Michelson Interferometry Summer School. 9 Aug. 1999
Cal Tech.

Beam Combination & Fringe Measurement.

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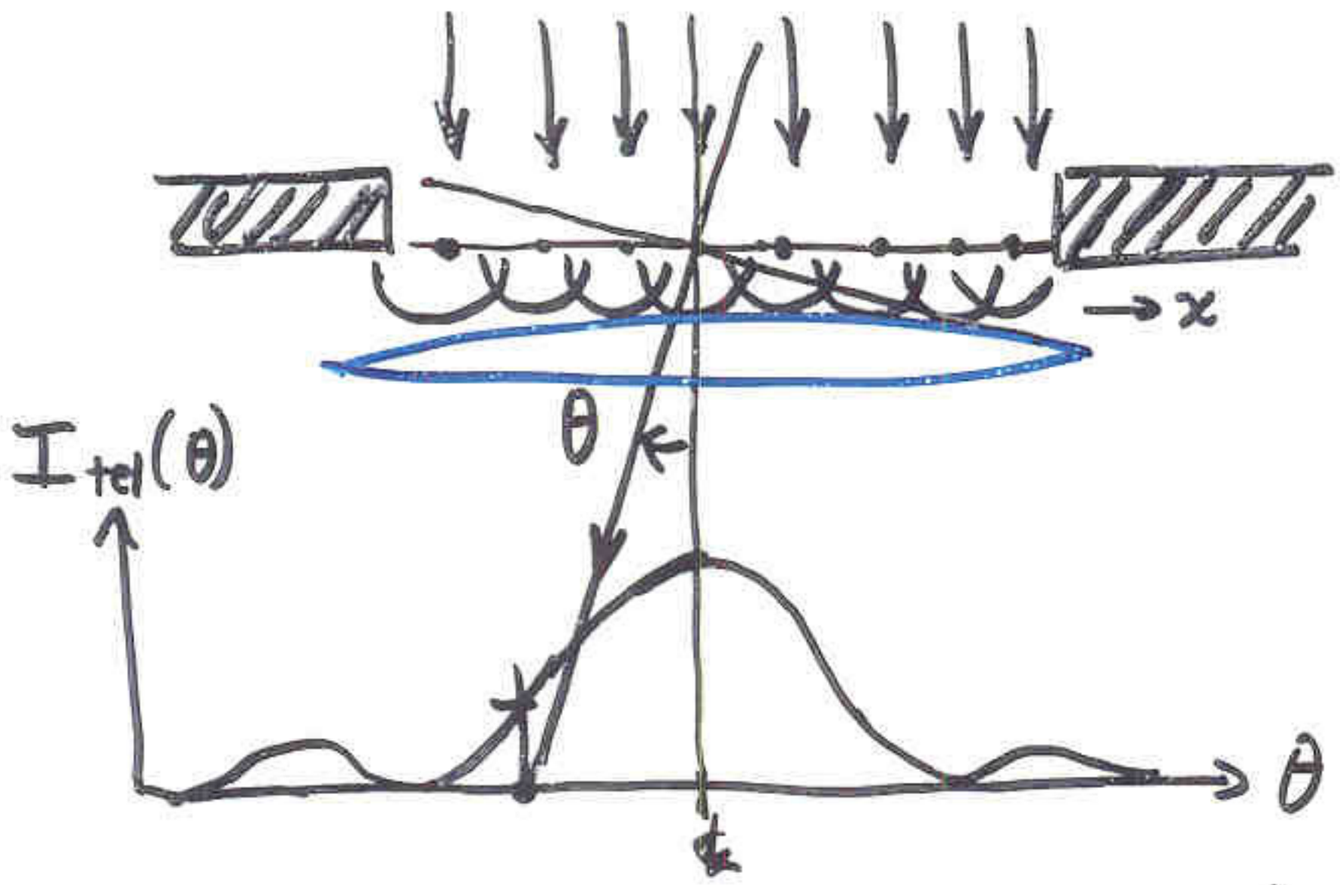
Goals of this talk:

1. Learn how to calculate interference of wavefronts for any optical system.
2. Learn how to separate the astrophysics from instrumental effects.

N.B. Homodyne detection (direct combination of beams) will be discussed. ($\therefore \lambda \lesssim 10\mu m$)

Heterodyne detection is discussed elsewhere at this school. ($\therefore \lambda \gtrsim 10\mu m$)

Derive single-telescope response to point source.



1-Dimensional aperture width = D

<u>amplitude.</u> $$A_{tel}(\theta) = \sum_{\text{wavelets}} = \int_D e^{i(\text{phase at } x)}\,dx$$

$$= \int_{-D/2}^{+D/2} e^{i\left(2\pi \frac{x\theta}{\lambda}\right)}\,dx \Big/ \int dx$$

$$= \frac{\lambda}{2\pi i\theta}\left[e^{+i\pi\theta D/\lambda} - e^{-i\pi\theta D/\lambda}\right] / D$$

$$= \frac{\sin(\pi\theta D/\lambda)}{(\pi\theta D/\lambda)}$$

<u>intensity.</u> $$I_{tel}(\theta) = |A_{tel}|^2 = \left[\frac{\sin(\pi\theta D/\lambda)}{\pi\theta D/\lambda}\right]^2$$

<u>1st zero.</u> $I_{tel}(\theta_{tel}) = 0$ when $\theta_{tel} = \lambda/D$

<u>circular aperture.</u> $$\int_{\text{circle}} \cdots \Rightarrow I_{tel}(\theta) = \left[\frac{2J_1(\pi\theta D/\lambda)}{\pi\theta D/\lambda}\right]^2$$

$$\theta_{tel} = 1.22\,\lambda/D$$

Single-telescope again.

add constant phase ϕ.

$$A_{tel}(\theta) = \int_{-D/2}^{+D/2} e^{i\left(2\pi \frac{x\theta}{\lambda} + \phi\right)} dx \quad / \int dx = \frac{\sin(\pi\theta D/\lambda)}{\pi\theta D/\lambda} \cdot e^{i}$$

I_{tel} is unchanged.

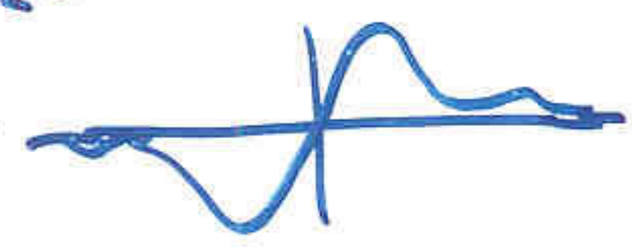
add off-axis angle θ_0.

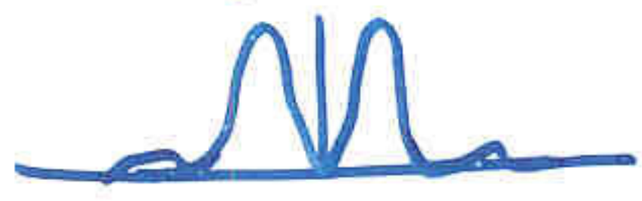
$$A_{tel}(\theta) = \int_{-D/2}^{+D/2} e^{i\left(2\pi x(\theta - \theta_0)/\lambda\right)} dx \quad / \int dx = \frac{\sin\left(\pi(\theta-\theta_0)D/\right.}{\pi(\theta-\theta_0)D/\lambda}$$

I_{tel} is shifted to center at θ_0.

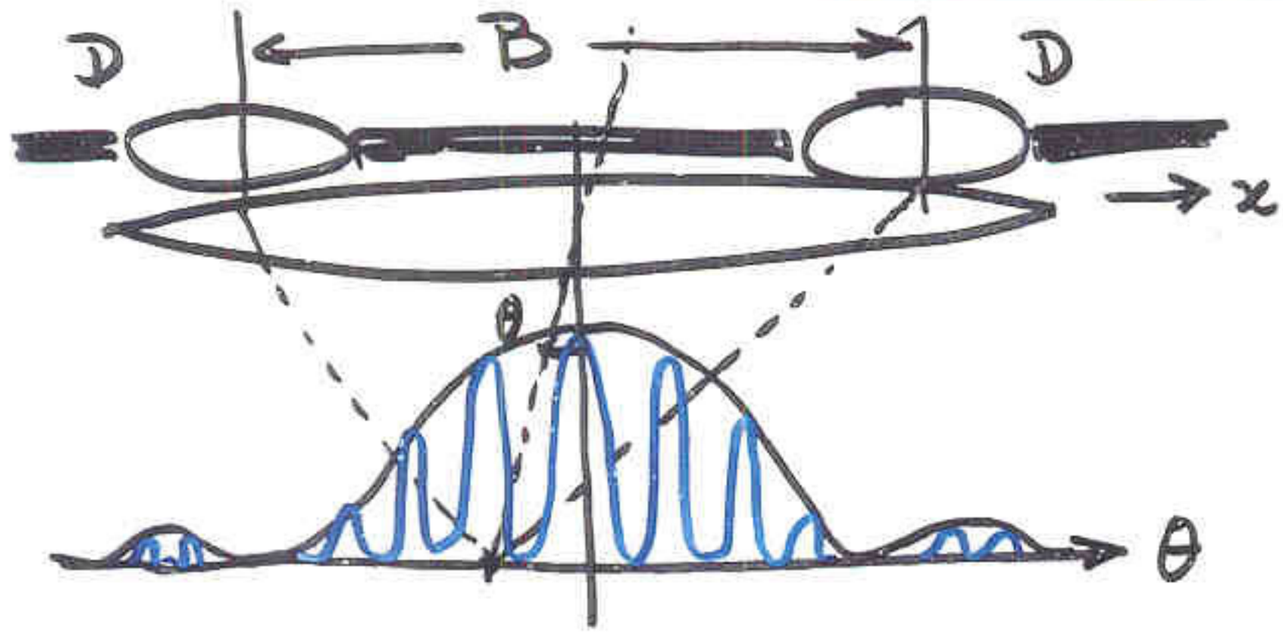
add phase step (across 1/2 aperture) of π.

$$A_{tel}(\theta) = \left[\int_{0}^{+D/2} e^{i(2\pi x\theta/\lambda)} dx + \int_{-D/2}^{0} e^{i(2\pi x\theta/\lambda - \pi/2)} dx\right] / \int$$

$=$ 

$I_{tel}(\theta) =$  i.e. 2 speckles.

Derive interferometer (2-tel.) response.



<u>amplitude.</u> $$A_{int}(\theta) = \sum_{wavelets} = \int e^{i(\text{phase at } x)}\,dx$$

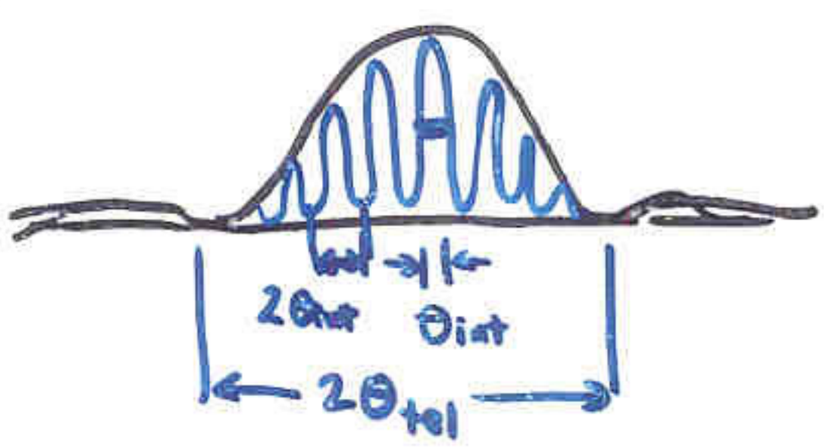
$$= \left[\int_{+\frac{1}{2}B-\frac{1}{2}D}^{+\frac{1}{2}B+\frac{1}{2}D} \sim\sim + \int_{-\frac{1}{2}B-\frac{1}{2}D}^{-\frac{1}{2}B+\frac{1}{2}D} \sim\sim \right] / 2D$$

$$= \frac{\sin(\pi\theta D/\lambda)}{\pi\theta D/\lambda} \cdot \cos(\pi\theta B/\lambda)$$

<u>intensity.</u> $$I_{int}(\theta) = I_{tel}(\theta) \cdot \tfrac{1}{2}\left[1 + \cos(2\pi\theta B/\lambda)\right]$$

<u>1st zero.</u> $I_{int}(\theta_{int}) = 0$ when $\theta_{int} = \frac{\lambda}{2B}$ = width of fringe.

<u>number of fringes in packet.</u> $$(\text{no. fringes}) = \frac{2\theta_{tel}}{2\theta_{int}} = \frac{1.22\lambda/D}{0.50\lambda/B} = 2.44\frac{B}{D}$$



Derive binary star response.

If 2 stars are separated by θ_{bin}, then intensity is

<u>intensity.</u>

$$I_{bin}(\theta) = I_{int}(\theta - \theta_{bin}/2) + I_{int}(\theta + \theta_{bin}/2)$$

$$\approx I_{tel}(\theta)\tfrac{1}{2}\Big[1 + \cos\big(2\pi(\theta + \theta_{bin}/2)/\lambda\big) + 1 + \cos\big(2\pi(\theta - \theta_{bin}/2)/\lambda\big)\Big]$$

$$= I_{tel}(\theta)\cdot\Big[1 + \underbrace{\Big(\cos\frac{\pi\theta_{bin}B}{\lambda}\Big)}_{V_{bin}}\cdot\cos\frac{2\pi\theta B}{\lambda}\Big]$$

<u>visibility of binary.</u>

$$V_{bin} = \cos(\pi\theta_{bin}B/\lambda)$$

<u>zero.</u>

$$V_{bin} = 0 \quad \text{when} \quad \theta_{bin} = \frac{\lambda}{2B}.$$

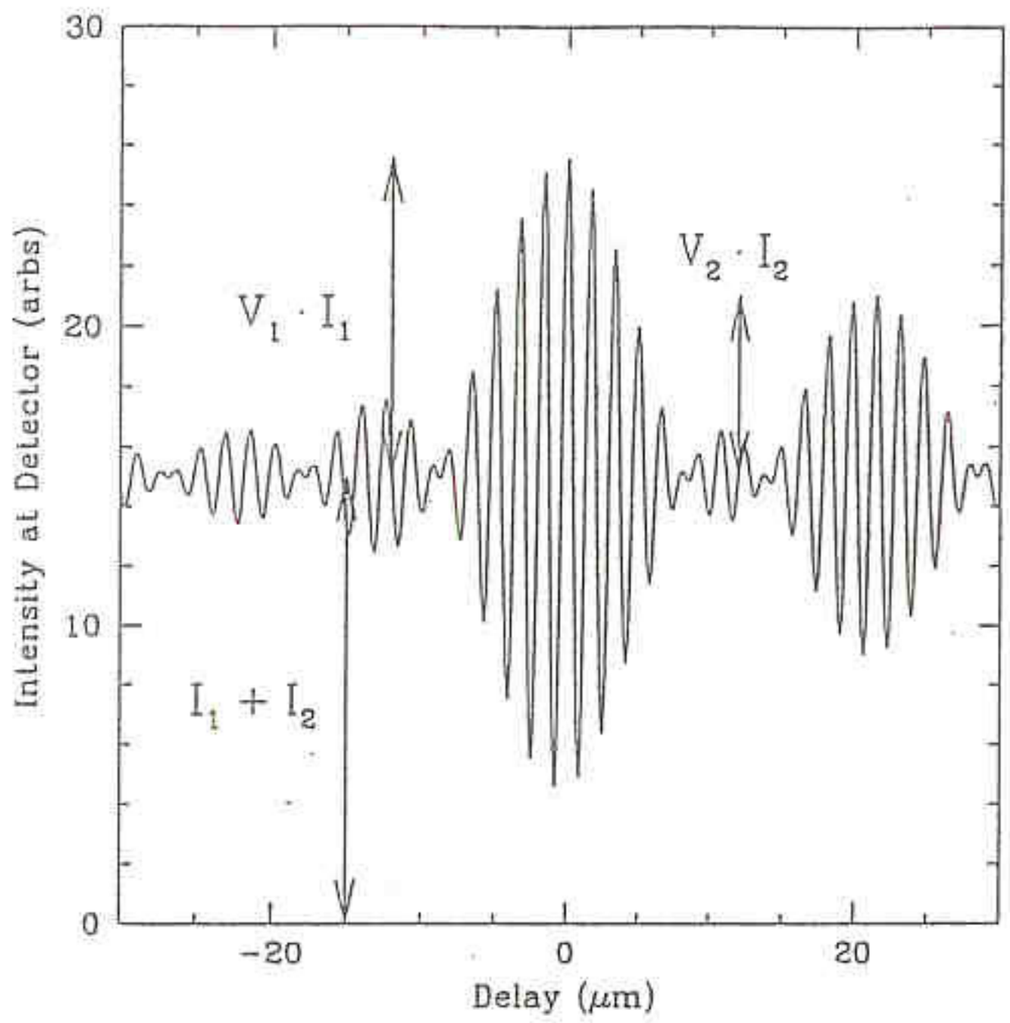
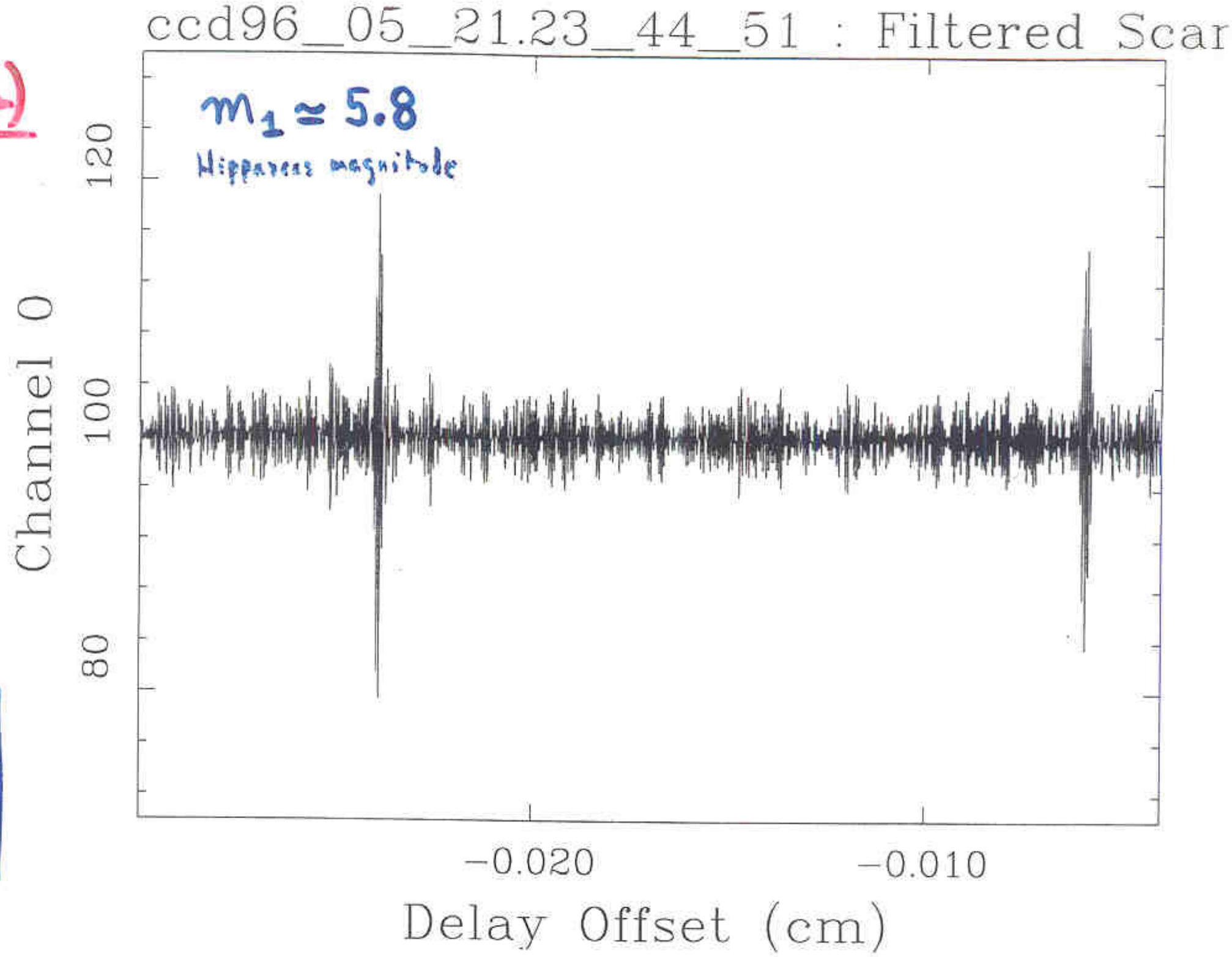


Figure 4.1. Wide binary fringe signal. This figure illustrates the problem discussed in the text, since the response has been calculated for two unresolved binary components ($V_1 = V_2 = 1.0$), yet if the amplitude of either fringe packet were measured with respect to the total mean flux, they would appear to have reduced visibility. theoretical calc.

