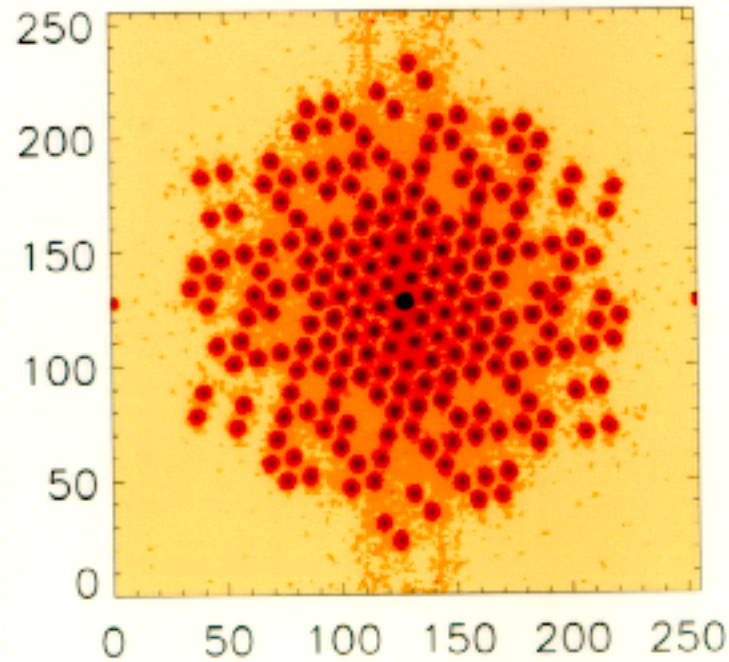


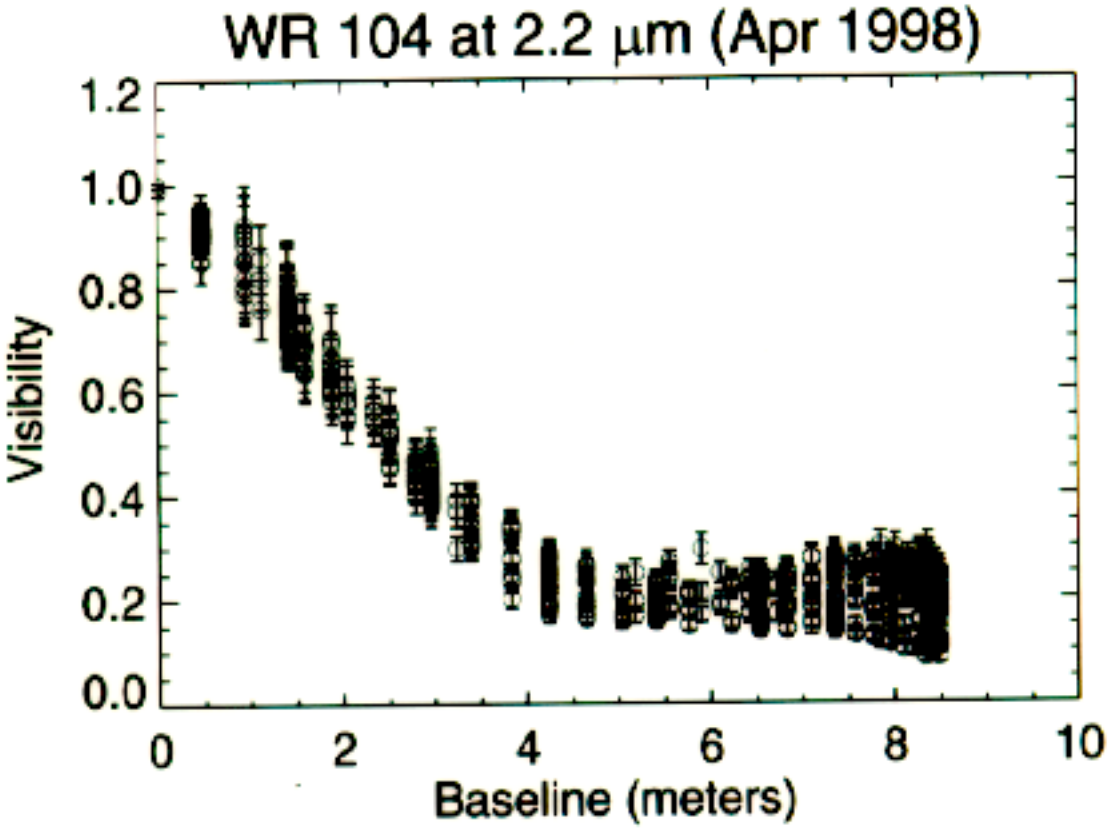
	Keck Pupil Mask	
	DRAWN BY: Peter Tuthill	DATE: 25-Feb-97
	SCALE: 1:50	MATERIAL: Aluminum
	15 Hole Golay (35cm holes)	

