Hertzsprung – Russell diagram

The observations

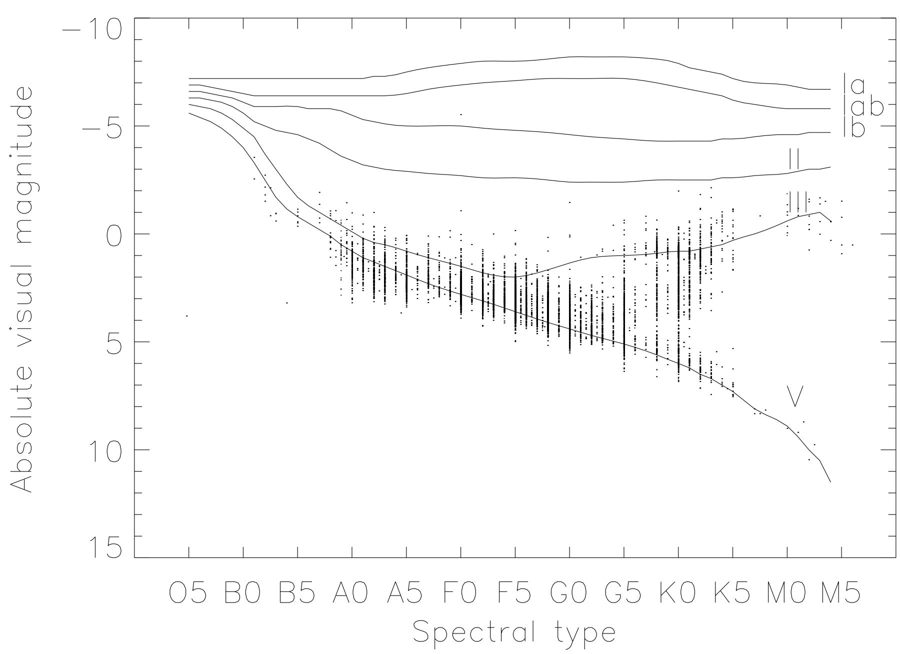
- Apparent visual magnitudes
- Trigonometric parallaxes
- Spectral type classification, a measure of the effective temperature

The result

- Stars occupy a limited parameter space in absolute (intrinsic) stellar brightness and spectral type
- Two branches exist, main sequence stars and giants. The narrow lines of the latter are indicative of a lower surface gravity. In combination with their greater brightness, this indicates the intrinsically larger sizes of the giants.

The interpretation

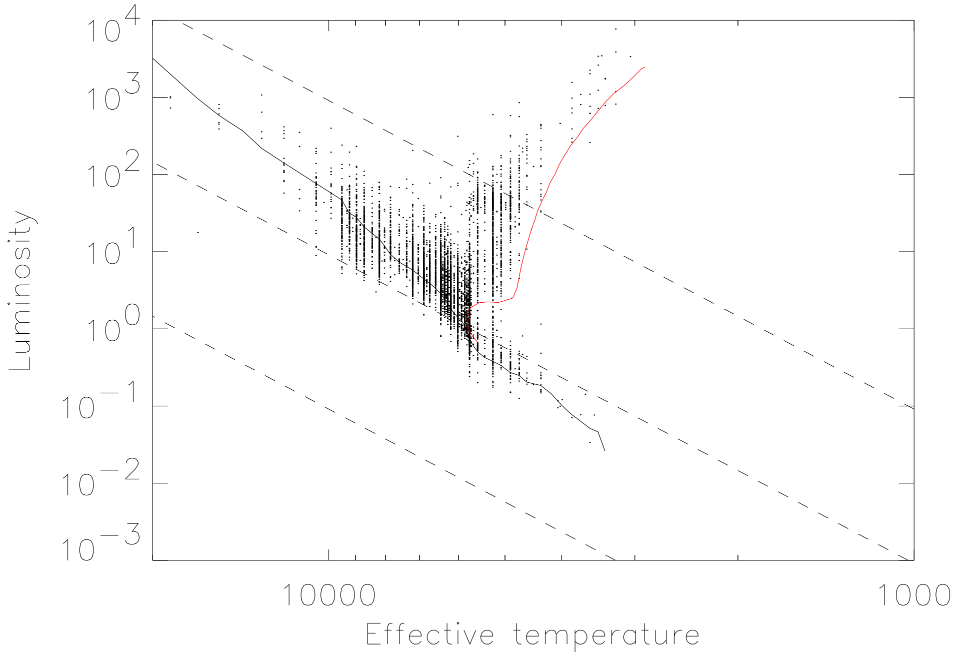
- A physical model of stellar structure based on nuclear fusion of matter with composition (X, Y, Z)
- Blackbody radiation
- Lifetime and main-sequence location of a star governed my its mass



Stellar model and evolution

Fundamental stellar properties

- **Mass**
- **Chemical composition**
- **Angular momentum**
- Luminosity
- Effective temperature
- Radius



Theoretical Hertzsprung-Russell diagram. Solid line is Zero-age-mainsequence (ZAMS). Dashed lines are blackbody stars of constant radius, the middle line corresponding to the solar radius. The red line is an example for an evolutionary track of a star of one solar mass.