HR 5727 / HIC 75312

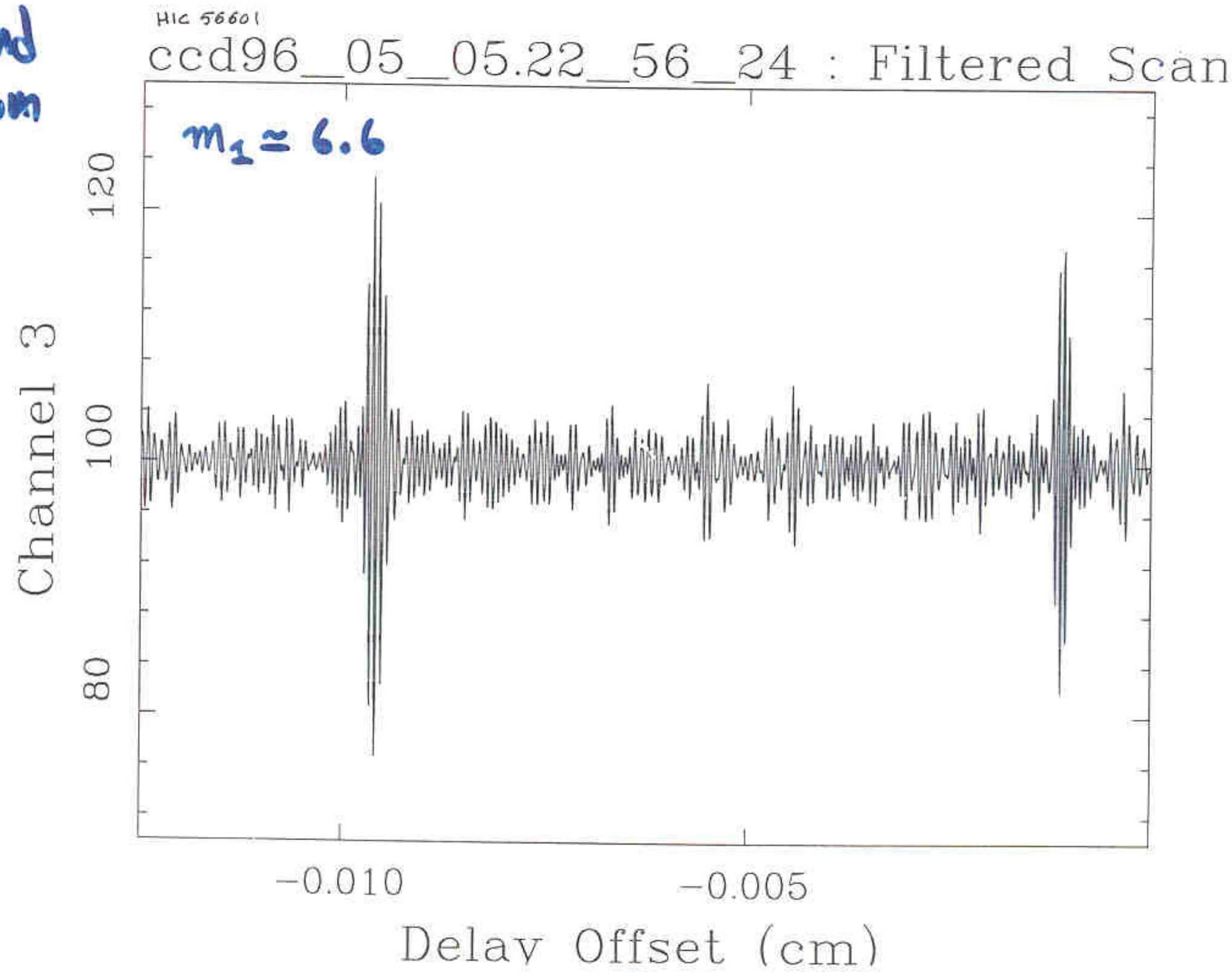
VRI
(.5-1.0 μm)



Binary Stars

IOTA
wide-band
≈ 0.5-1.0 μm

HIC 56601



Derive uniform disk response.

Add up (incoherent) fringe patterns from square disk:

<u>intensity</u>. $$I_{USD}(\theta) = \sum_{sq.\,disk} (\text{intensities}) = \int_{sq.\,disk} I_{int}(\theta-\theta_x)\cdot d\theta_x\, d\theta_y \Big/ \int d\theta$$

$$= \int_{-\theta_{disk}/2}^{+\theta_{disk}/2} I_{tel}(\theta)\cdot \tfrac{1}{2}\left[1+\cos 2\pi(\theta-\theta_x)B/\lambda\right]\cdot \theta_{disk}\cdot d\theta_x \Big/ \theta_{disk}^2$$

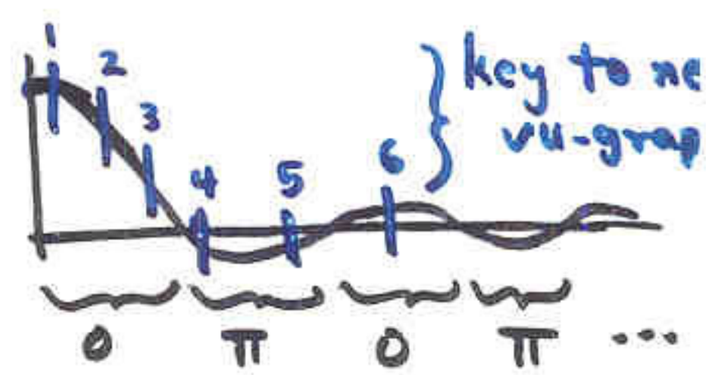
$$\simeq I_{tel}(\theta)\cdot\tfrac{1}{2}\Big[1+\underbrace{\left(\frac{\sin \pi B\theta_{disk}/\lambda}{\pi B\theta_{disk}/\lambda}\right)}_{V_{USD}}\cdot\cos\frac{2\pi\theta B}{\lambda}\Big]$$

<u>visibility</u>. $$V_{USD} = \frac{\sin(\pi B\theta_{disk}/\lambda)}{\pi B\theta_{disk}/\lambda}, \text{ square disk.}$$

$$V_{UD} = \frac{2J_1(\pi B\theta_{disk}/\lambda)}{\pi B\theta_{disk}/\lambda}, \text{ round disk.}$$

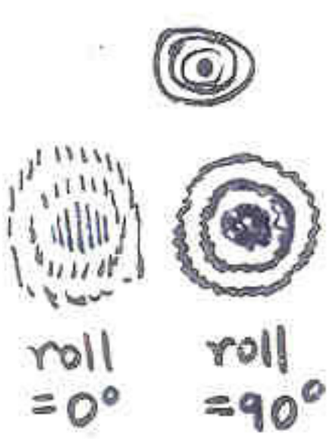
<u>1st zero</u>. $V_{UD} = 0$ when $\theta_{disk} = 1.22\,\lambda/B$, $B = 1.22\,\lambda/\theta_{disk}$

<u>phase</u>. $$\text{phase} = \begin{cases} 0 & \text{inside odd lobes} \\ \pi & \text{inside even lobes} \end{cases}$$



<u>example 1.</u> see next vu-graph from Born & Wolf.

<u>example 2.</u>



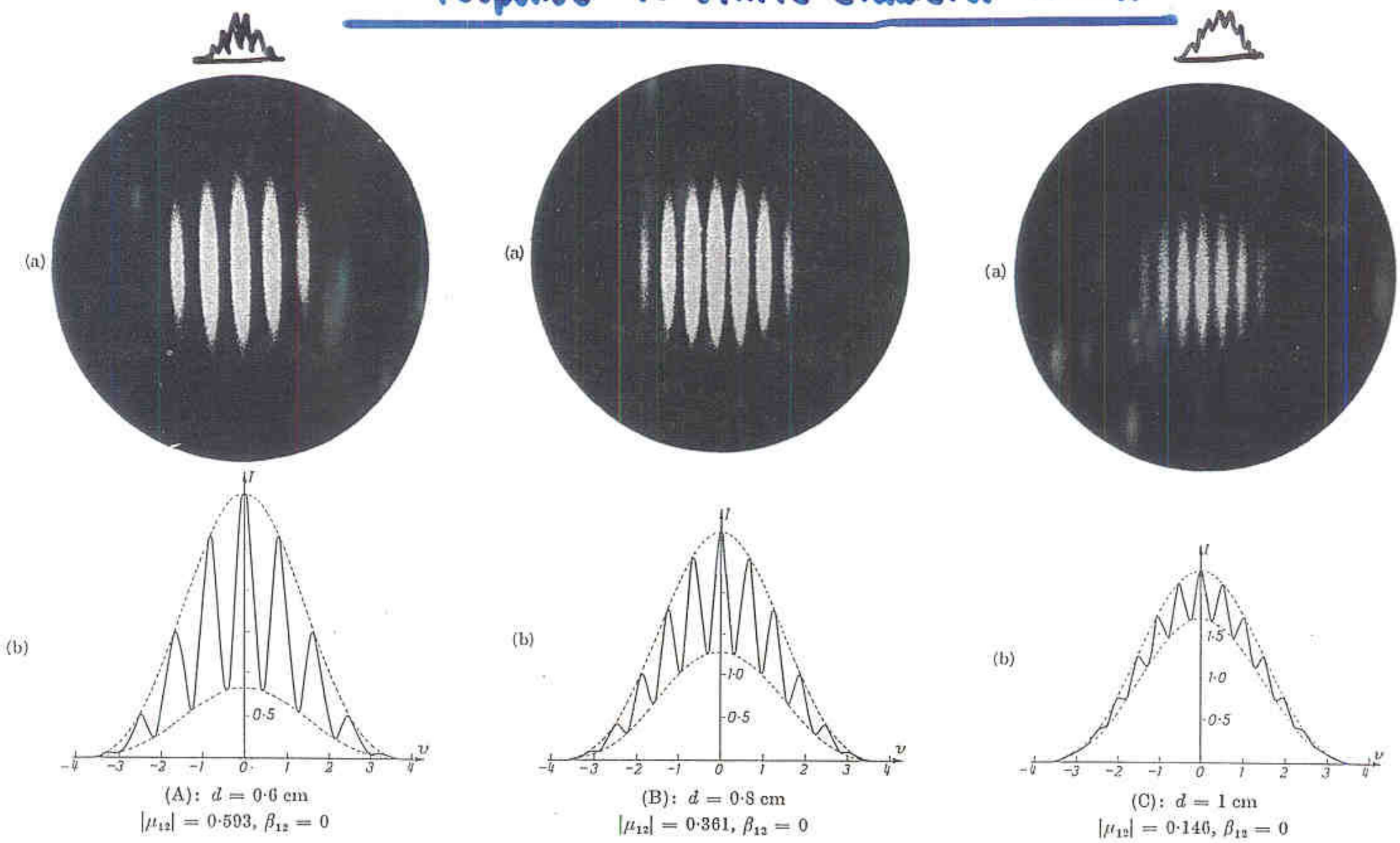
SIM pocket demonstration card.

$D \approx 0.07$ mm so $\theta_{tel} = 1.22\,\lambda/D \approx 2000.''$ ≈ sun, moon.

$B \approx 0.25$ mm so $\theta_{int} = \frac{\lambda}{2B} \approx 200.''$ ≈ Mag-light at 6'-18 inches.

(# fringes in packet) $= 2.44\,B/D \approx 8.$

Measured & theoretical 2-aperture response to finite diameter source.



(A): $d = 0{\cdot}6$ cm
$|\mu_{12}| = 0{\cdot}593$, $\beta_{12} = 0$

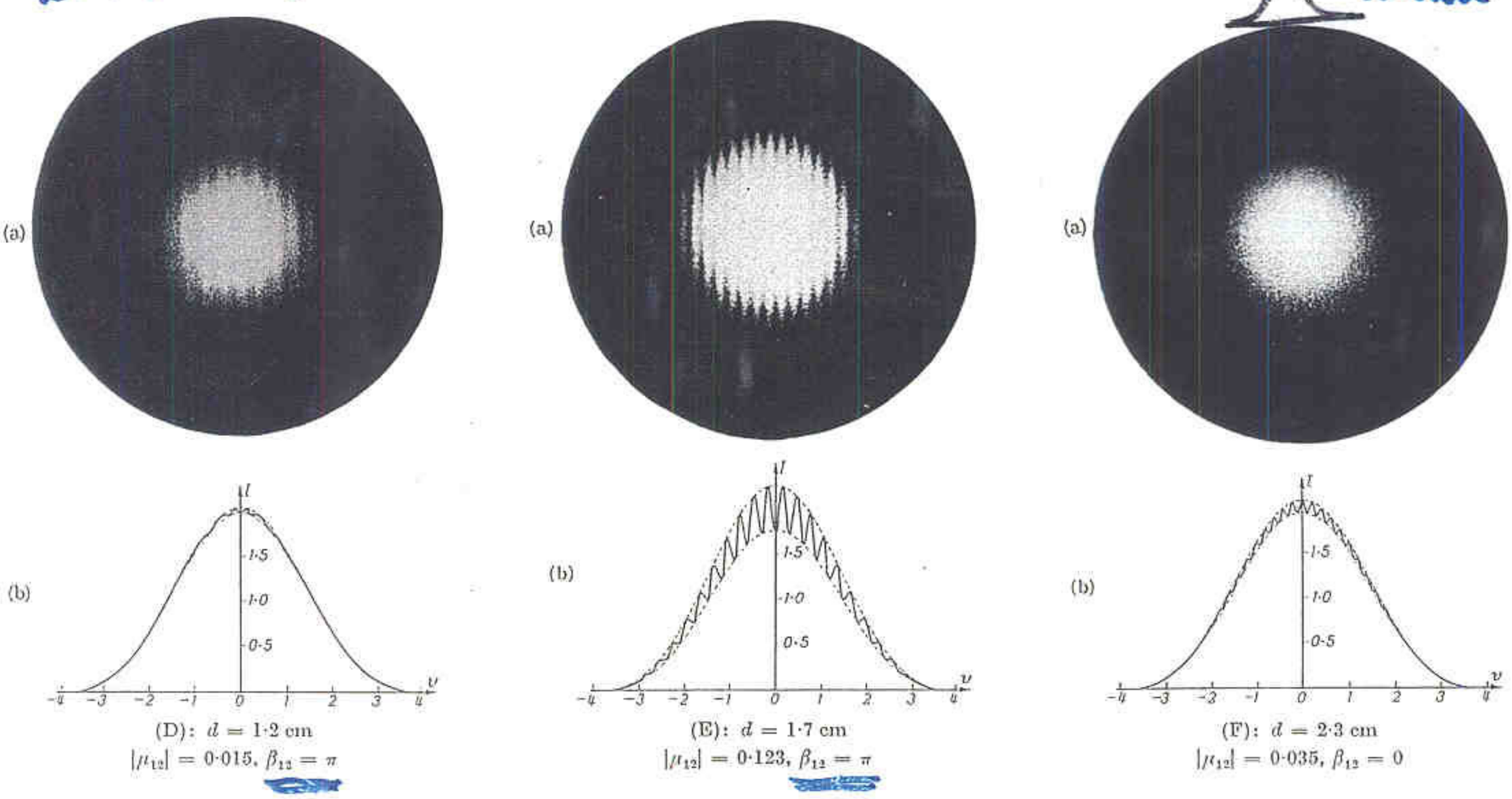
(B): $d = 0{\cdot}8$ cm
$|\mu_{12}| = 0{\cdot}361$, $\beta_{12} = 0$

(C): $d = 1$ cm
$|\mu_{12}| = 0{\cdot}146$, $\beta_{12} = 0$

near 1st zero

within 1st sidelobe

2nd sidelobe



(D): $d = 1{\cdot}2$ cm
$|\mu_{12}| = 0{\cdot}015$, $\beta_{12} = \pi$

(E): $d = 1{\cdot}7$ cm
$|\mu_{12}| = 0{\cdot}123$, $\beta_{12} = \pi$

(F): $d = 2{\cdot}3$ cm
$|\mu_{12}| = 0{\cdot}035$, $\beta_{12} = 0$

Fig. 10.6. Two-beam interference with partially coherent light.

(a) Observed patterns, (b) theoretical intensity curves. Focal length of lenses L_0, L_1, and L_2 of Diffractometer: $f_0 = 20$ cm, $f_1 = f_2 = R = 152$ cm. Diameter of $L_0 = 5$ cm. Distance from L_0 to σ_1: 40 cm. Separation of L_1 and L_2: 14 cm. Distance of mirror M from $L_2 = 85$ cm. Diameter $2\rho_1$ of pinhole σ_1: $0{\cdot}9 \times 10^{-2}$ cm. Diameter $2a$ of apertures at P_1 and P_2: 0·14 cm. Mean wavelength $\bar{\lambda} = 5790$ Å.

[After B. J. Thompson and E. Wolf, *J. Opt. Soc. Amer.*, 47 (1957), 895.]

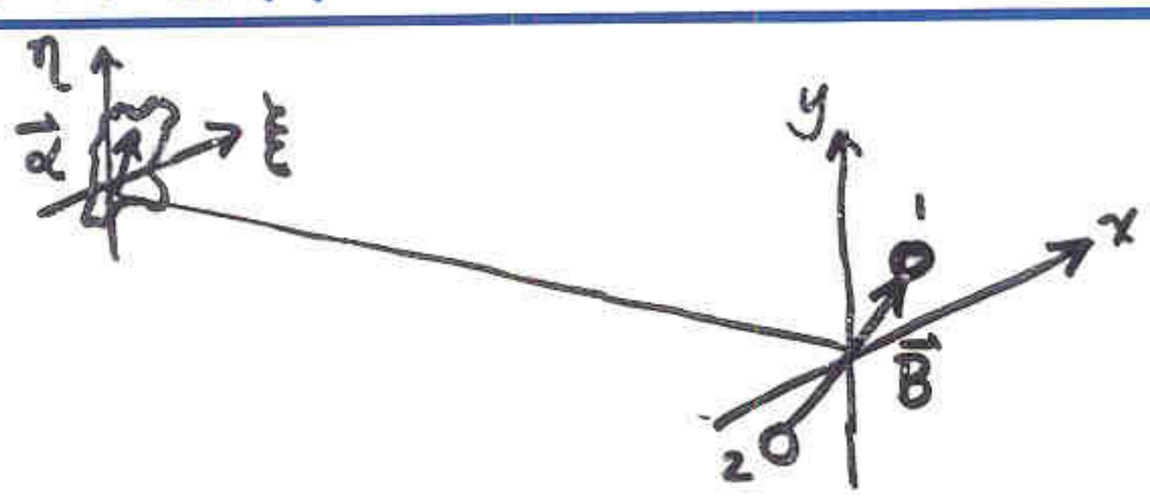
Beam pattern on sky.