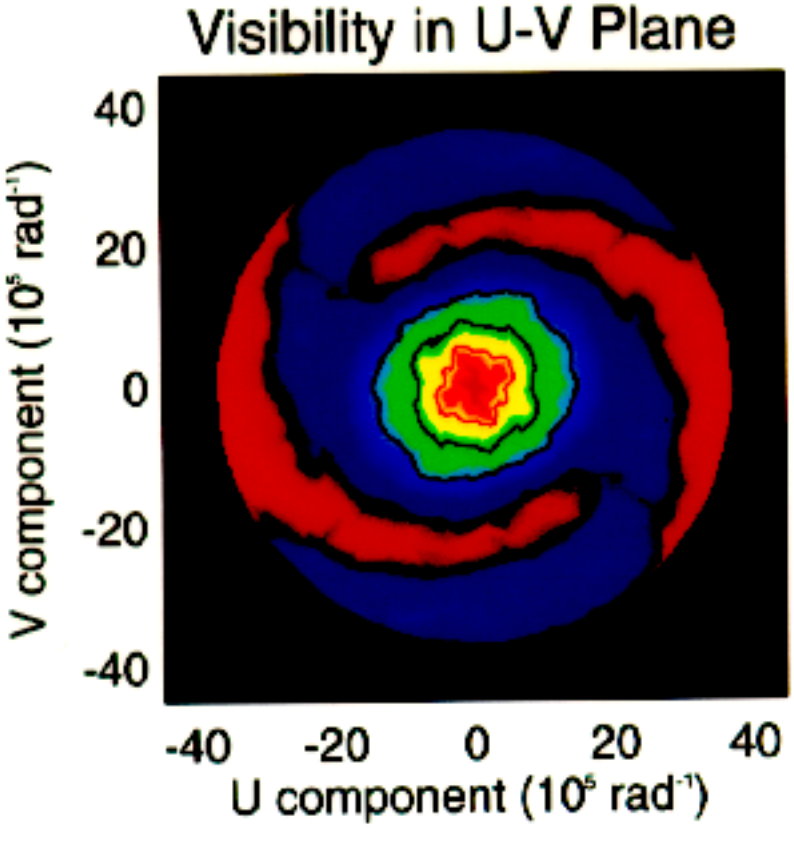
Speckle Pattern

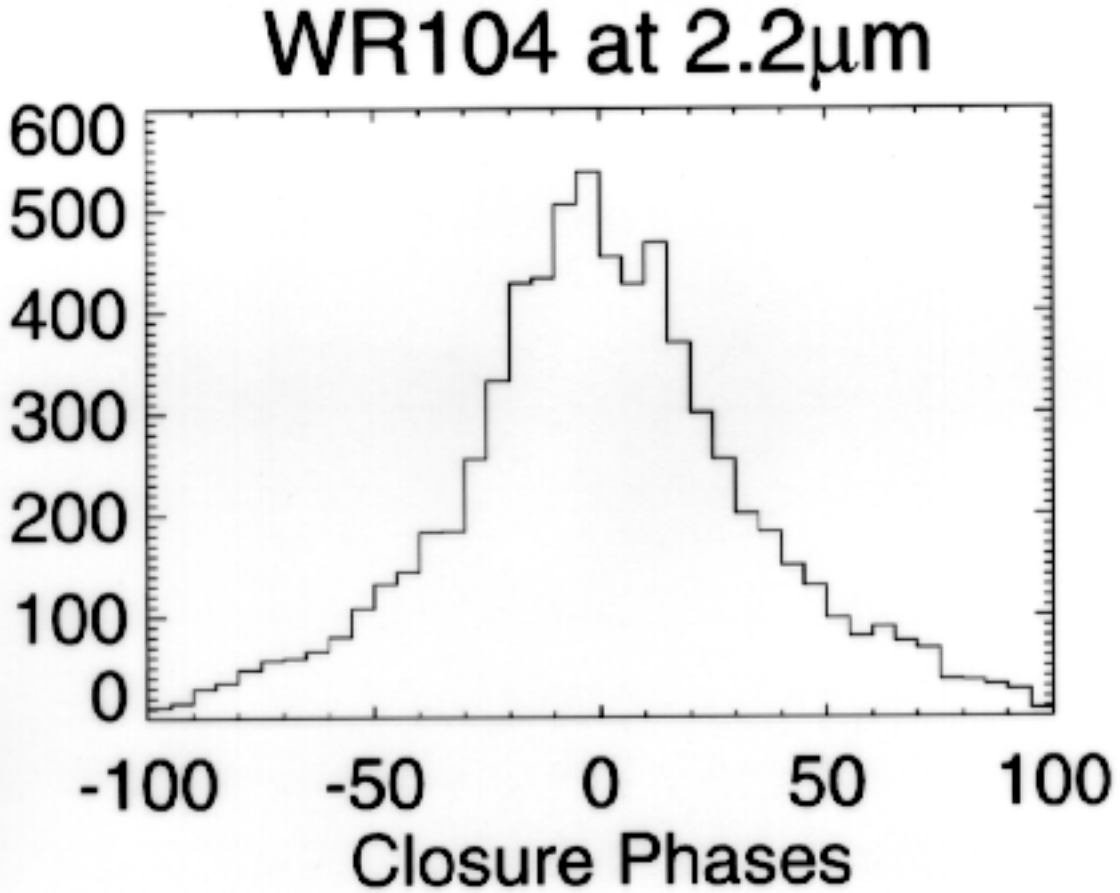