Relationships between fundamental parameters

Stefan–Boltzmann law (black body radiation)

$$\log L/L_\odot = 2\log R/R_\odot + 4\log T_e/T_{e\odot}$$

Surface gravity g

$$\log M/M_\odot = 2\log R/R_\odot + \log g/g_\odot$$

Solar values: $T_e = 5780$ K, $\log g_\odot = 4.44$ (cgs units).

Zero–age main sequence

$$\log L/L_\odot = 3.8\log M/M_\odot + 0.08$$

$$\log R/R_\odot = \begin{cases} 0.917\log M/M_\odot - 0.020 & (-1.0 < \log M/M_\odot < 0.12 \\ 0.640\log M/M_\odot + 0.011 & (0.12 < \log M/M_\odot < 1.3) \end{cases}$$

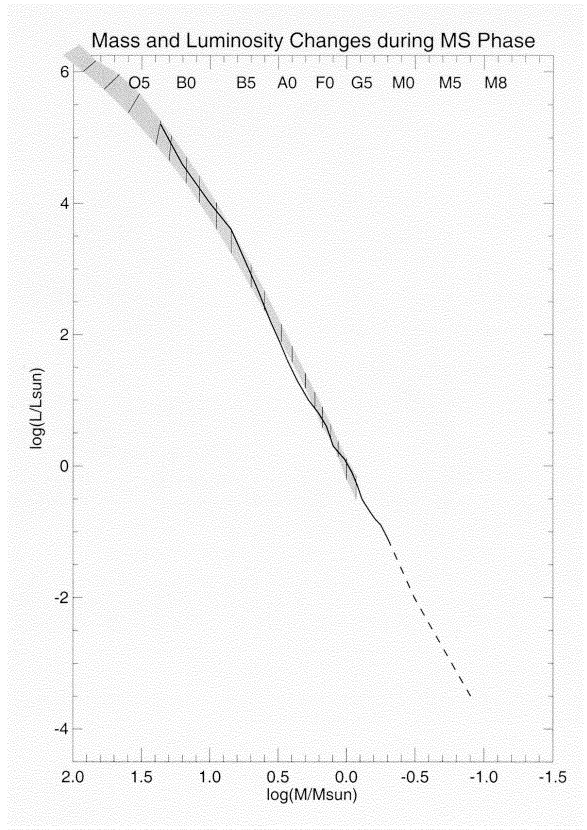
Bolometric luminosity

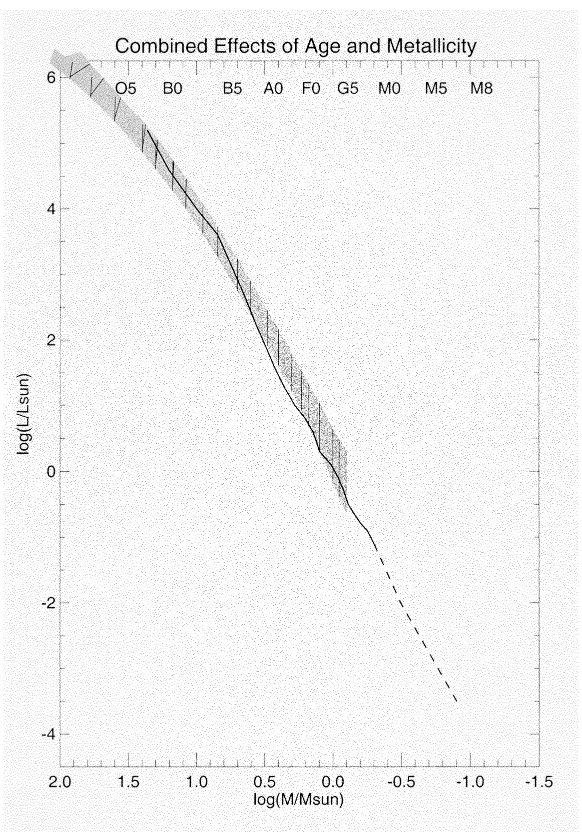
$$M_{\rm bol} \equiv M_V - BC = -2.5\log(L/L_\odot) + 4\overset{\rm m}{.}74$$