Think like a radio astronomer.

The antenna pattern is considered to be projected out from the receiver horn ~~a~~ antenna ~~a~~ array onto the sky. As you move the antenna, or change the phase at an array element, the pattern sweeps across the sky. The received signal is the convolution of the moving pattern and the sources in the sky.

A sinusoidal pattern picks out the Fourier component at that spacing of fringes.

van Cittert – Zernike theorem (1934, 1938).



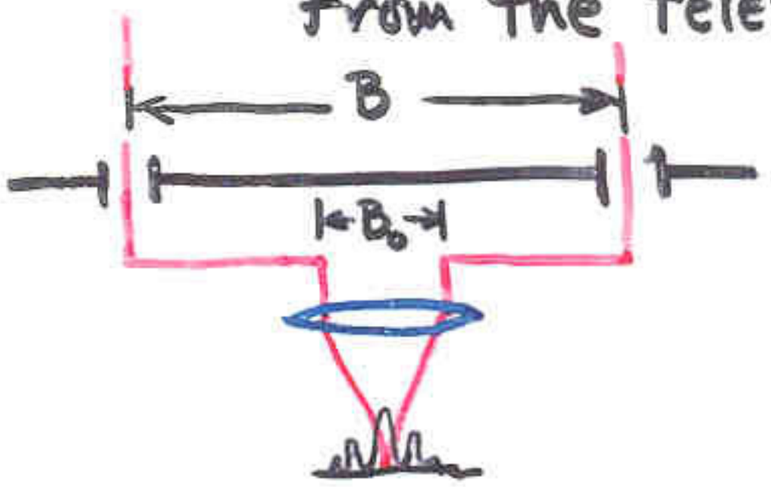
<u>complex degree of coherence.</u> $$\mu_{12}(\vec{B}) = \int_{FOV} I(\vec{\alpha}) \cdot e^{-ik\vec{B}\cdot\vec{\alpha}} \cdot d\vec{\alpha} \Big/ \int_{FOV} I(\vec{\alpha}) \cdot d\vec{\alpha}$$

<u>degree of coherence.</u> $|\mu| \equiv V$ = visibility ; <u>phase</u> = $\arg(\mu)$.

<u>inverse relation.</u> $$I(\vec{\alpha}) \Big/ \int_{FOV} I(\vec{\alpha}) d\vec{\alpha} = \int_{\text{all } \vec{B}} \mu(\vec{B}) \cdot e^{+ik\vec{B}\cdot\vec{\alpha}} \cdot d\vec{B}$$

Michelson's stellar interferometer.

Suppose we decouple the collecting apertures at B from the telescope feed apertures at B_0.



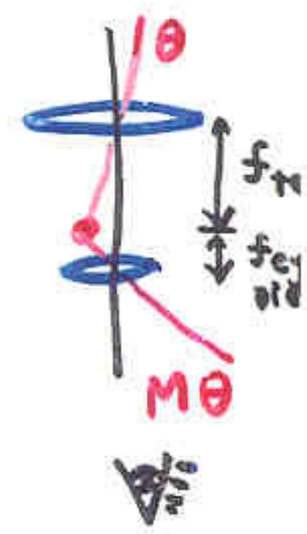
The coherence is measured by B.

The display pattern is set by B_0.

$$I_{int}(\theta) = \underbrace{I_{tel}(\theta)}_{\text{envelope vs } \theta} \cdot \frac{1}{2}\left[1 + \underbrace{\left(\frac{\sin \pi B \theta_{disk}/\lambda}{\pi B \theta_{disk}/\lambda}\right)}_{\text{degree of modulation indep. of } \theta} \cdot \underbrace{\cos\left(2\pi \theta B_{tel}/\lambda\right)}_{\text{modulation vs } \theta \text{ with period indep. of } B \text{ and } \theta_{disk}}\right]$$

<u>magnification.</u> So can make B_0 any convenient value. Michelson used $B_0 = 1.14$

so the fringe width is $\theta_{int} = \frac{\lambda}{2B_0} = 0.045$ arcsec.

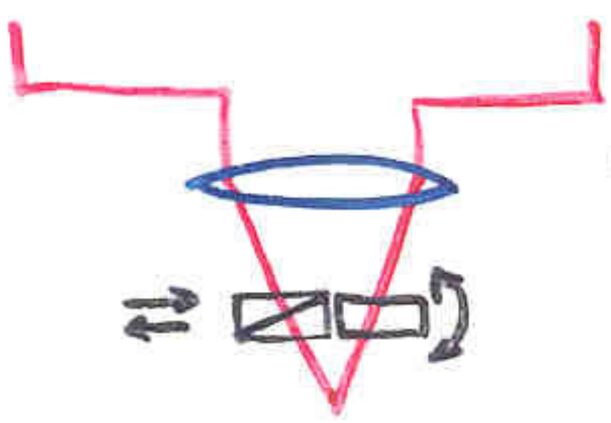


Assume his eye had $\theta_{eye} \simeq 1.22 \frac{\lambda}{5\,mm} \simeq 25.$ arcsec.

Magnify θ_{int} to match θ_{eye} with eyepiece M,

$$M = \theta_{eye}/\theta_{int} \simeq \frac{25}{.045} \approx 600.$$

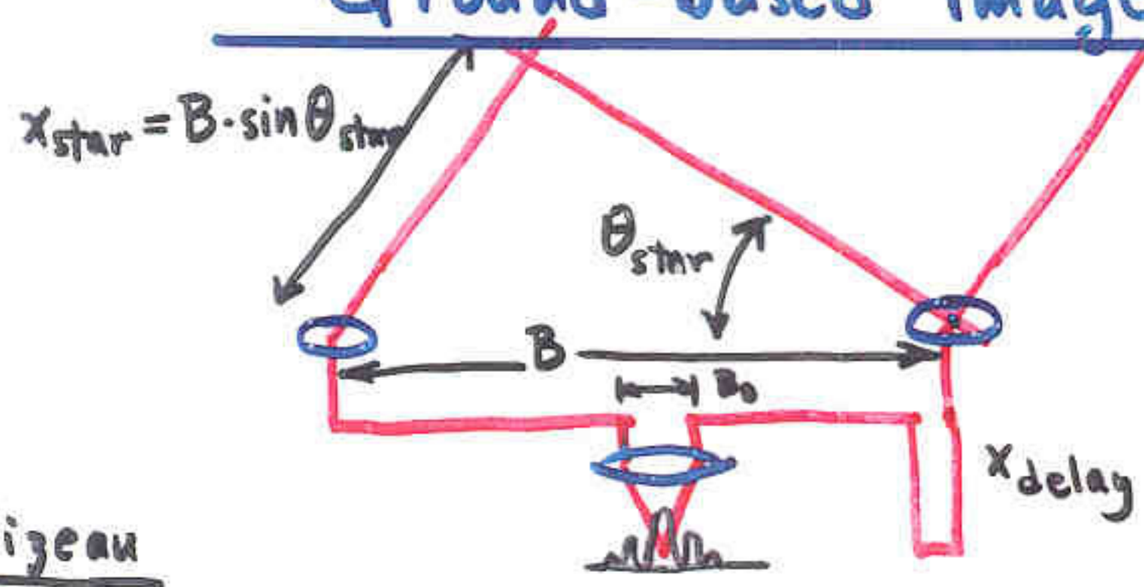
<u>details.</u>



Tilt plate gives angle motion,
& superposes images,
i.e., makes wavefronts parallel at entrance pupil.

Wedge plates give variable thickness,
to compensate for tilt plate's thickness,
i.e., makes all color wavefronts arrive at same time as in other beam.

Ground-based image-plane interferometer.



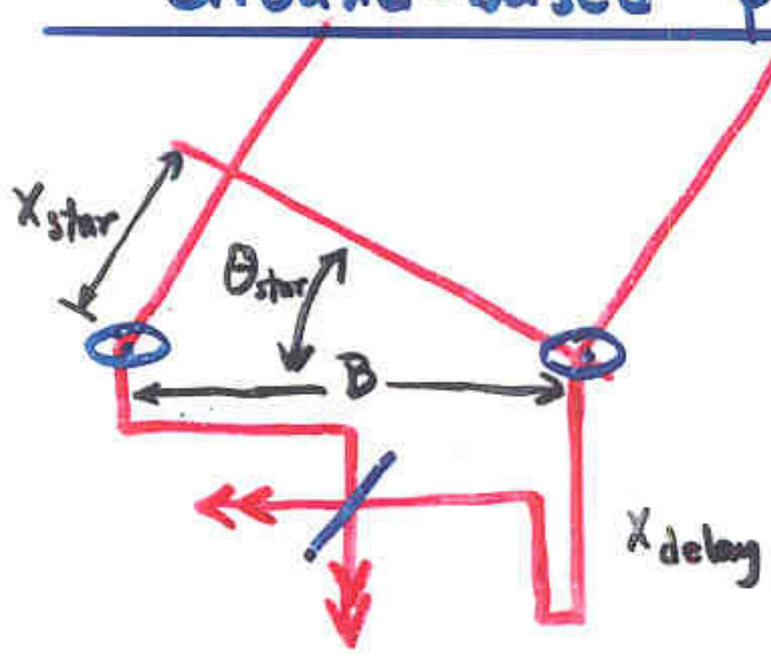
Phase difference between beams

$$\phi = \frac{2\pi}{\lambda}\left(x_{delay} - x_{star}\right)$$

Figeau type.

$$I_{int}(\theta) = \underbrace{I_{tel}(\theta)}_{\text{envelope vs } \theta} \cdot \frac{1}{2}\left[1 + \underbrace{V}_{\text{visib. of star}} \cdot \cos\frac{2\pi}{\lambda}\left(\underbrace{\theta B_0}_{\text{fringe modulation}} + \underbrace{x_{delay} - x_{star}}_{\text{fringe phase or position.}}\right)\right]$$

Ground-based pupil-plane interferometer.



Phase difference between beams

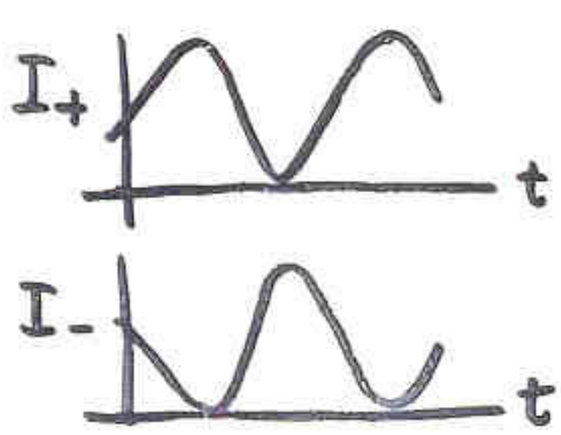
$$\phi = \frac{2\pi}{\lambda}\left(x_{delay} - x_{star}\right) + \frac{\pi}{2}$$

conservation of energy at lossless beamsplitter

Michelson type.

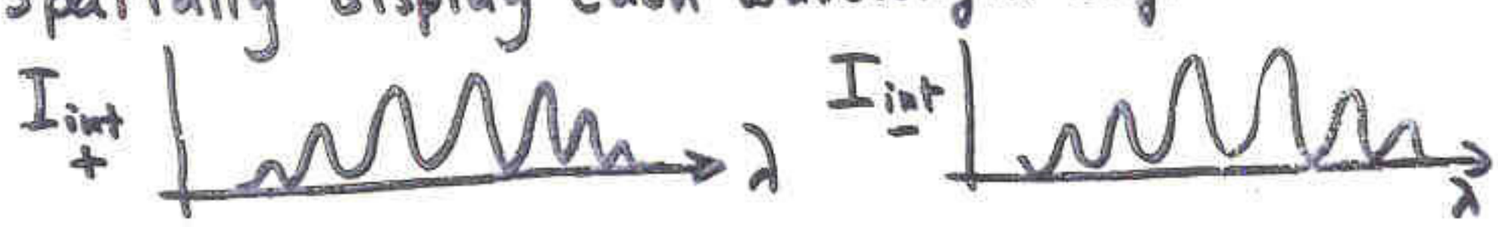
$$I_{int}(t) = I_{tel}(\theta) \cdot \frac{1}{2}\left[1 \pm V \cdot \sin\frac{2\pi}{\lambda}\left(x_{delay} - x_{star}\right)\right]$$

Note: B_0 here is zero; use time modulation $x_{delay}(t) - x_{star}(t) = vt$ and 1 pixel each for $I_\pm$.



Channel spectrum.

Spatially display each wavelength segment of I_{int}, with delay $\approx$ few λ.



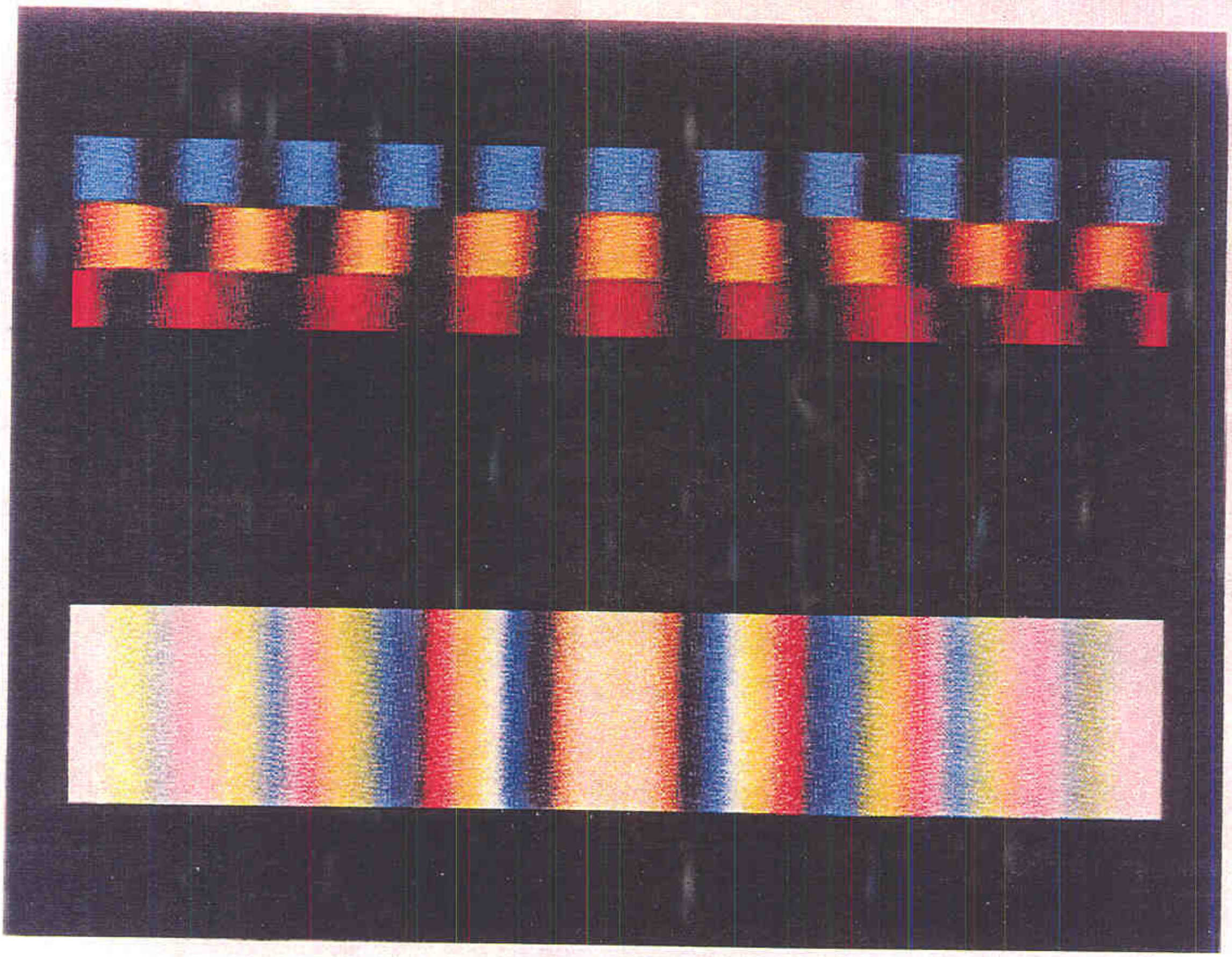
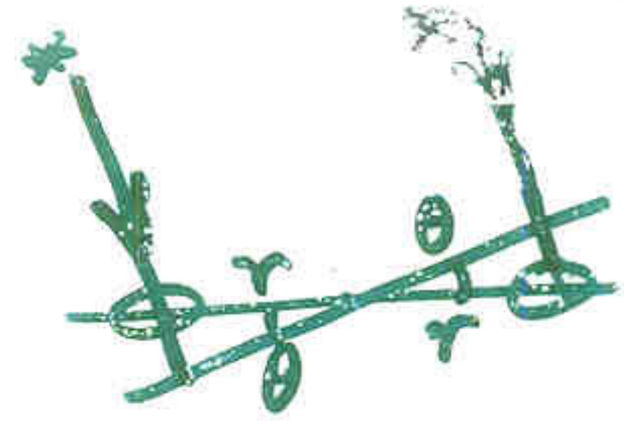


PLATE II

Color plate, A. Michelson.



Standard Nulling

Assume that an ideal achromatic null can be arranged. Then the intensity is

$$I_\pm = \left| e^{ik\theta r} \pm e^{-ik\theta r} \right|^2$$

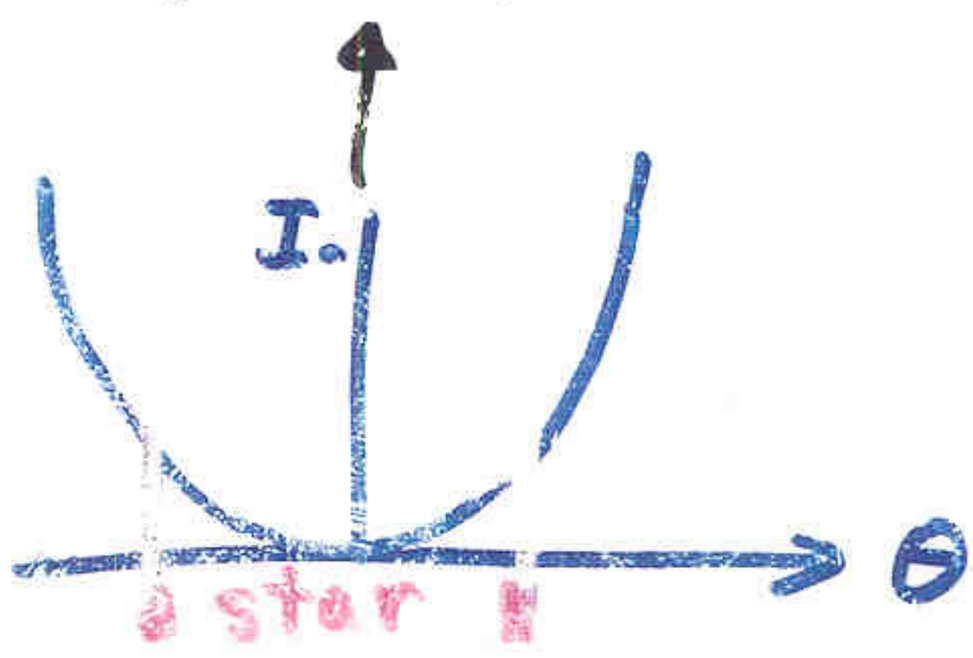
$$= 2\cdot\left[1 \pm \cos 2k\theta r\right]$$

The complementary outputs are the bright & null fringes, resp.