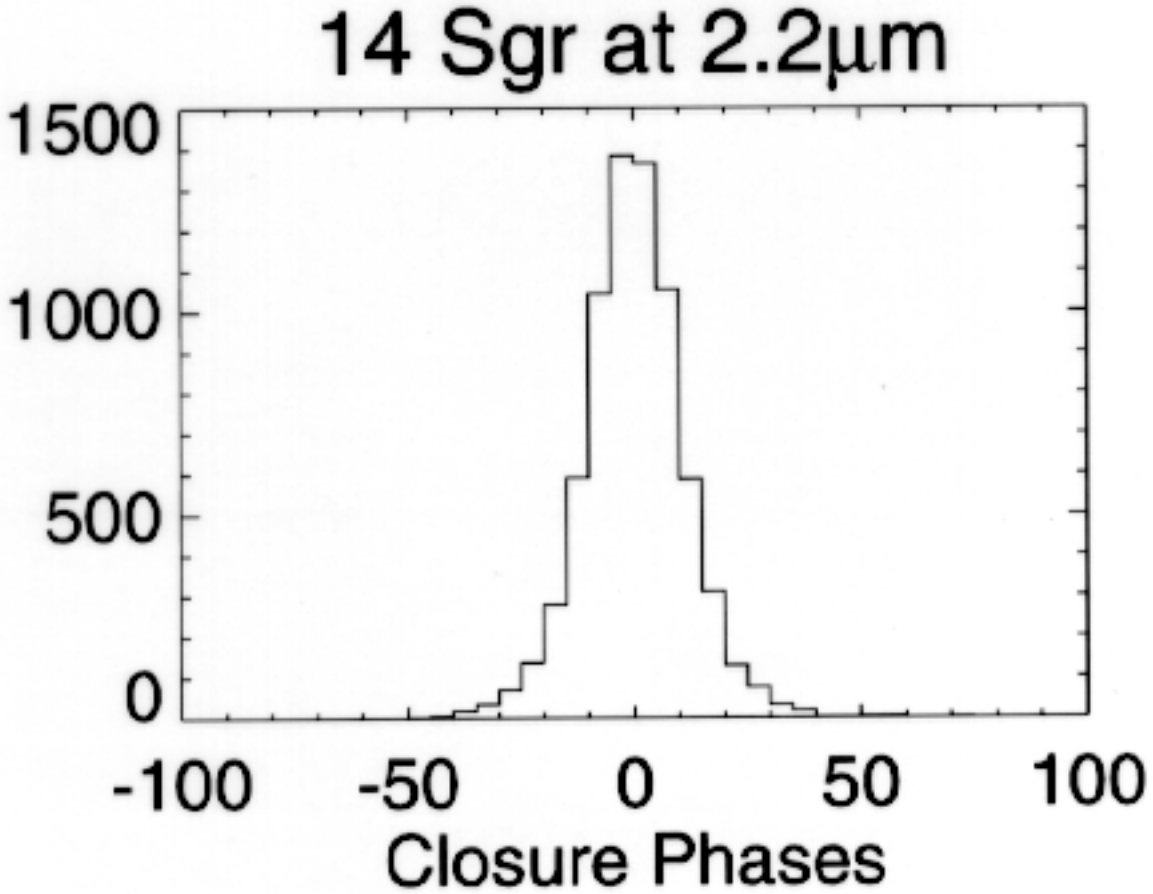
Power Spectrum

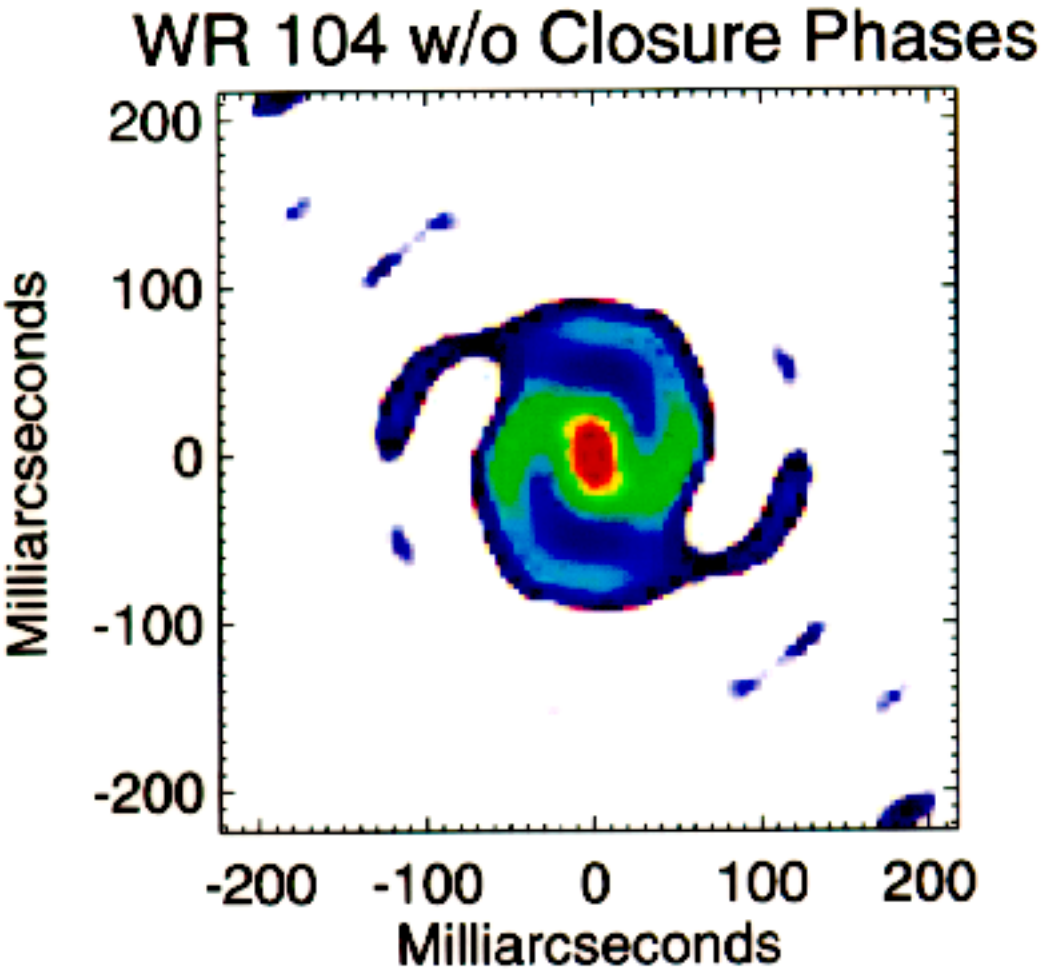


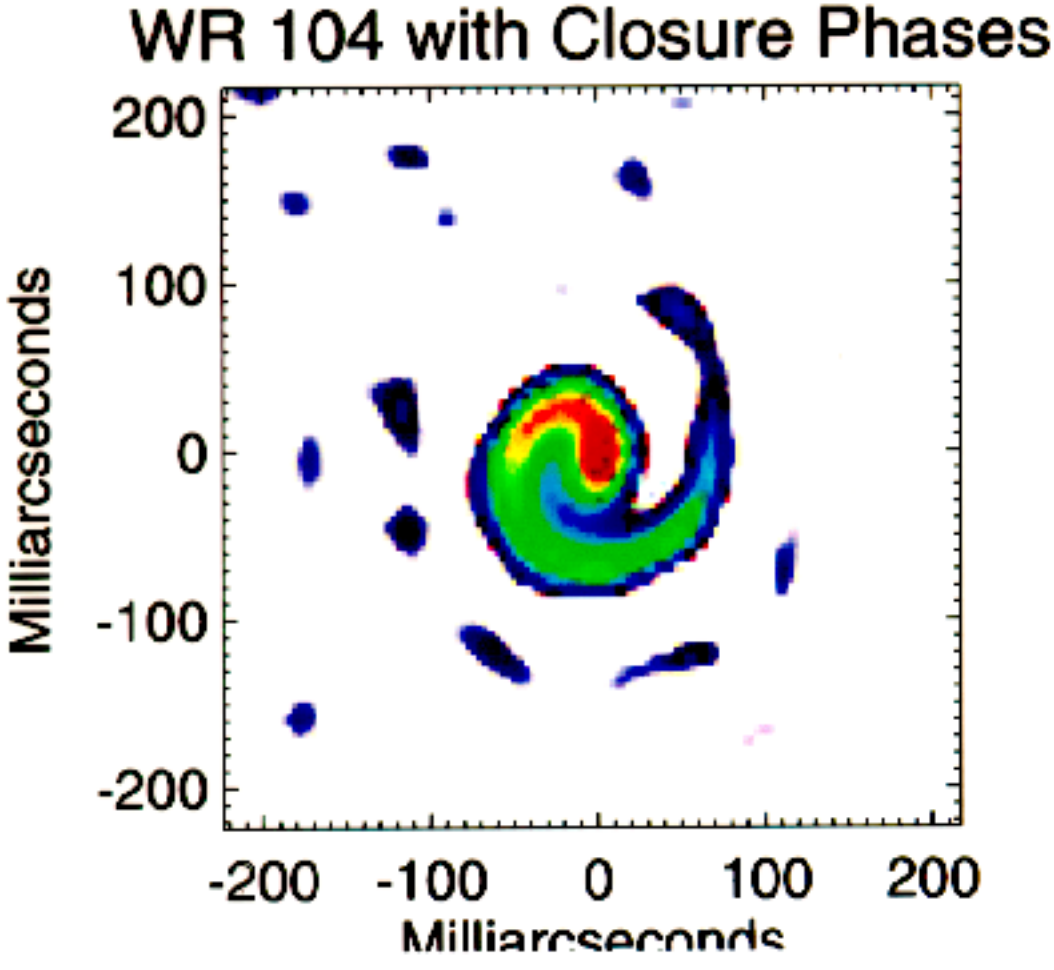

WR 104 at 2.2 μm (Apr 1998)
Visibility
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
Baseline (meters)

