Signal to Noise in Detection and Interferometry

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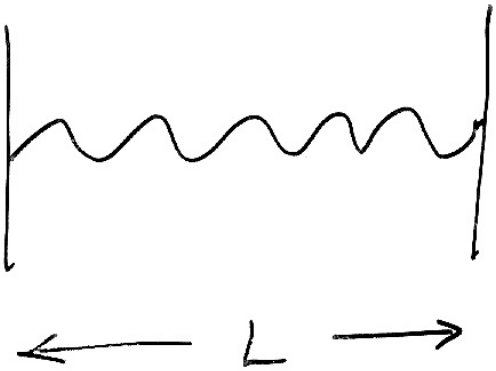
Summer School for Interferometry, CIT, Pasadena

Aug. 8, 1999

number of quanta per mode in thermal equilibrium

$$\bar{n} = \frac{\sum_{m=0}^{\infty} m e^{-\frac{mh\nu}{kT}}}{\sum_{m=0}^{\infty} e^{-\frac{mh\nu}{kT}}} = \frac{1}{e^{h\nu/kT} - 1}$$

Power flow



$$\frac{c}{\nu} = \lambda = \frac{2L}{n} \qquad \therefore \quad n = \frac{2L}{c}\nu$$

Assume 1 photon per mode, Then $\Delta n = \frac{2L}{c}\Delta\nu$

Round trip time $t = \frac{2L}{c}$ $\therefore \frac{\Delta n}{t} = \Delta\nu$

or, for 2 polarizations, the total

photon flux $= 2h\nu\Delta\nu$ or, at thermal equilibrium $= \frac{2h\nu\Delta\nu}{e^{h\nu/kT} - 1}$

For fractional loss ε in path to $T \to 0$,

$$\text{photon flux} = \frac{2 h\nu \Delta\nu \varepsilon}{e^{h\nu/kT} - 1}$$

For signal power $P_\nu \Delta\nu$ in each polarization

$$S/N = \frac{2 P_\nu \Delta\nu \, t}{h\nu \sqrt{\frac{2\Delta\nu\varepsilon t}{e^{h\nu/kT}-1}}}$$

$$= \frac{P_\nu}{h\nu} \sqrt{\left(e^{h\nu/kT} - 1\right) \frac{2\Delta\nu t}{\varepsilon}}$$

where t is time length of measurement in seconds.

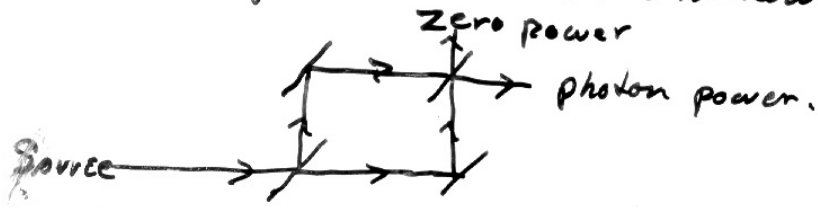
N.B. This is for a photodiode
For a photoconductor, S/N is less by $\sim \frac{1}{\sqrt{2}}$

Heterodyne Detection

Uncertainty principle requires $\Delta\phi\,\Delta n \geq 1/2$

or, for $\Delta n = \sqrt{n}$, $\quad \Delta\phi \geq \frac{1}{2\sqrt{n}}$

This is true for absolute phase, not for relative phase of two waves, with absolute phase of both unknown. eg. interferometer



"Squeezing"

Heterodyne detection

current $i \propto (E_{LO} + E_s + E_0)^2$

$\approx E_{LO}^2 + 2E_{LO}E_s + 2E_{LO}E_0$

where E_{LO} = field of local oscillator

E_s = " " signal

E_0 = zero point fluctuation field

Power $\propto E_{LO}^2 (E_s^2 + E_0^2)$

where usually $E_a^2 \gg E_s^2$ and it produces 1 photon/sec/unit bandwidth

$\therefore$ Noise power $= 2h\nu\,\Delta\nu$ for two sidebands of width $\Delta\nu$.

For signal power P_ν per unit bandwidth

$$\text{Heterodyne } S/N = \frac{P_\nu}{h\nu}\sqrt{2\Delta\nu t}$$

where t is time length of measurement in seconds.

∴ Fundamental laws give S/N advantage to direct detection of $\sqrt{\frac{e^{\frac{h\nu}{kT}}-1}{\varepsilon}}$

if $\frac{e^{h\nu/kT}-1}{\varepsilon} > 1$

Table

λ	$\sqrt{e^{\frac{h\nu}{kT}}-1}$ for $T = 283^\circ K$
1 cm	0.071
100 μm	0.81
11 μm	10
3 μm	4.8×10^3
0.6 μm	2.6×10^{18}

Practicalities

Bandwidth
pros and cons
factor of $\sqrt{\frac{100}{1/9}}$
for $100\ cm^{-1}$ bandwidth, $\Delta T < 10^{-6}\ ^\circ K$
for direct detection sensitivity
Spectral problems

Non fundamental noise
Leakage radiation — telescope noise
" current in detectors
L.O. scattering and cure

Present limits for direct detection at 10 μm
For 1 sec ave.
limit $\sim 2 \times 10^{-15}$ watts. for minimum $\Delta\nu$
= theoretical value for $\Delta\nu \simeq 100\ cm^{-1}$
for $\Delta\nu$ actually $= 100\ cm^{-1}$, limit $\sim 5 \times 10^{-14}$ W.
" " $= 10\ cm^{-1}$, " $\sim 1.3 \times 10^{-14}$ W

Other direct vs. heterodyne differences
Isolation of plane waves or point sources
by heterodyne detection
Isolation of fringe signal
Decrease vs. no decrease of signal with
multiple telescopes
Difference in practical averaging time
Complexities of each.