Near the null, the intensity is quadratic

$$I_-(\theta) \approx 2\cdot\left[1 - \left(1 - \frac{(2k\theta r)^2}{2!} + \cdots\right)\right]$$

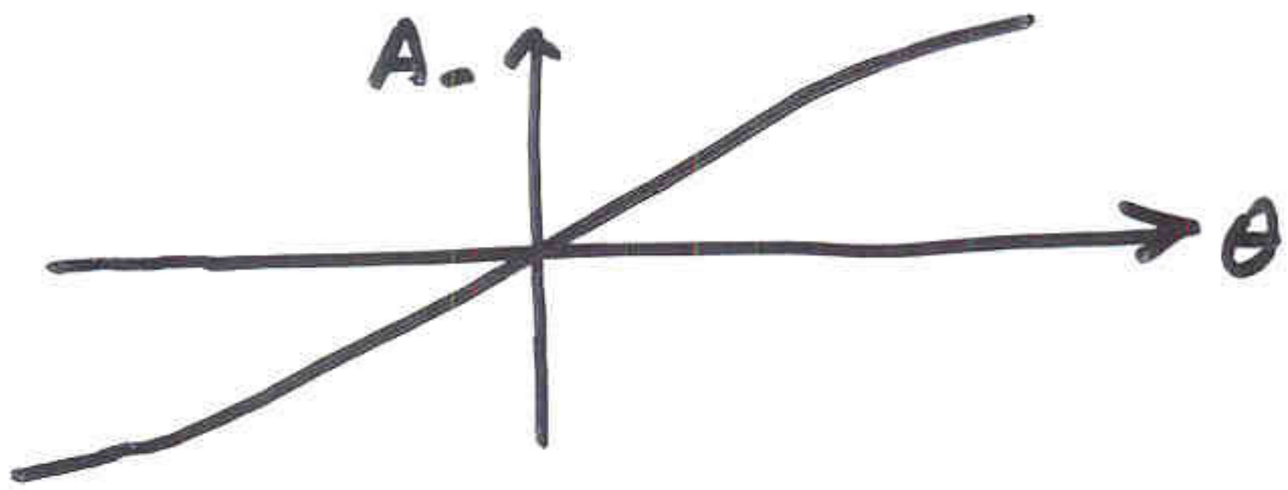
$$\simeq (2k\theta r)^2 - \cdots$$



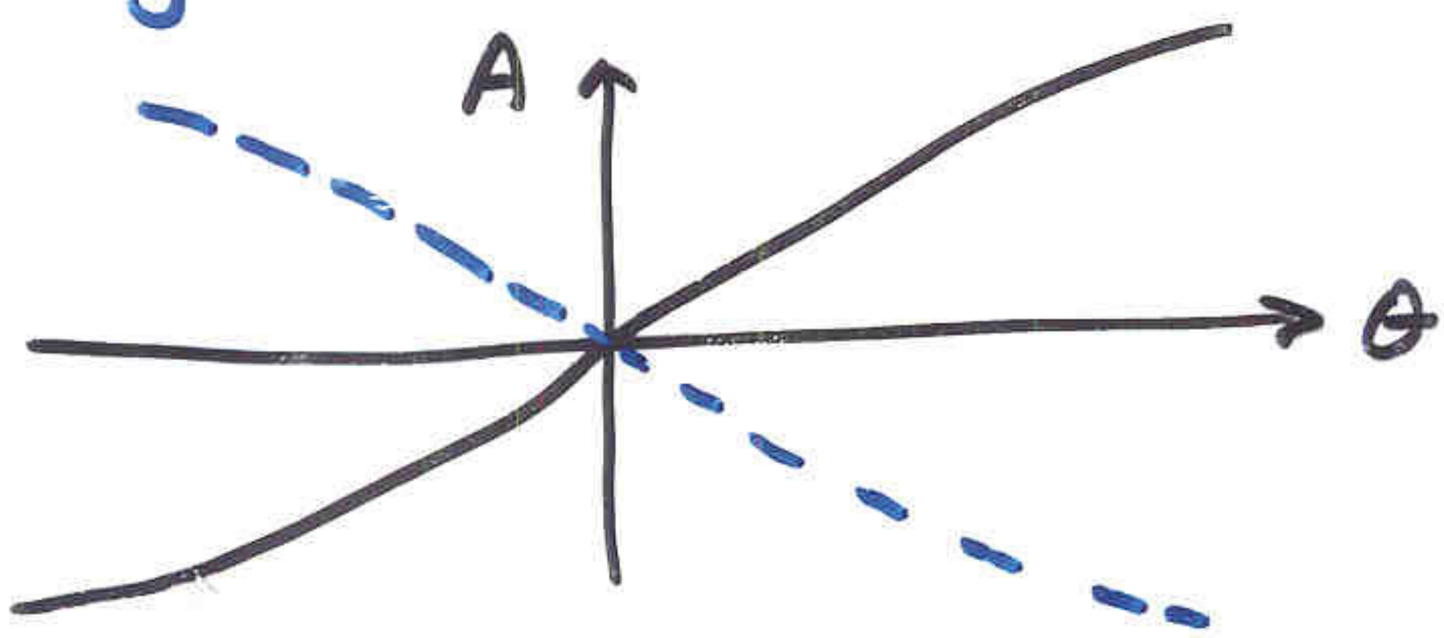
Super Nulling.

The amplitude (electric vector) in a standard null is

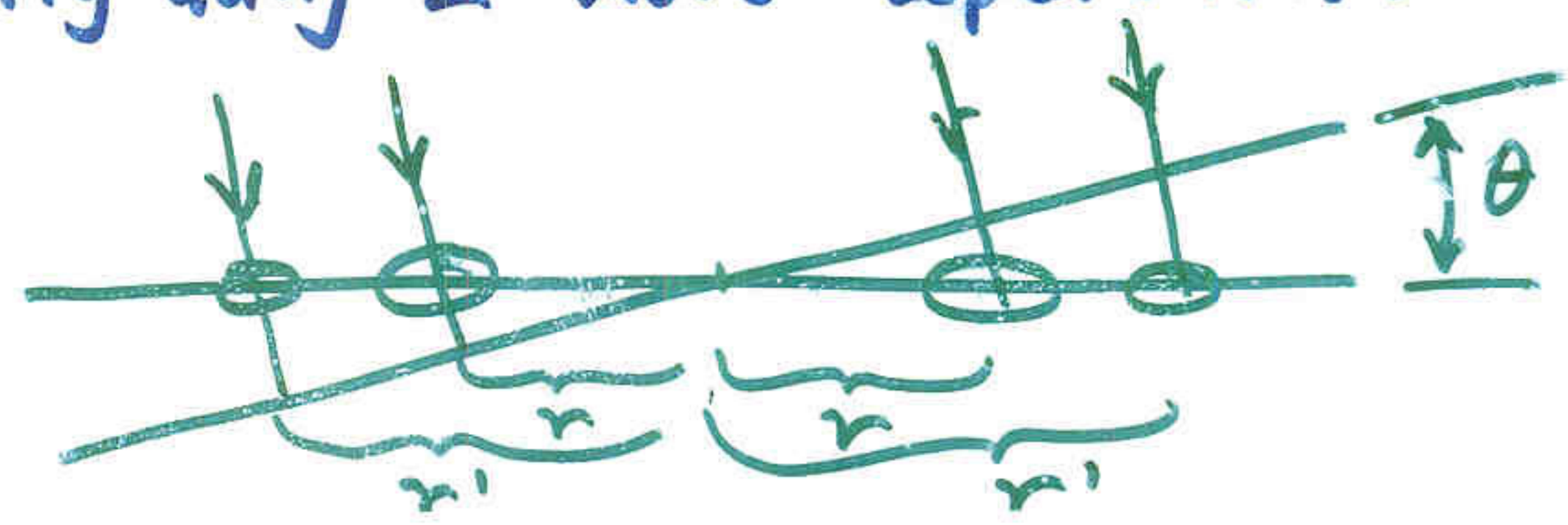
$$A_- = e^{+ik\theta r} - e^{-ik\theta r}$$
$$= 2i \cdot \sin(k\theta r).$$



If we could cancel this amplitude with one of opposite sign, we could make a very wide null.



Try using 2 more apertures:



(super nulling)

The amplitude from all 4 apertures is, assuming the outer ones have relative weight ε:

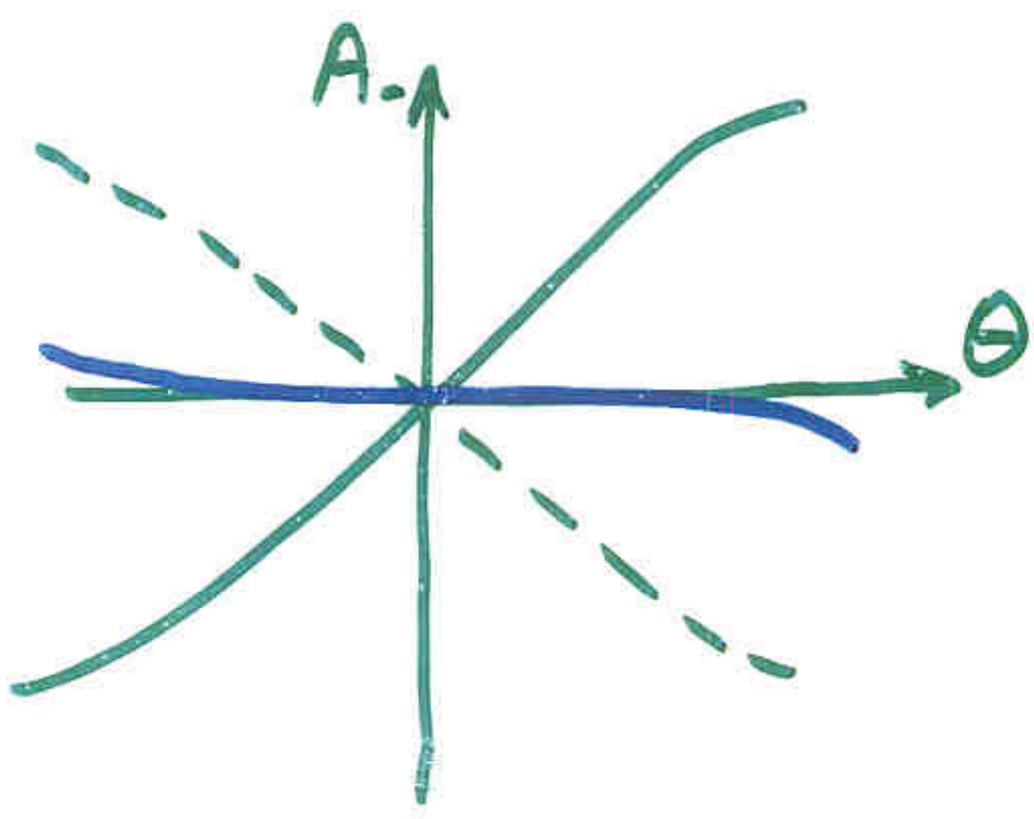
$$A_- = -i e^{+ik\theta r'} + e^{+ik\theta r} - e^{-ik\theta r} - (-\varepsilon e^{-ik\theta r'})$$

$$= 2i \cdot \left\{ \sin(k\theta r) - \varepsilon \sin(k\theta r') \right\}$$

where we have added exactly π retardation to flip the sign.

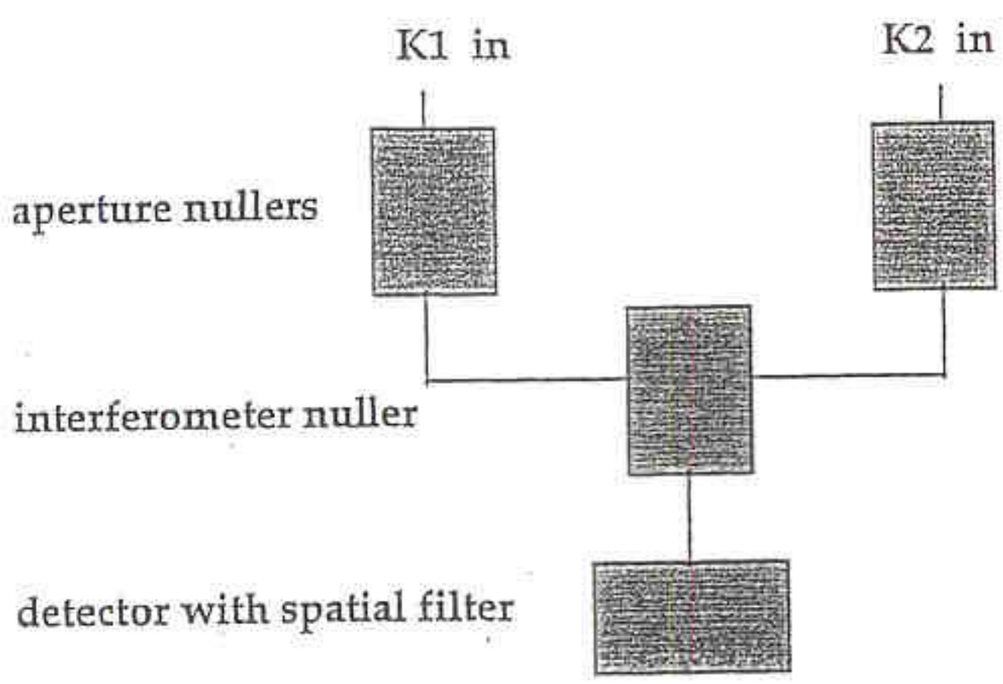
$$A_- \simeq \underbrace{k\theta(r - \varepsilon r')}_{=0 \text{ if } \varepsilon = r/r'} - \frac{(k\theta)^3}{3!}(r^3 - \varepsilon r'^3) + \cdots$$

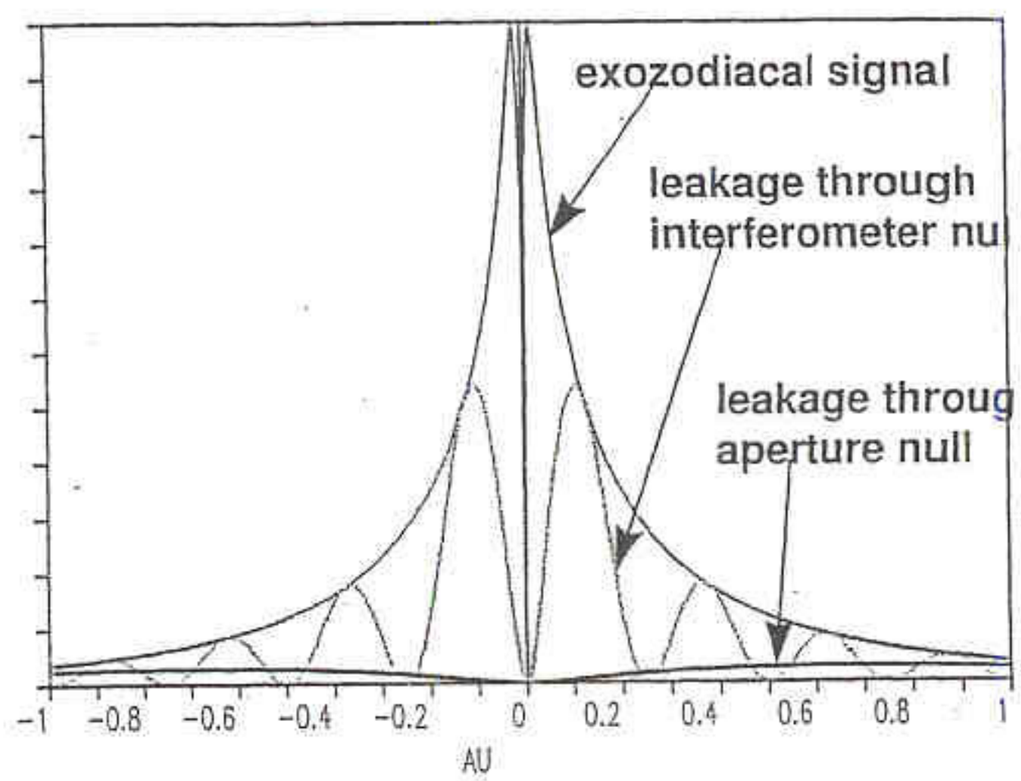
Then $A_- \simeq -\frac{(k\theta r)^3}{3!} \cdot (1 - 1/\varepsilon^2) + \cdots$



So 2 extra mirrors, or petals, will work, provided that phase control exists.

mas
200 mas


K1 in
K2 in
aperture nullers
interferometer nuller
detector with spatial filter


exozodiacal signal
leakage through
interferometer nu
leakage throug
aperture null
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
AU

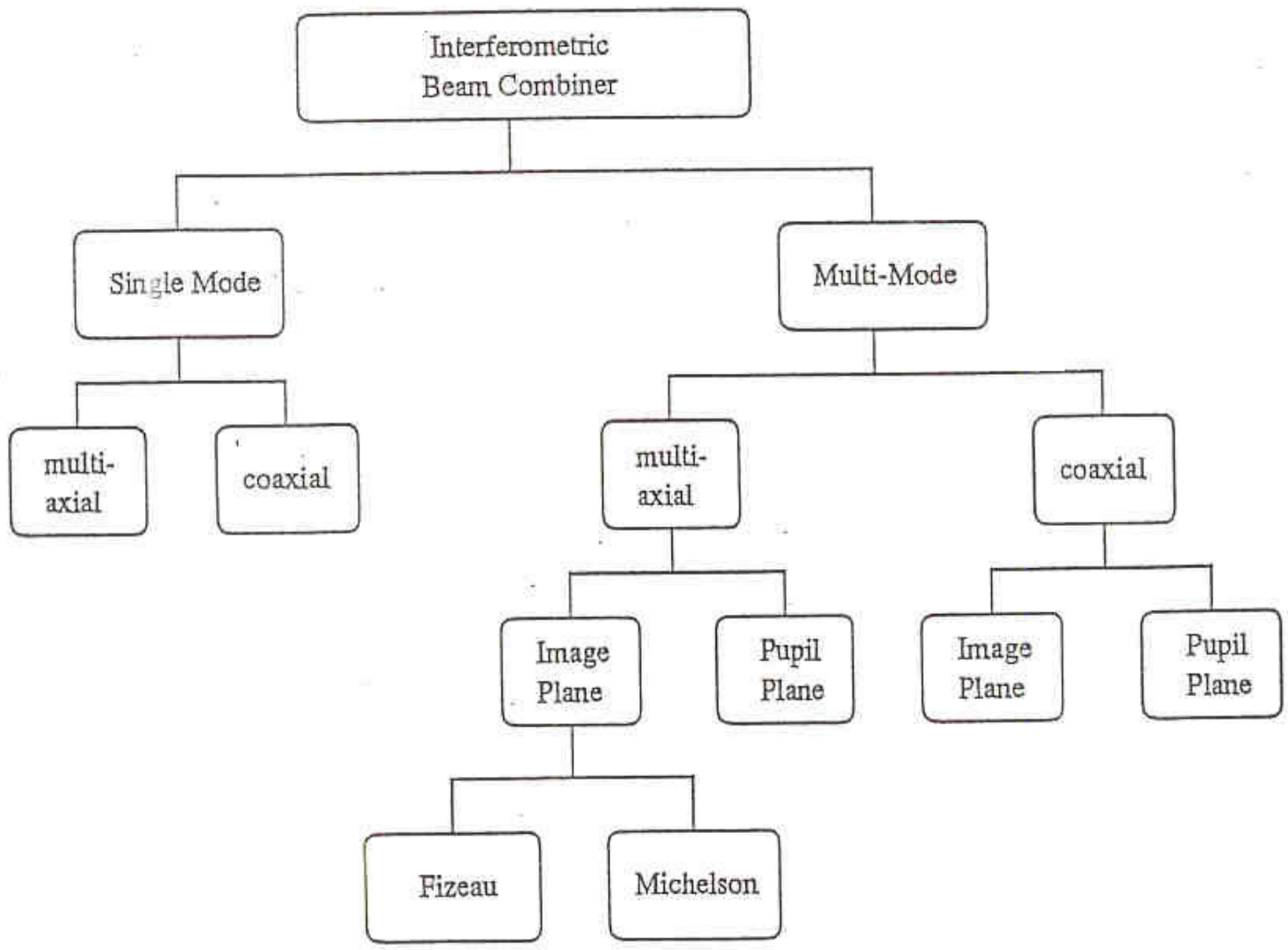
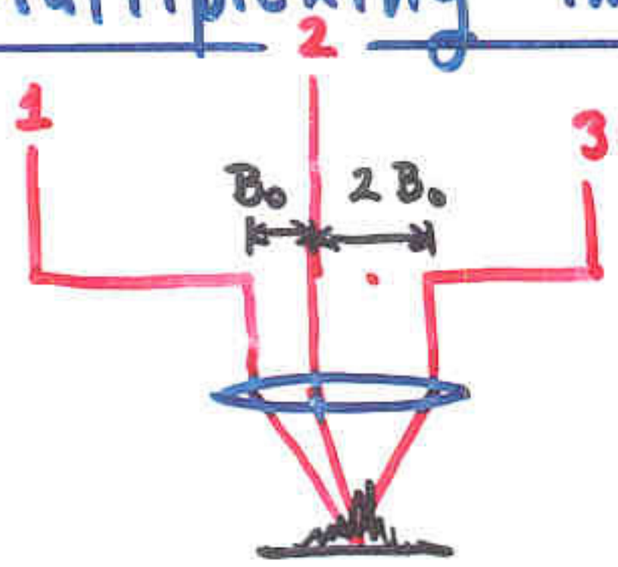


Figure 4.5: Decision tree for field mixing in an astronomical interferometer.