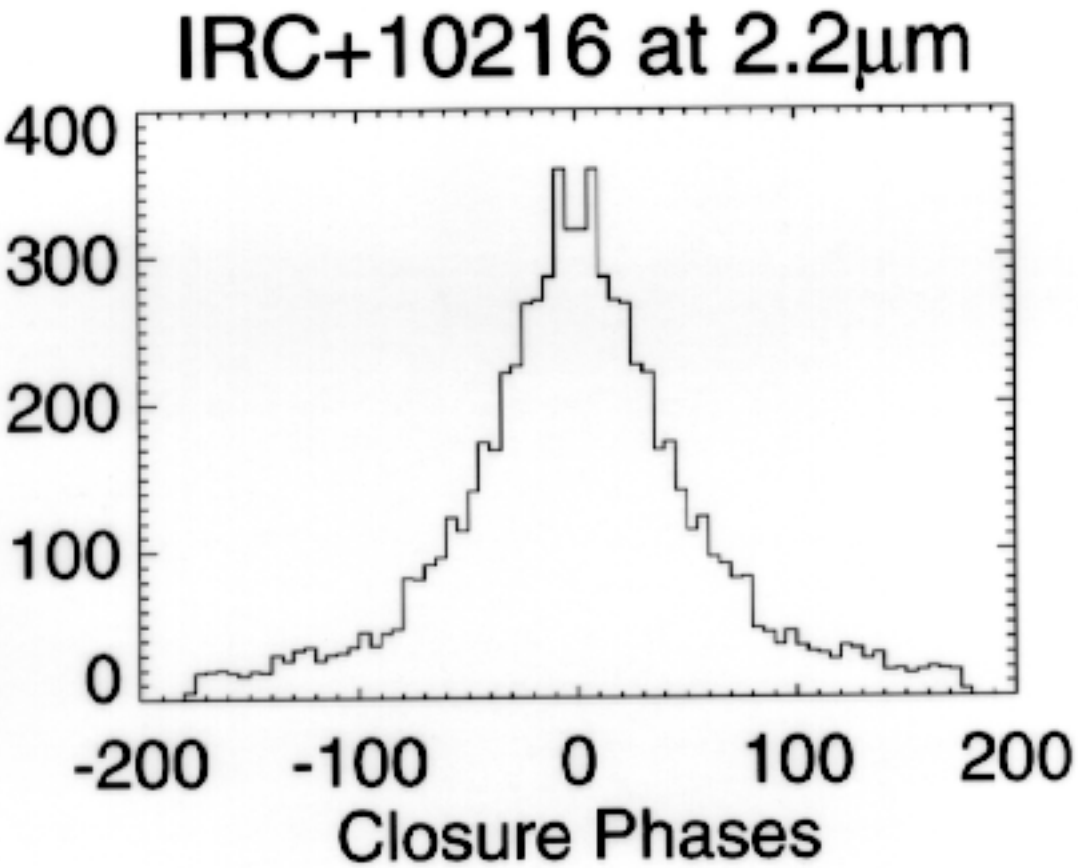

Visibility in U-V Plane
V component (10^5 rad^{-1})
40
20
0
-20
-40
-40
-20
0
20
40
U component (10^5 rad^{-1})