Multiplexing image-plane interferometer.

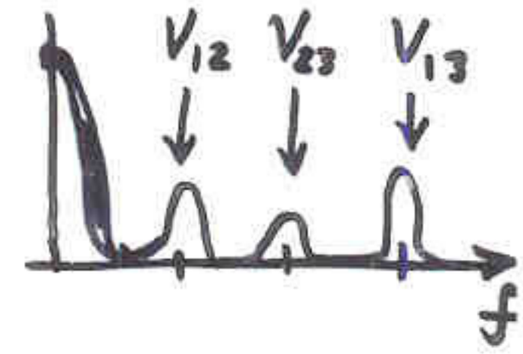


Use minimum redundancy array at combiner lens.

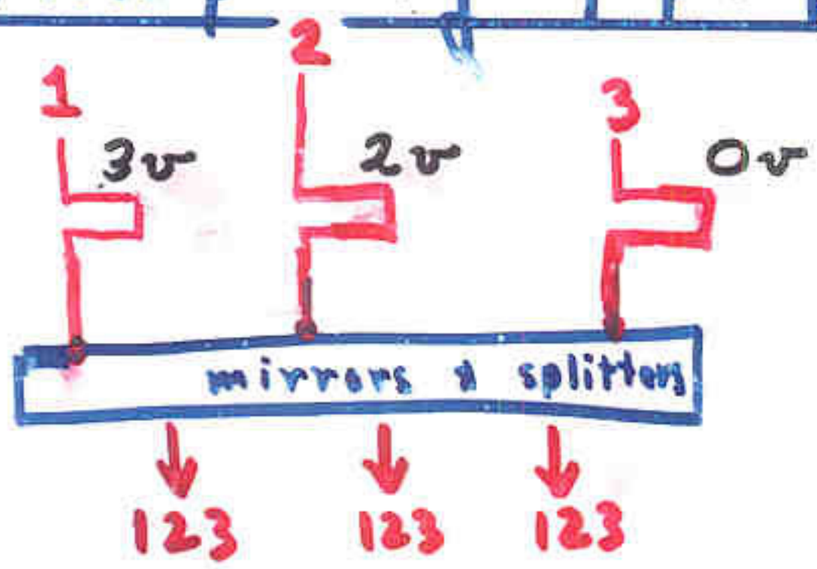
Get different spatial frequency for each baseline.

Fizeau type.

$|\mathrm{FFT}(\text{fringe pattern})|^2 =$ power spectral density



Multiplexing pupil-plane interferometer.

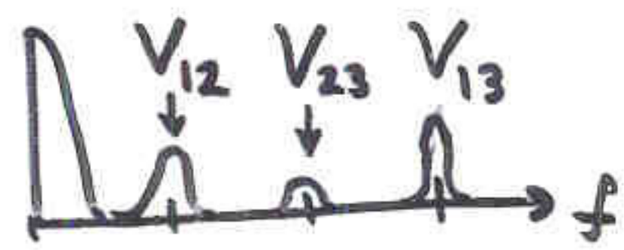


Use different delay-line speeds.

Mix beams & get different time frequencies in each.

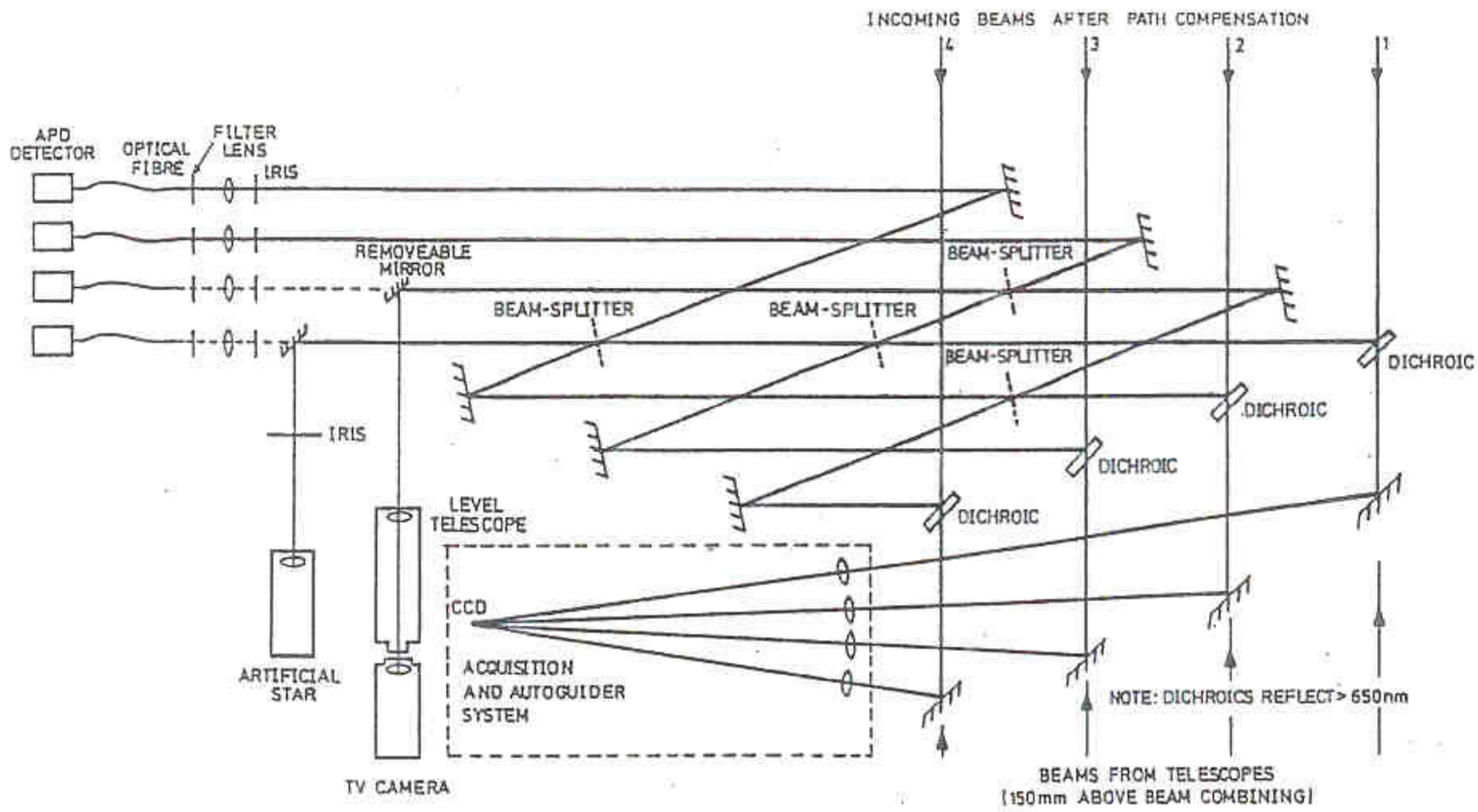
Michelson type.

$|\mathrm{FFT}(\text{each time sequence})|^2 =$ power

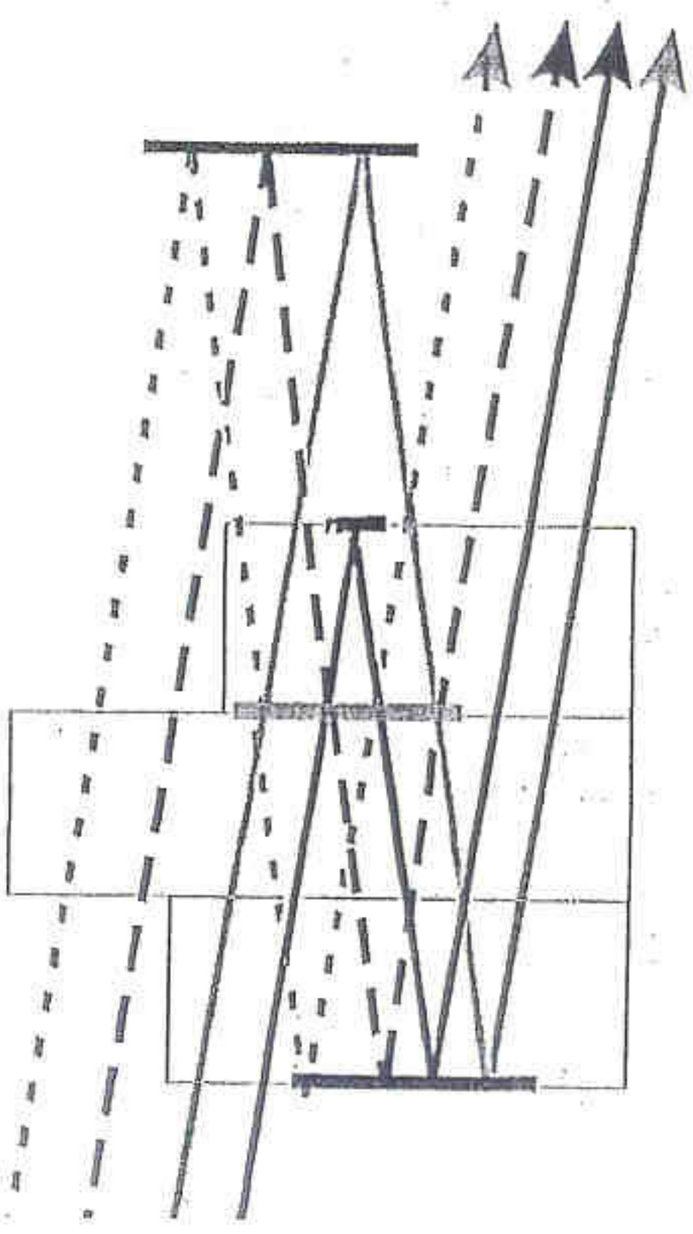


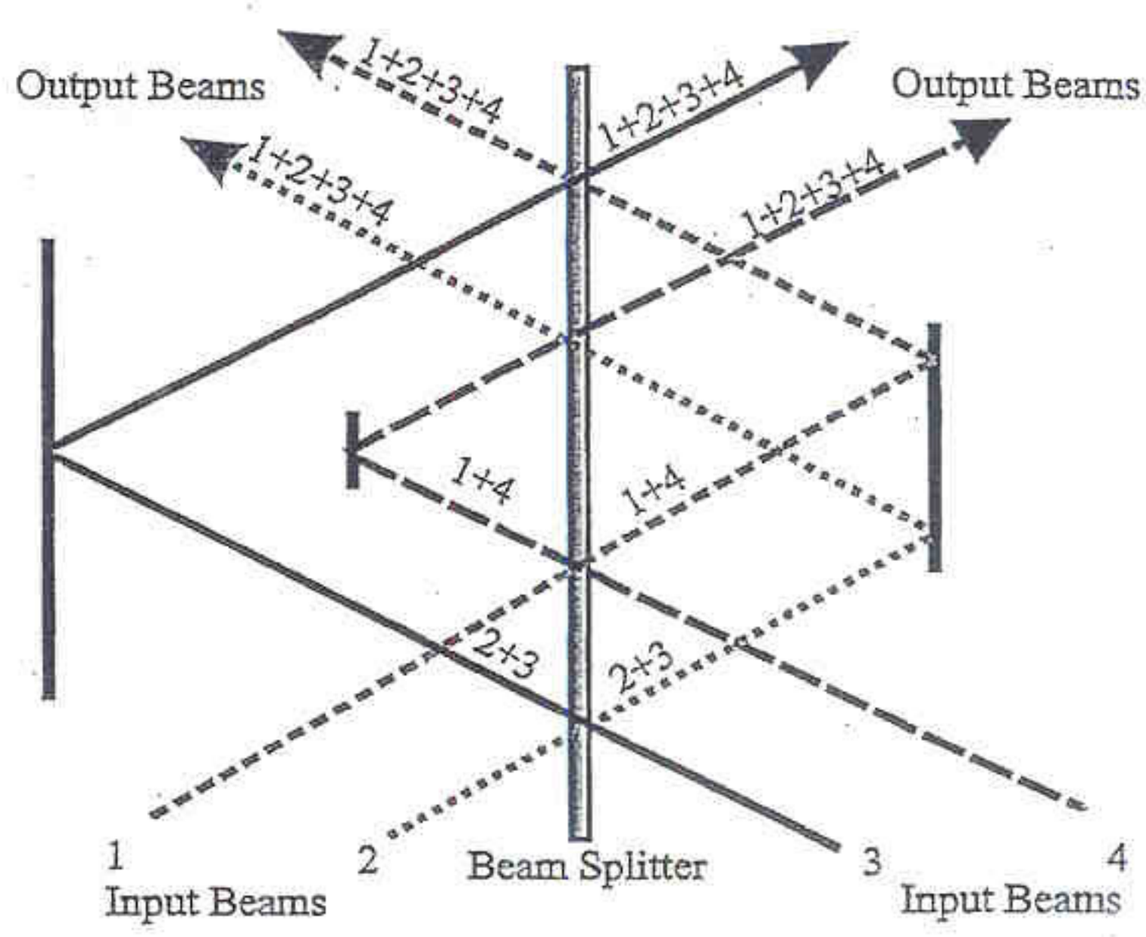
FOV. The schemes above have a FOV $\gtrsim \theta_{tel}$, i.e. small, because output pupils $\neq$ scaled input pupils.

Combination schemes for 4 beams.

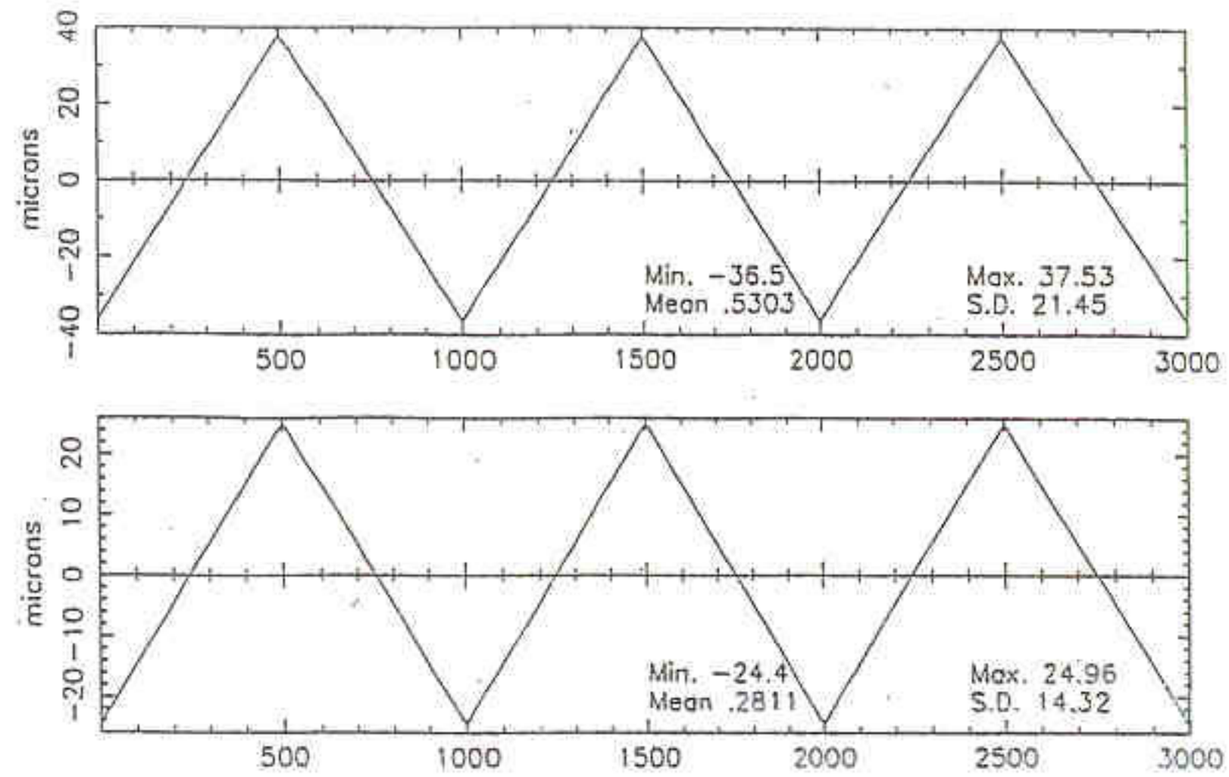


BEAM COMBINING TABLE

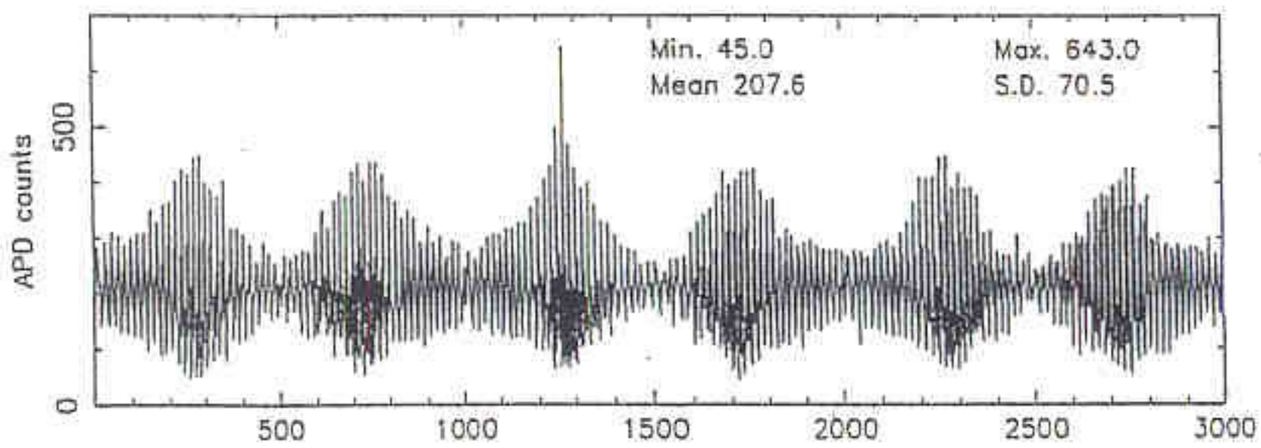




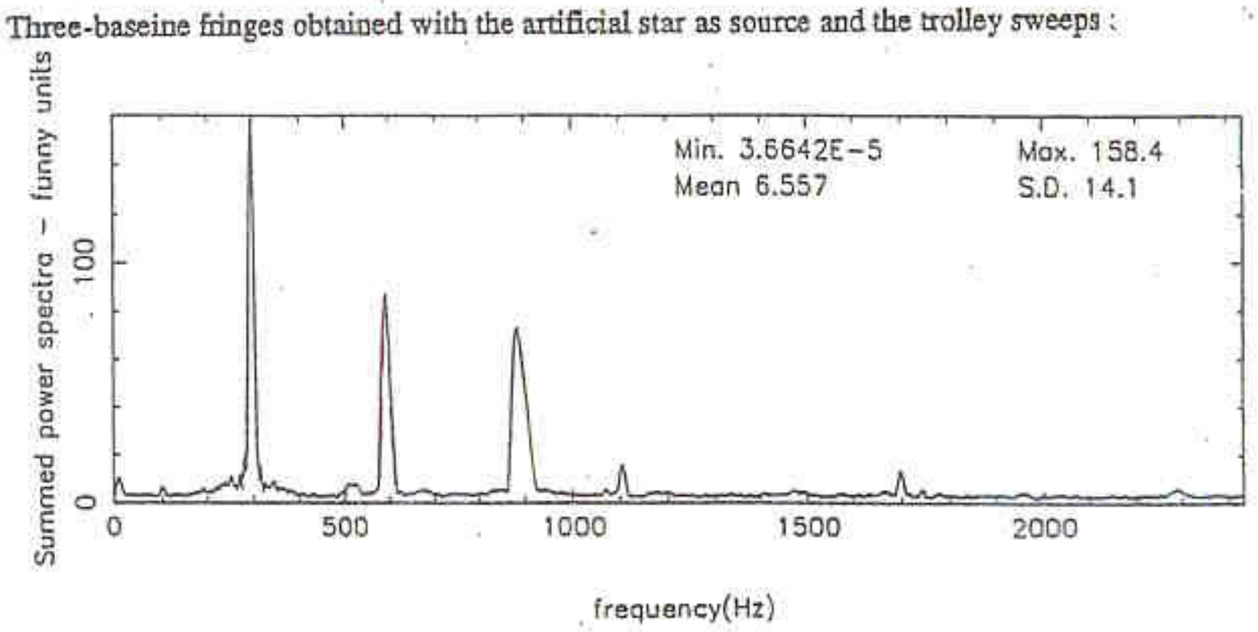
3 beams at COAST.



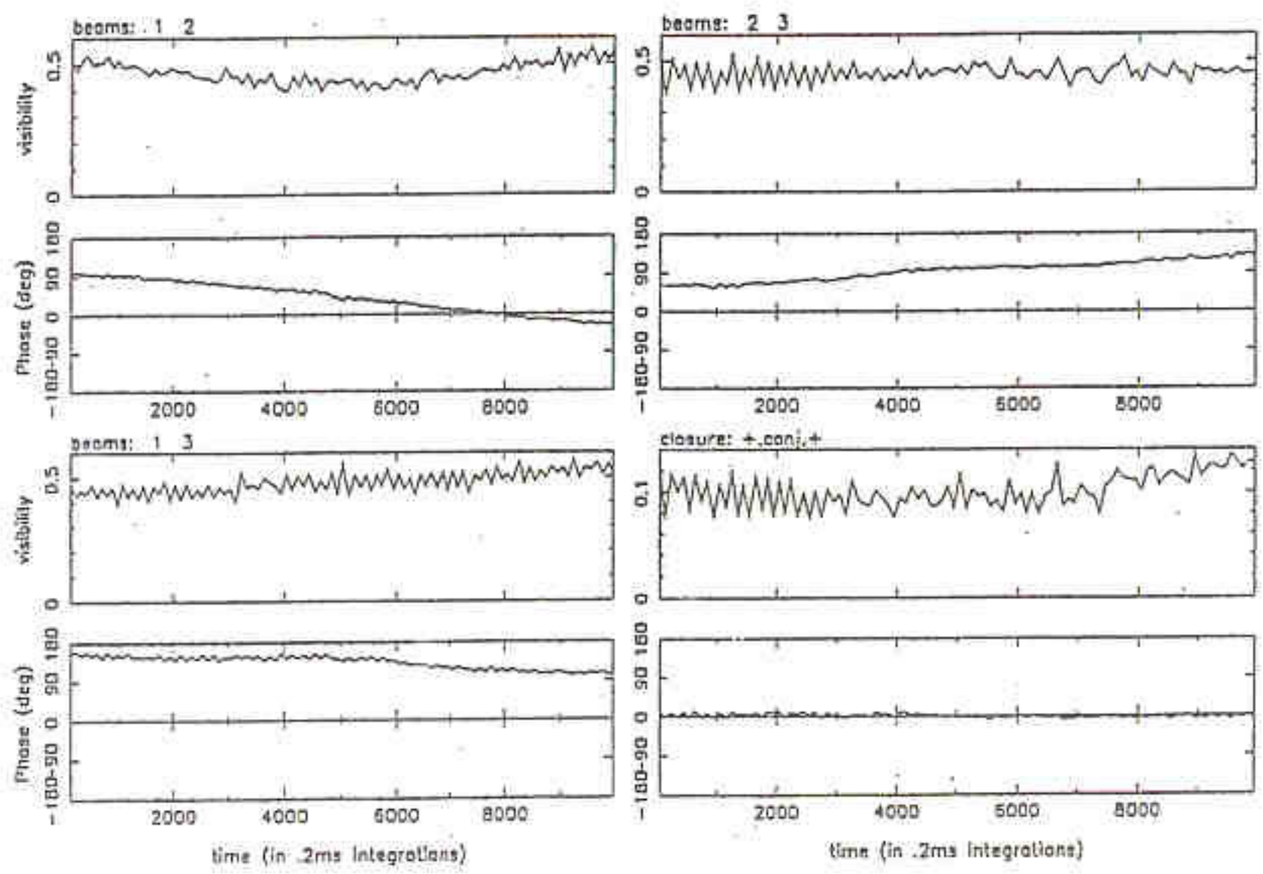
Large amplitude sawtooth sweeps of the roof mirrors on two trolleys. Note the speeds of motion are in the ratio 3:2.



Three-baseine fringes obtained with the artificial star as source and the trolley sweeps : Time unit



Power spectrum of fringes



Amplitude and phase of fringes on three baselines. Fourth quadrant shows the triple product amplitude and the closure phase

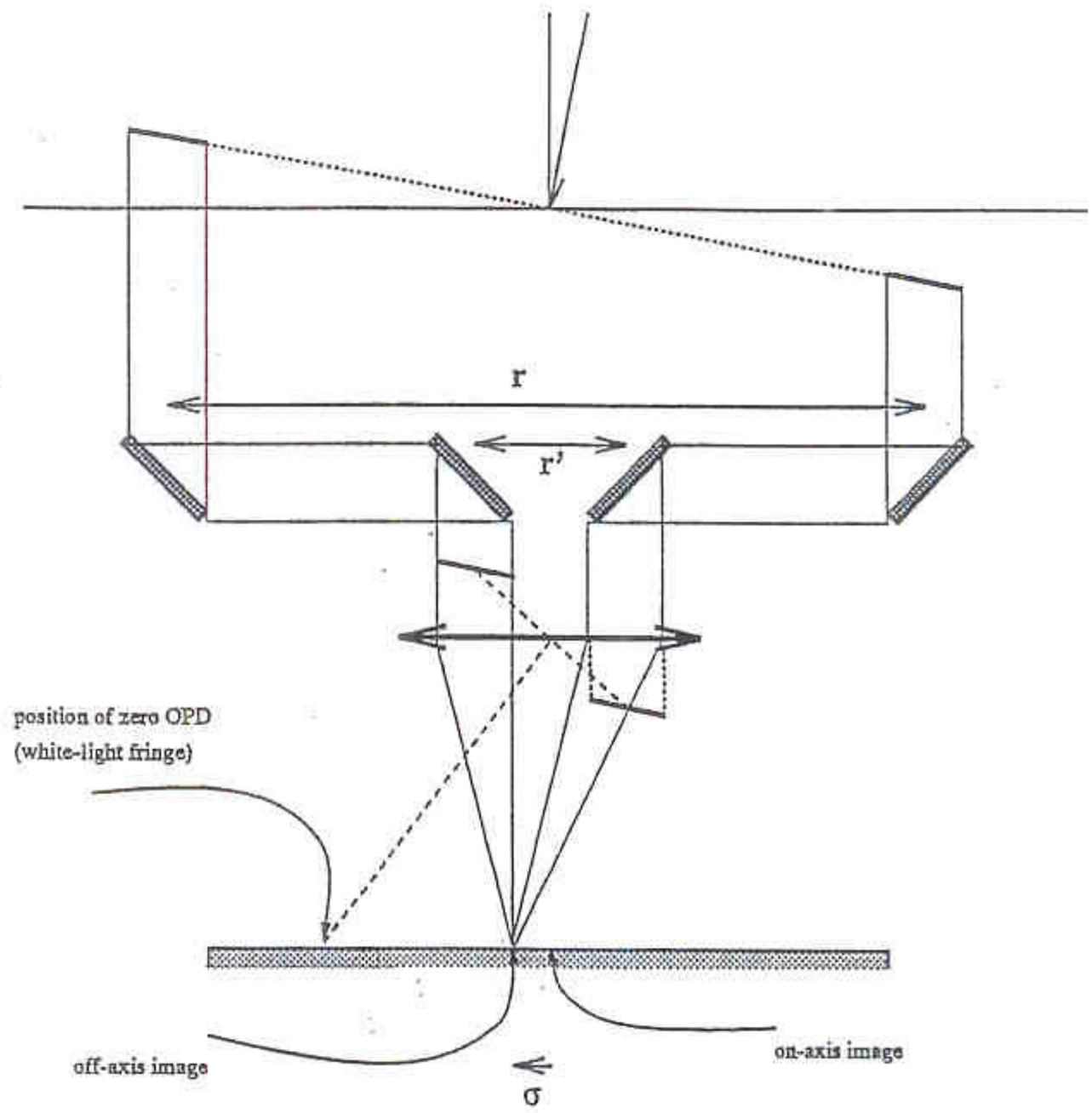


Figure 4.9: Geometry for 2 element Michelson interferometer.

input pupil ≠ scaled output pupil

⇒ poor FOV.

Instrumental effects.

Bandpass. If rectangular bandpass of width $\Delta\sigma$ (where $\sigma = \frac{1}{\lambda}$), then

$$V_{bandpass} = \frac{\sin \pi \cdot x \cdot \Delta\sigma}{\pi \cdot x \cdot \Delta\sigma} \quad \text{where } x \equiv X_{delay} - X_{star}.$$

Wavefront tilt. If wavefronts are tilted by angle α, then

$$V_{tilt} = \underbrace{\frac{\sin \pi \cdot D \cdot \alpha/\lambda}{\pi \cdot D \cdot \alpha/\lambda}}_{\text{square}} \quad \text{or} \quad \underbrace{\frac{2\, J_1(\pi D \alpha/\lambda)}{\pi D \alpha/\lambda}}_{\text{circular}}$$

For $V_{tilt} > 0.90$ need $\alpha < 0.3\,\lambda/D$.

Relative intensity. If the relay optics and/or beamcombiner have intensity ratio ρ, then

$$V_{rel.\,int.} = \frac{2}{\rho^{1/2} + \rho^{-1/2}}.$$

Non-flatness of surfaces. If the wavefronts have rms perturbations δ, then

$$V_{surfaces} \approx e^{-(2\pi\delta/\lambda)^2}.$$

If there are N surfaces of δ_0 each, then

$$\delta \approx N^{1/2} \cdot \delta_0.$$

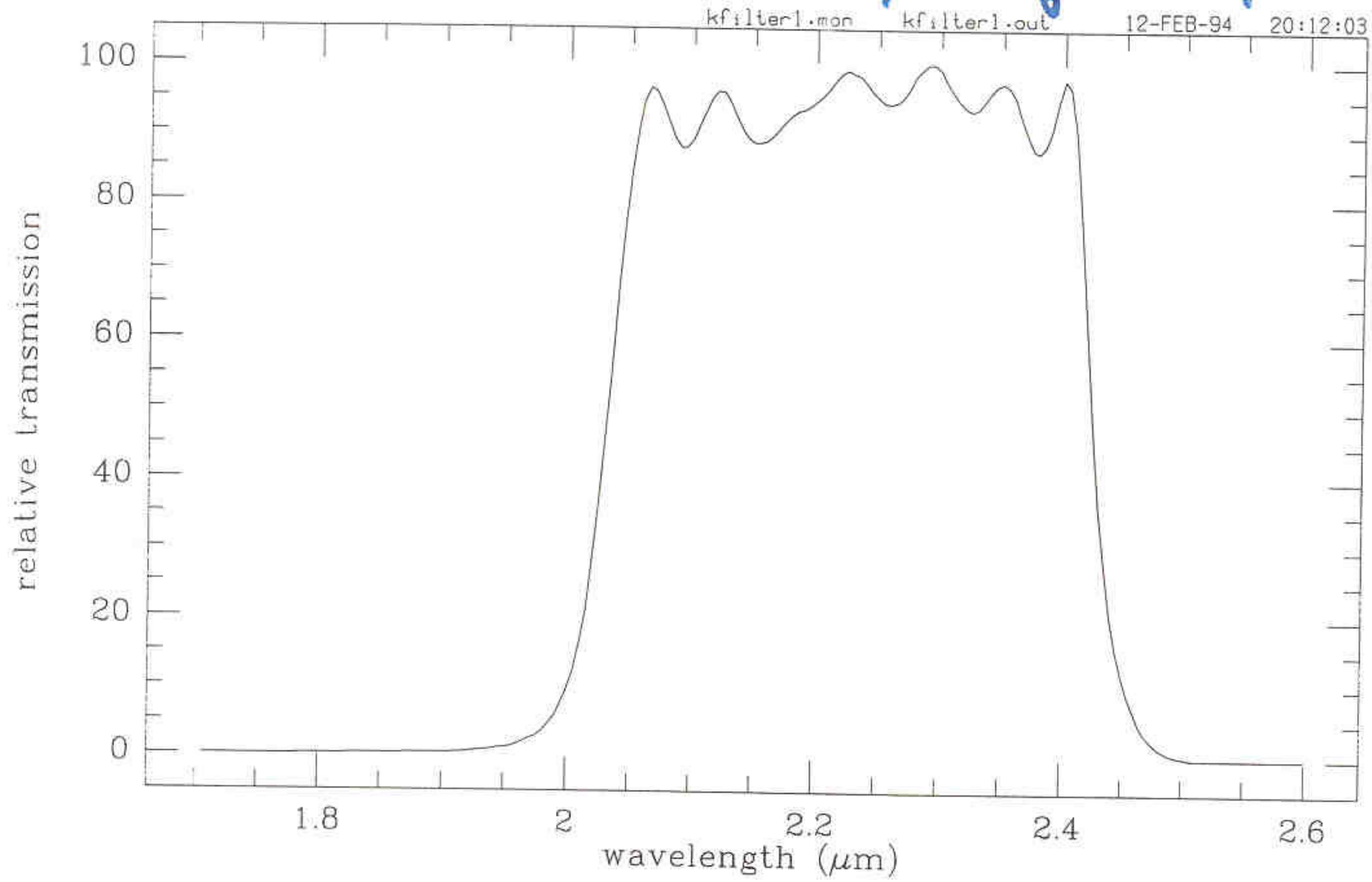
If $\delta = \lambda/20$ is rms of each beam, then

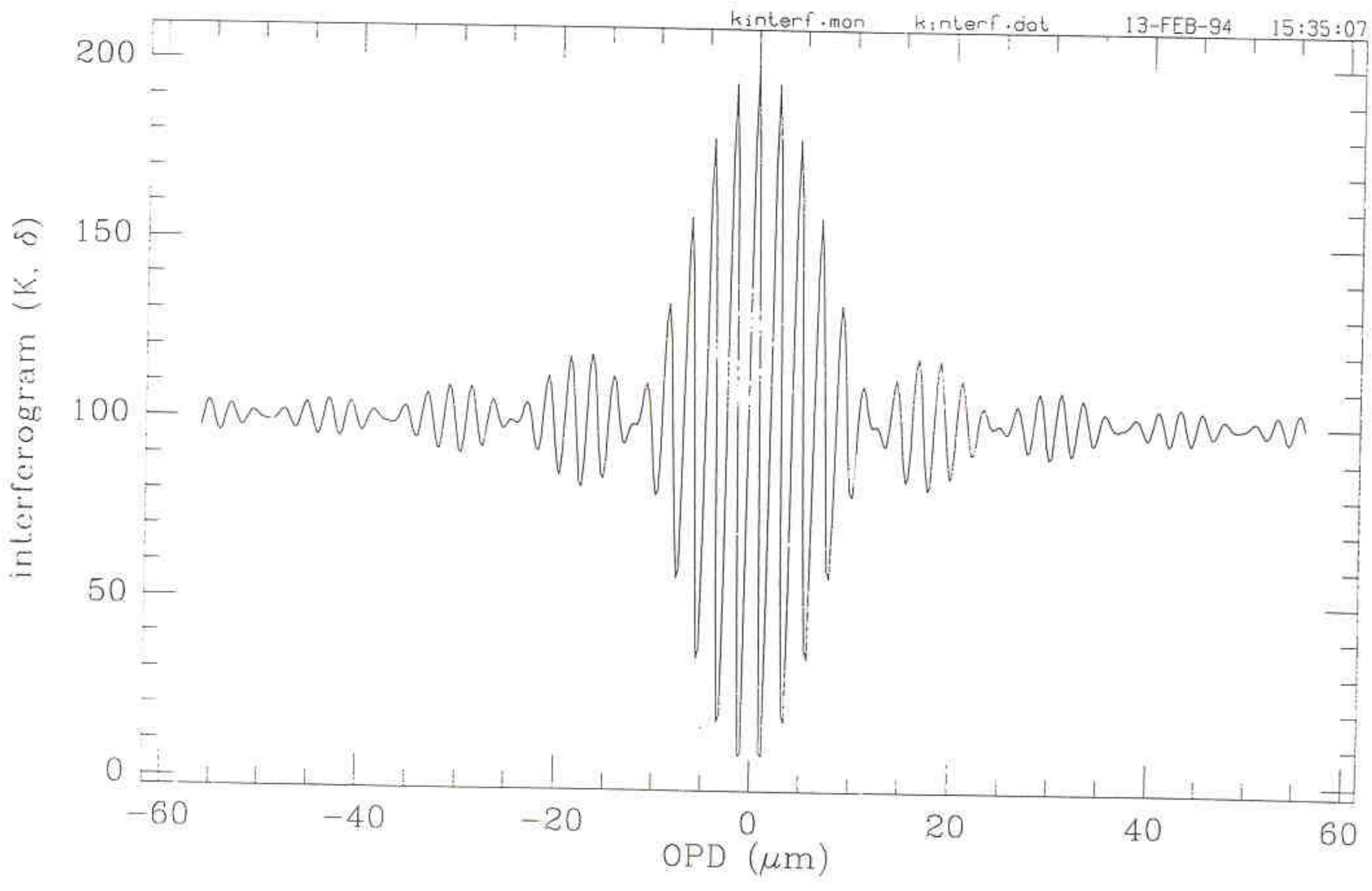
$$V(\lambda/20) \approx e^{-(\pi/10)^2} \simeq 0.90.$$

Shear. No effect, unless beams no longer overlap.