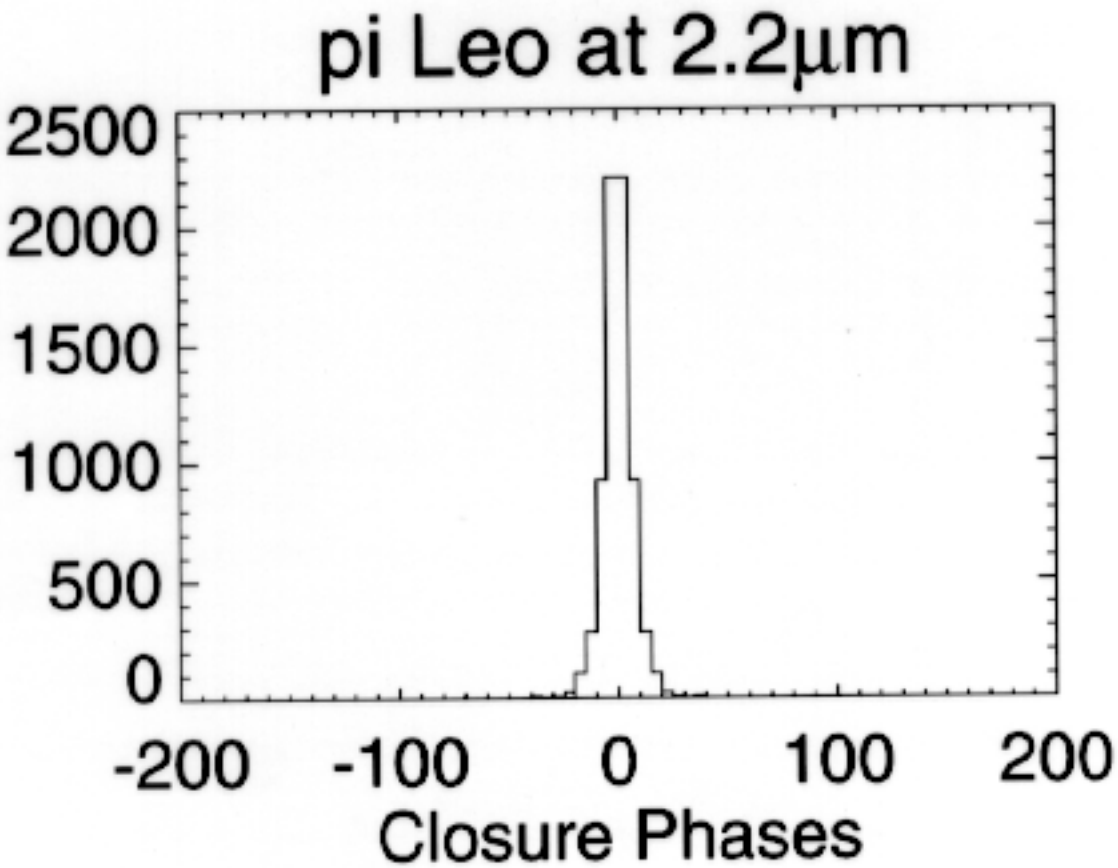

WR104 at 2.2μm
600
500
400
300
200
100
0
-100
-50
0
50
100
Closure Phases

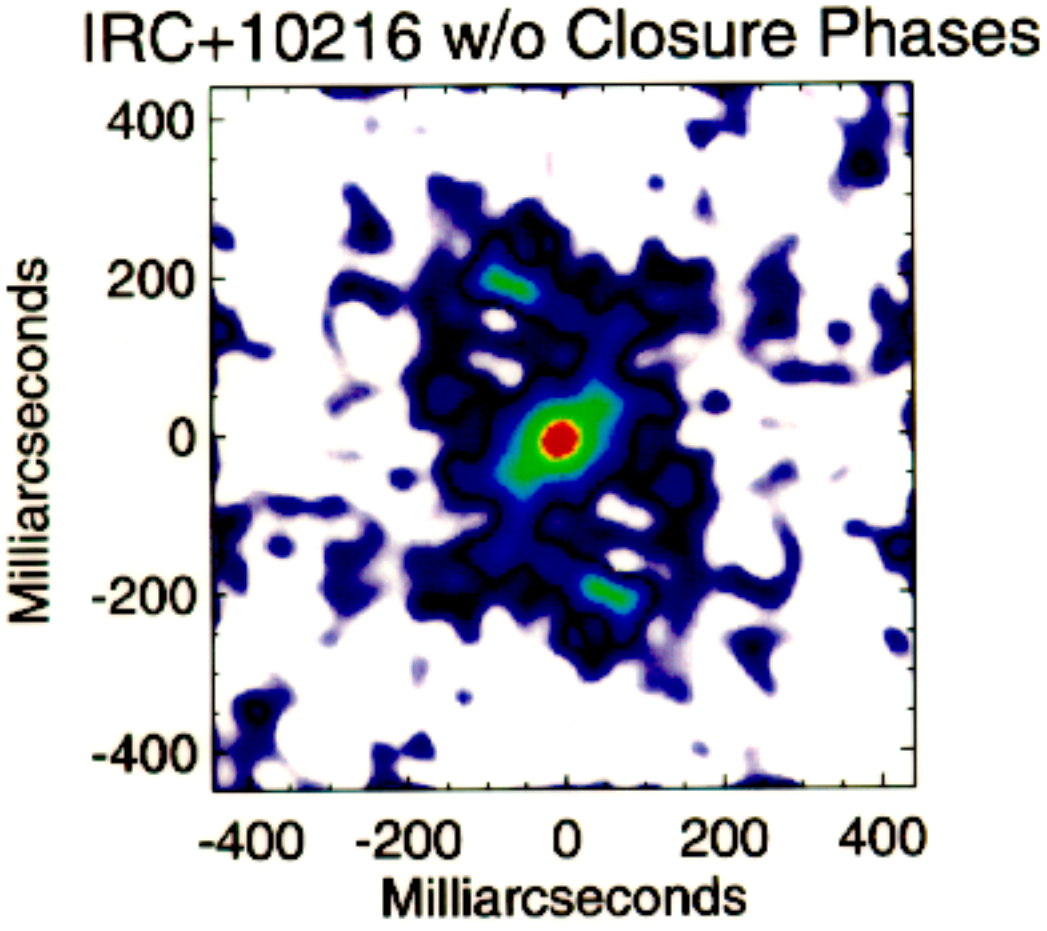

14 Sgr at 2.2μm
1500
1000
500
0
-100
-50
0
50
100
Closure Phases

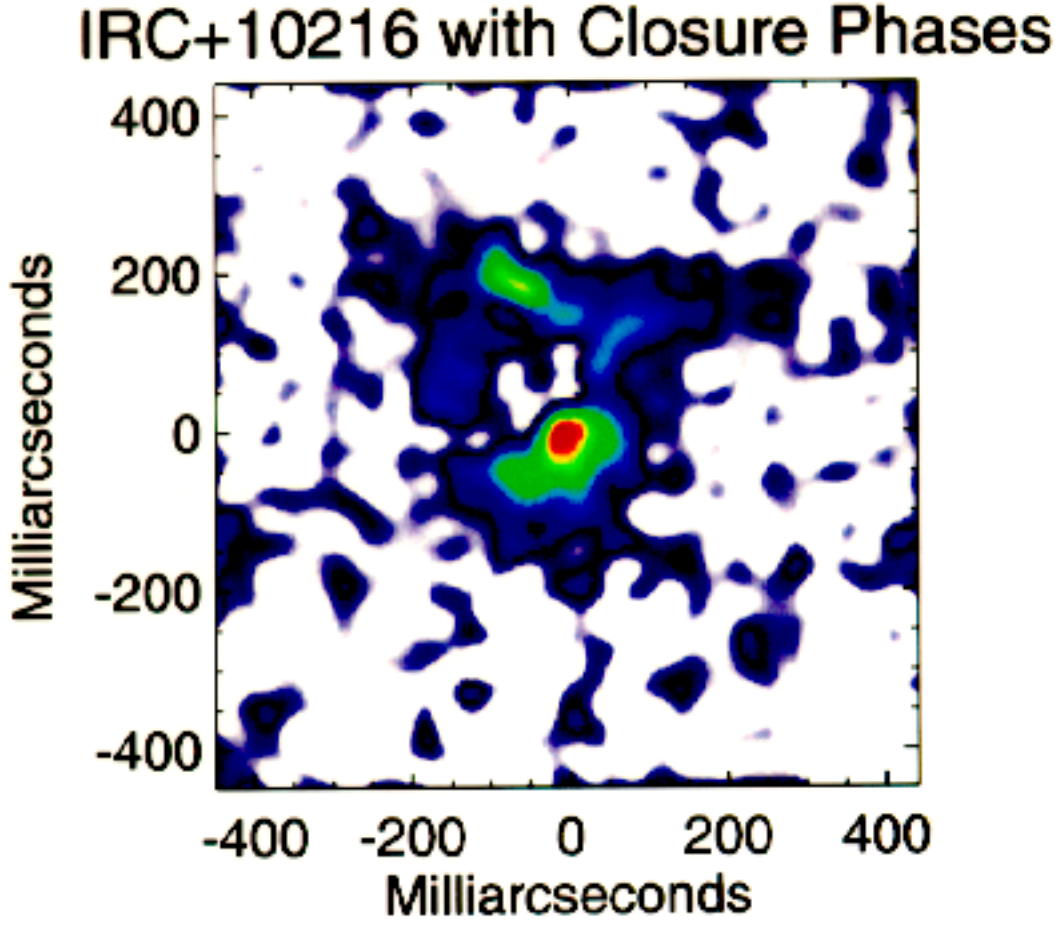

WR 104 w/o Closure Phases
200
100
0
-100
-200
Milliarcseconds
-200
-100
0
100
200
Milliarcseconds


WR 104 with Closure Phases
200
100
0
-100
-200
Milliarcseconds
-200
-100
0
100
200


IRC+10216 at 2.2μm
400
300
200
100
0
-200
-100
0
100
200
Closure Phases


pi Leo at 2.2μm
2500
2000
1500
1000
500
0
-200
-100
0
100
200
Closure Phases


IRC+10216 w/o Closure Phases
400
200
0
-200
-400
Milliarcseconds
-400
-200