Different telescope diameters. See rel. int. calc.

K-band filter & corresponding wave packet.





Measuring visibility.

<u>$\lambda/4$ steps.</u> Change x by $(\lambda/4)\cdot(0,1,2,3)$, measure $I(x)$,

calculate $V \sim \dfrac{0-2+1-3}{0+1+2+3}$

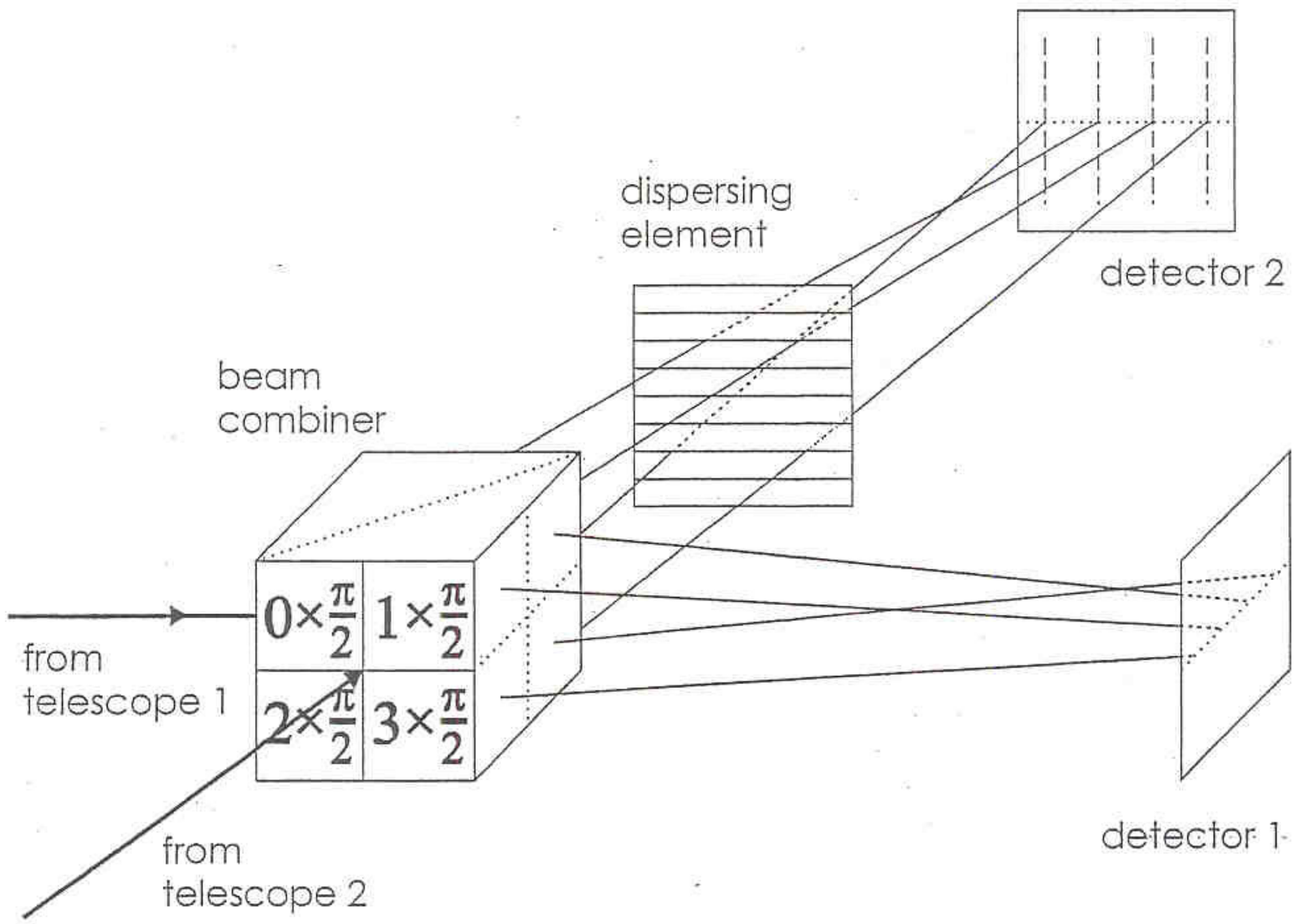
.

<u>many λ sweep.</u> Change $x = vt$ and repeat in triangle wave.

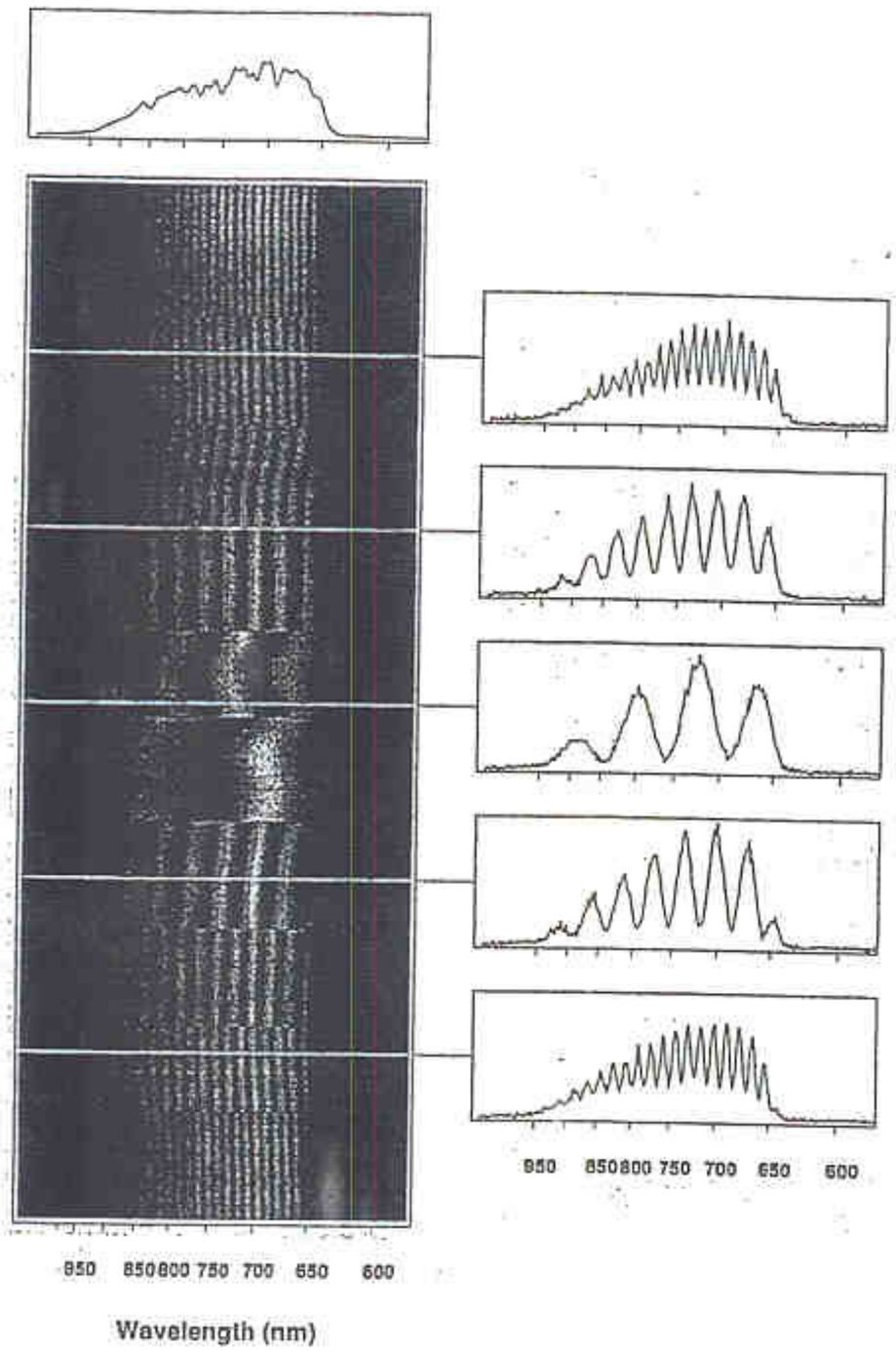
- fit theoretical wave packet to time data;
- calculate $|FFT|^2$ & ratio high freq. to low;
- wavelet analysis.

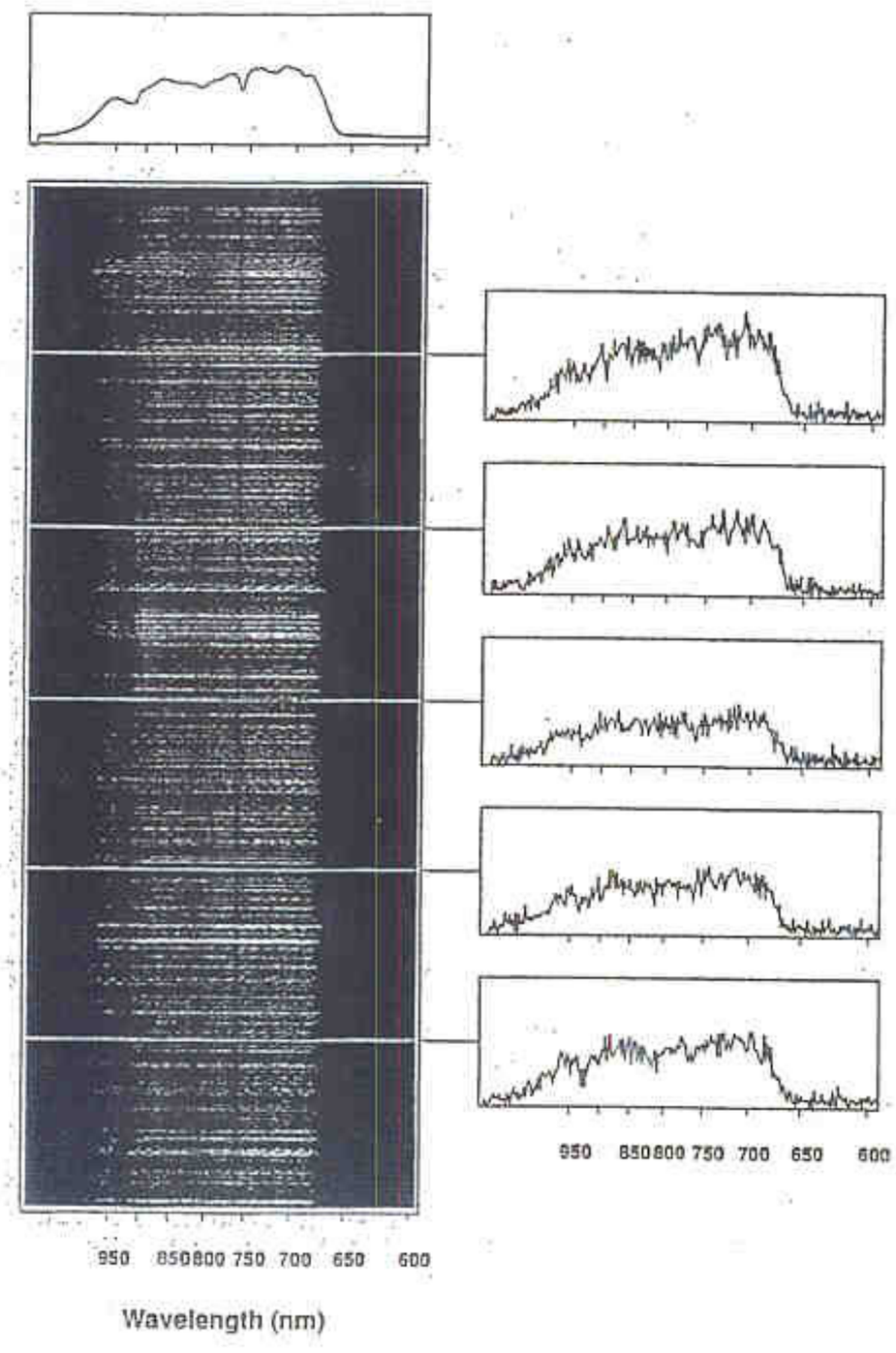
<u>dispersed (channel) spectrum.</u> Allow atmosphere to give few λ path variation of x

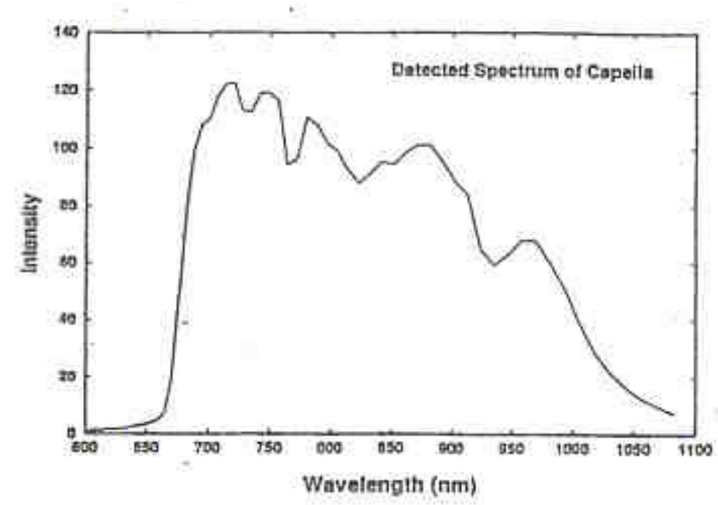
calculate peak-to-valley variations at each λ.


dispersing
element
detector 2
beam
combiner
$0\times\frac{\pi}{2}$
$1\times\frac{\pi}{2}$
$2\times\frac{\pi}{2}$
$3\times\frac{\pi}{2}$
from
telescope 1
from
telescope 2
detector 1

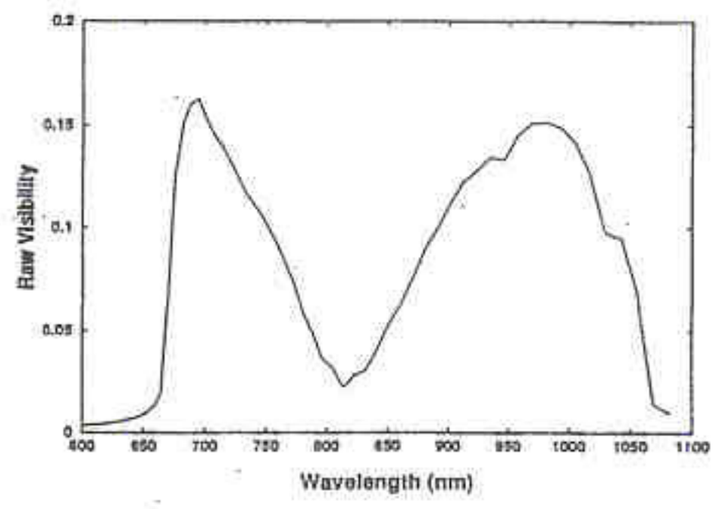
Channel spectra - COAST.





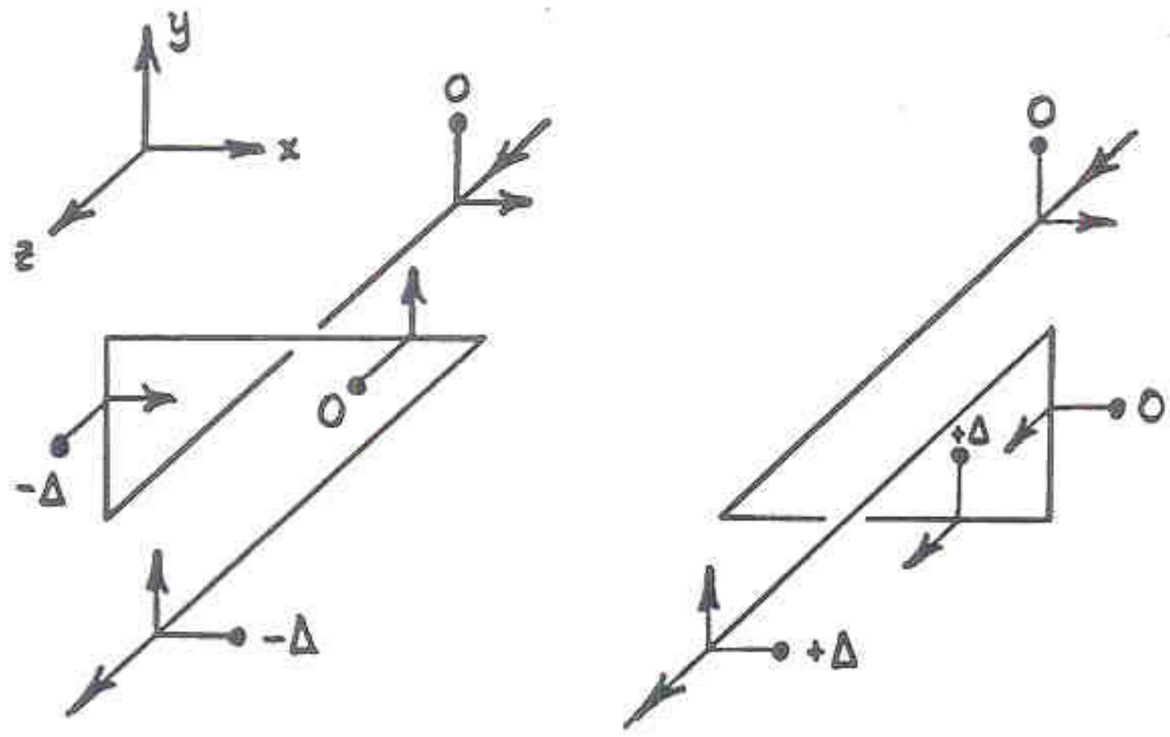


(a)



(b)

Polarization & Visibility.



(a) order = z y x z. (b) order = z x y z.

Fig. 1. Two ray paths, each starting and ending in the same directions, but following a different order of reflections, give rise to a relative phase shift in one component with respect to the other, in this case by an amount 2Δ, where Δ is the relative phase shift at a single 45 degree reflection.