0
200
400
Milliarcseconds


IRC+10216 with Closure Phases
400
200
0
-200
-400
Milliarcseconds
-400
-200
0
200
400
Milliarcseconds

The Future

Outstanding Problems with Mapping

- Should fit directly to closure phases; outside of self-cal paradigm (computationally expensive but straightforward)
- Baseline-dependent residuals (limitation of χ^2; use multi-resolution approach)
- Point source(s) embedded in extended nebulosity (model/mapping hybrids, i.e. use *a priori* info)
- Artifacts of (highly) uneven Fourier coverage

Which Triangles?

- Given a sparse array, which closure phases are most important to measure?
- Better understanding of how closure phases contrain the parameter space of possibility Fourier phases

Important Points

- Closure phases (and amplitudes) are insensitive to <u>telescope-specific</u> errors such as atmospheric fluctuations
- They <u>can not</u> be used to calibrate baseline-dependent problems
- Non-zero closure phases indicate asymmetric structure; however, point-symmetric objects (such as disks) have Φ_{CP} of 0° or 180°
- Software and theoretical work <u>necessary</u> for optimal imaging with the latest generation of interferometers

- Despite the total scrambling of Fourier phases by a turbulent atmosphere, the use of closure phases and image constraints allow nearly complete recovery of all phase information