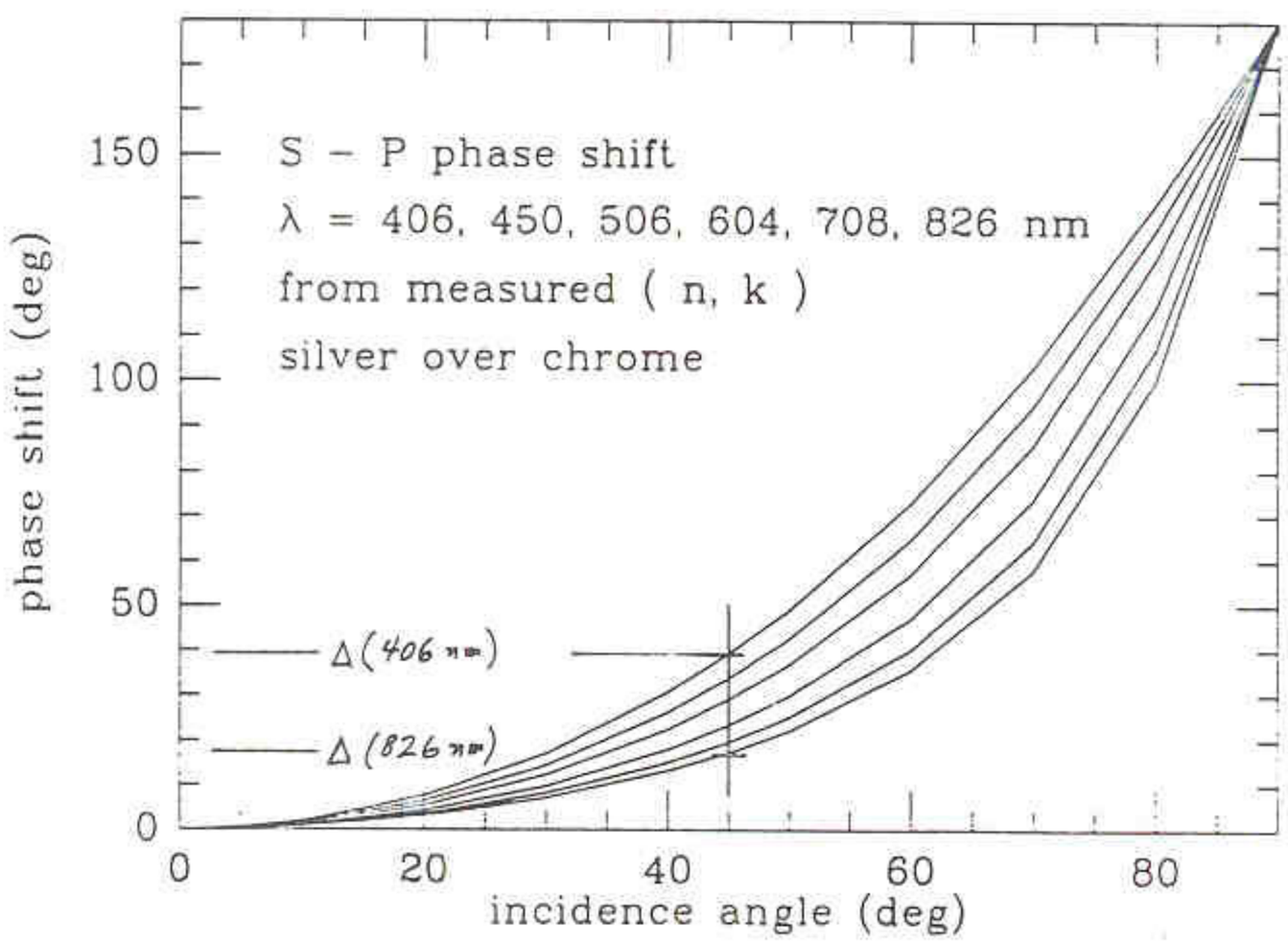


Fig. 2. Calculated s-p phase shift, based on measurements of the complex index of refraction of a silver-over-chrome mirror, at 6 wavelengths.

Visibility Reduction Factor

Beam 1: $\vec{A}_1 = (A_x, A_y)_1 = a_1 e^{ik\xi}(1, e^{i\phi_1})$

Beam 2: $\vec{A}_2 = (A_x, A_y)_2 = a_2 e^{ik(\xi+\ell)}(1, e^{i\phi_2})$

Combined: $I = |\vec{A}_1 + \vec{A}_2|^2$

$$I = \bar{I} \cdot \left[1 + \underbrace{\left(\frac{2a_1a_2}{a_1^2+a_2^2}\right)}_{\text{visibility term}} \cdot \underbrace{\cos\left(k\ell + \frac{\phi}{2}\right)}_{\text{modulation term}} \cdot \underbrace{\left|\cos\frac{\phi}{2}\right|}_{\substack{\text{polarization term} \\ \equiv \text{constant}}} \right]$$

where $\phi \equiv \phi_2 - \phi_1$ = relative phase shift between 2 beams.

∴ In unpolarized light, the measured visibility can be permanently degraded by purely instrumental polarization effects, in the amount

$$\left|\cos\frac{\phi}{2}\right|$$

which is a function of wavelength only.

Fiber optics - single mode.

Core.
- Core index n_1, radius a ($n_1 \sim 1.48$, $a \sim 2\mu m$).
- Cladding index n_2, radius b ($n_2 \sim 1.46$, $b \sim 60\mu m$).
- Incident / exit cone $\sin i = (n_1^2 - n_2^2)^{1/2} \equiv NA$ ($i \sim 14°$ f/2.0)

Dispersion.
- intermodal $= 0$ for SM fiber.
- material
- waveguide

} balance these to get zero.

Waveguide parameter. $V = \frac{2\pi}{\lambda_0} a (n_1^2 - n_2^2)^{1/2}$

$> 10 \Rightarrow$ geometric optics
$< 10 \Rightarrow$ wave optics.

Coupling.

} cyclic

z = coupling length $\approx$ 3 mm typically.

Mfg. couplings.
- twist & melt (fused)
- polish & mate (polished).

Cutoff λ_c. For $\lambda > \lambda_c$ the fiber is single-mode.

$$\lambda_c = \frac{2\pi a}{2.4048} (n_1^2 - n_2^2)^{1/2}.$$

If $NA \sim .24$, $\lambda_c \sim 0.6a$.

Etendue. $A\Omega = \pi a^2 \pi i^2 = \pi a^2 \pi (NA)^2 \approx (1.202\lambda_c)^2 \approx \lambda^2$.

Transmission. loss < 1 dB/km for silica & fluoride.

Integrated optics.

Fibers on a chip.

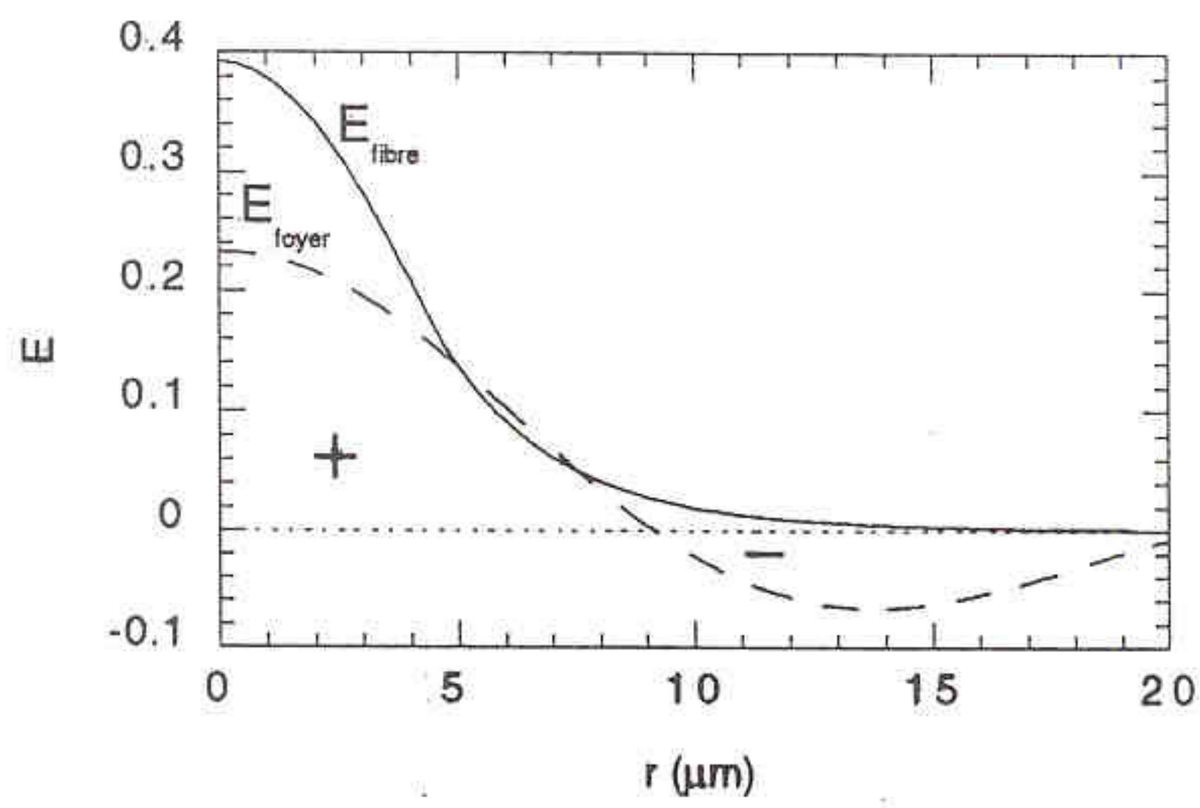
Injecting starlight in a fiber

- Efficiency set by the overlap integral:

in the focal plane: $\rho = \left| \iint E_{tel} \, E^*_{fibre} \right|^2$

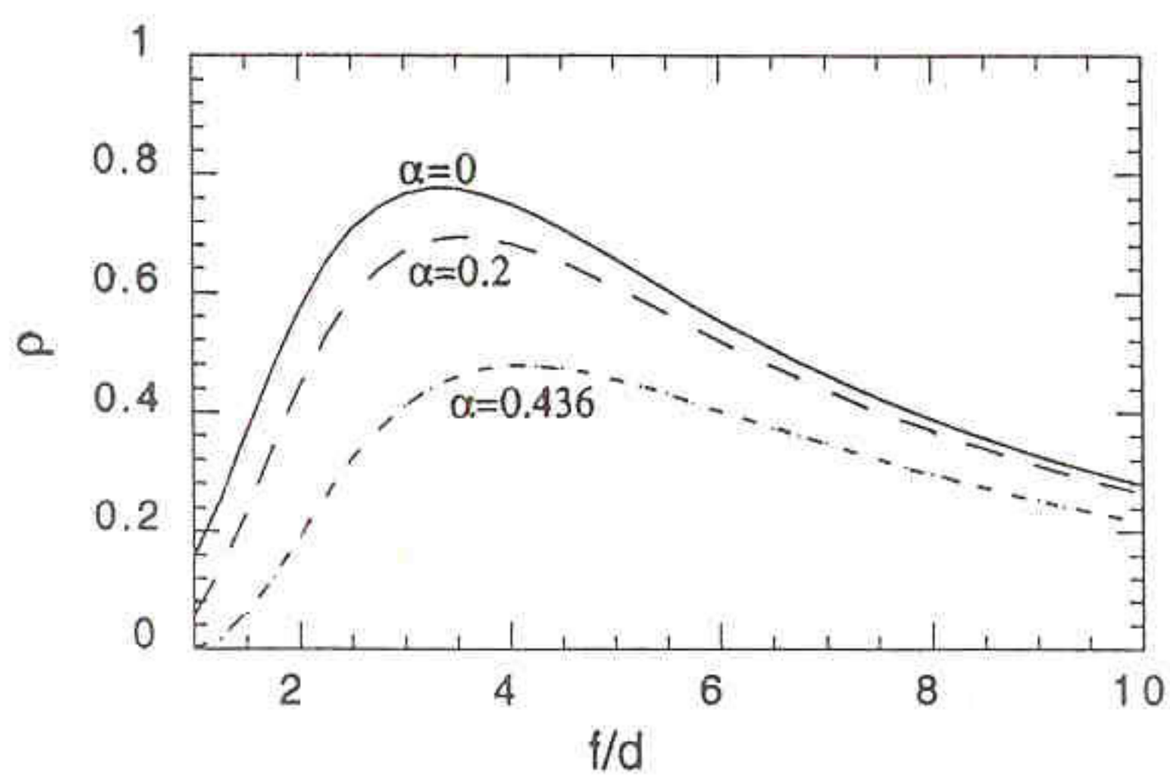
=> the field *amplitudes* must match

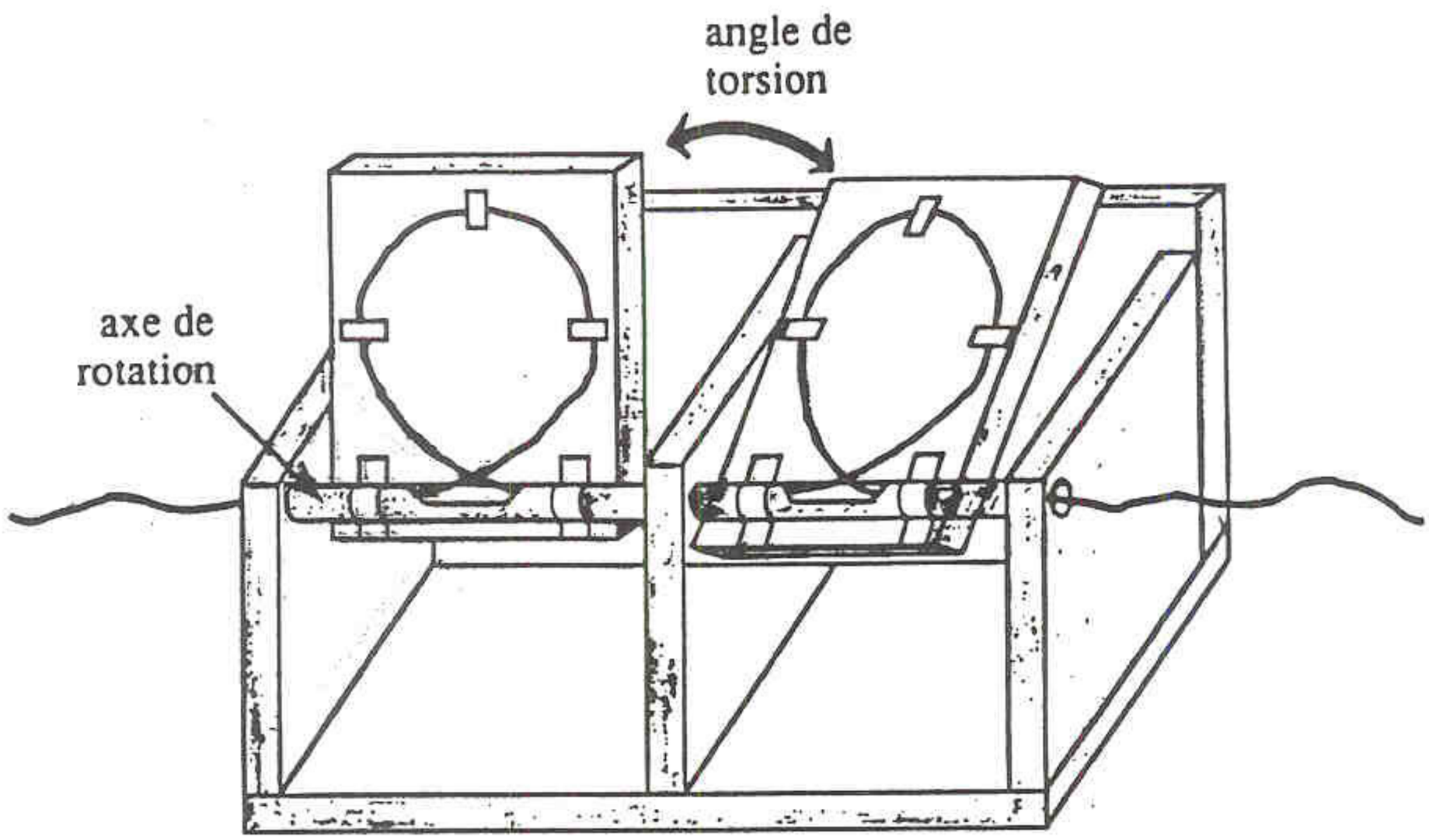
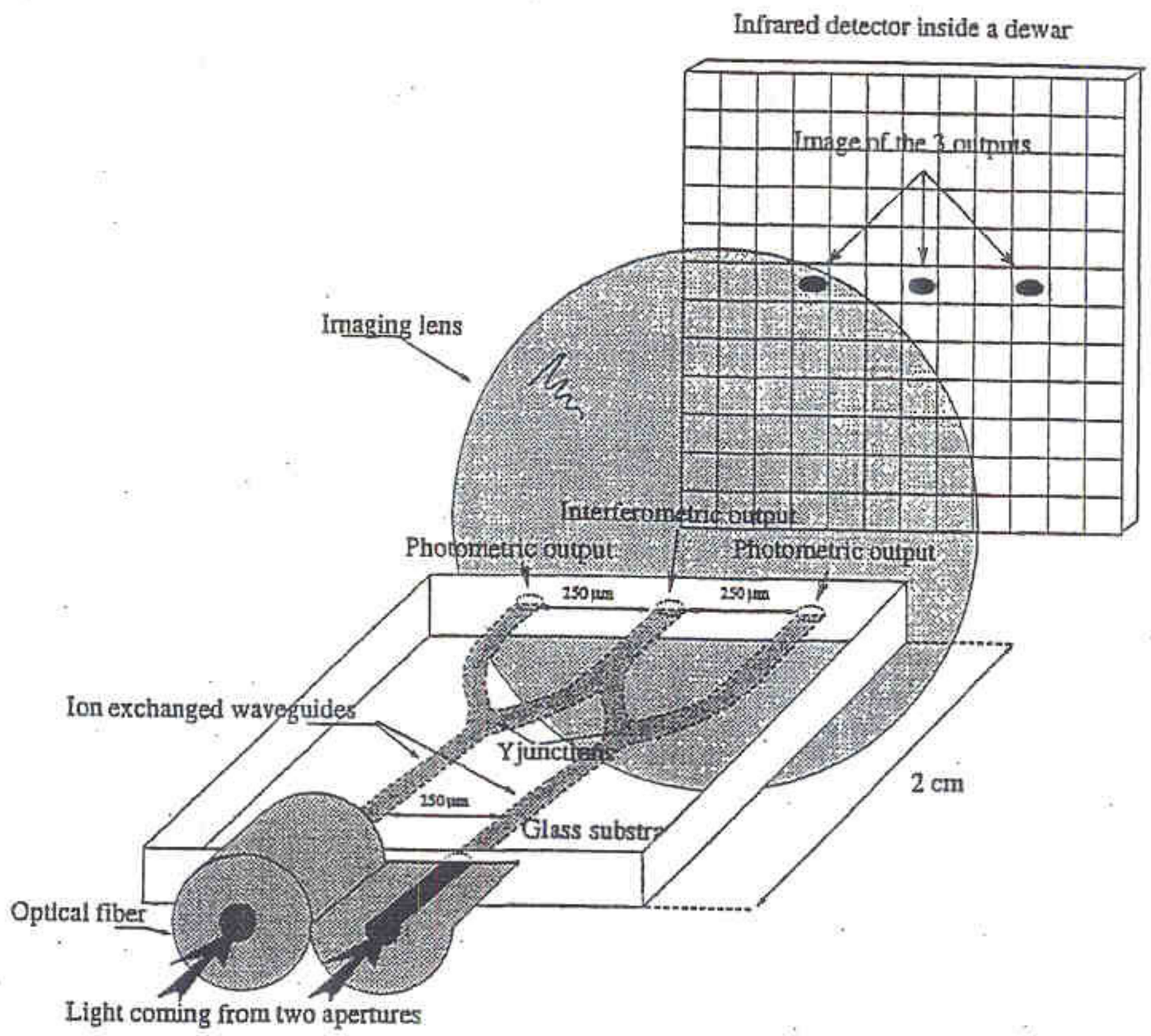
1- Diffraction limited beams



- The optimal f / d is the one that maximizes the overlap integral
- Maximum possible efficiency: $\rho_{\max} = 78\%$

(but less if the pupil has a central obstruction α)




angle de
torsion
axe de
rotation

Integrated optics.