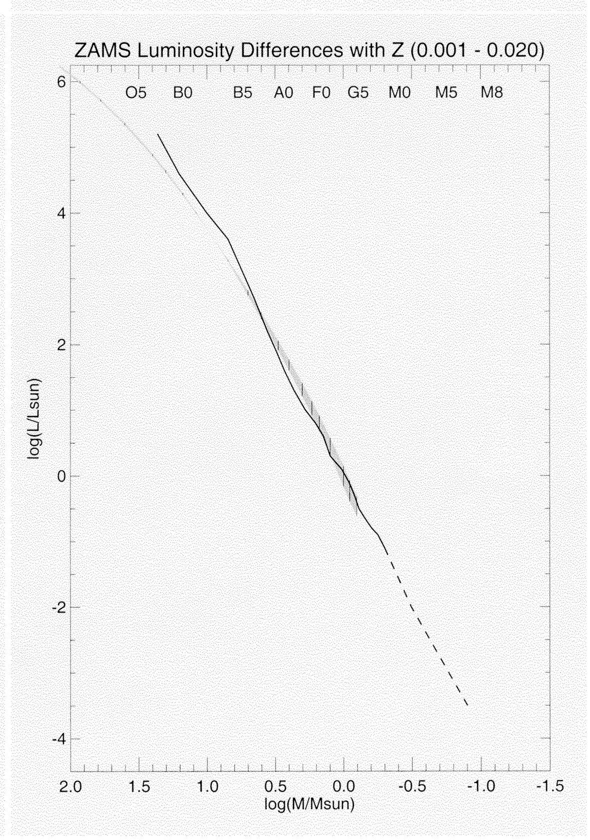
$$M_V = m_V + 5 + 5\log\pi$$

The width of the main-sequence

These diagrams show the luminosity and mass parameter space due to variations in age and metallicity (courtesy of W. Hartkopf, models from Schaller et al. 1992, mass-luminosity relation by Popper, 1980).

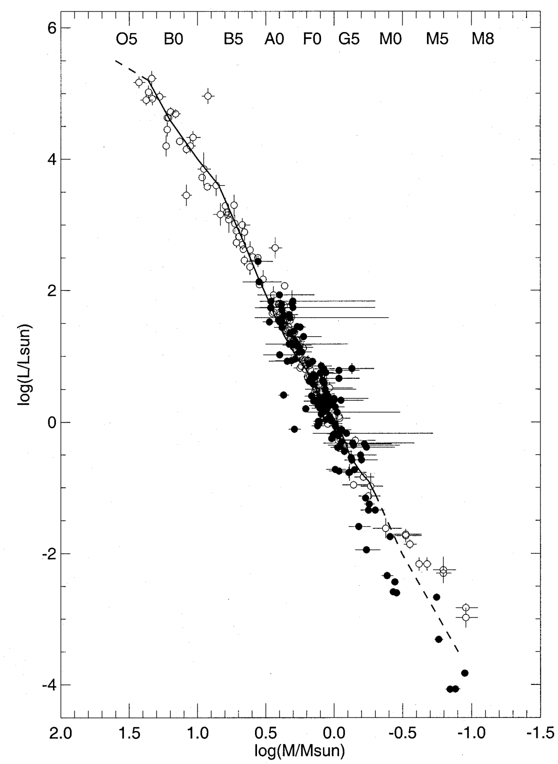


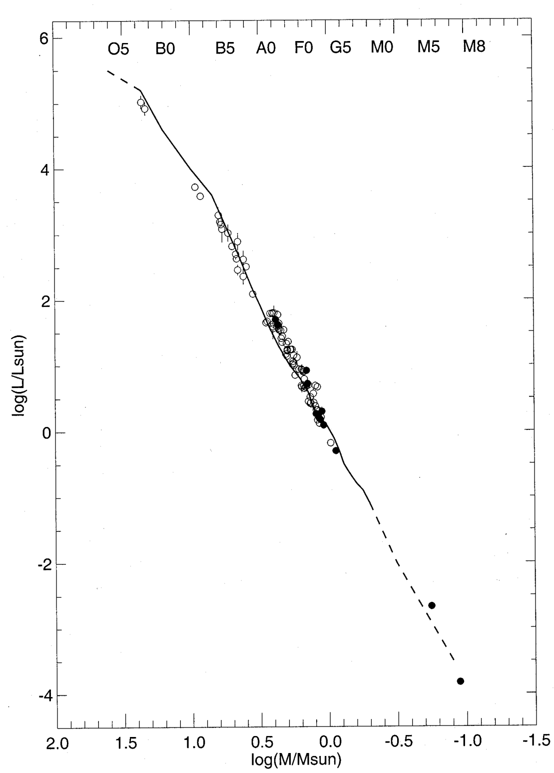


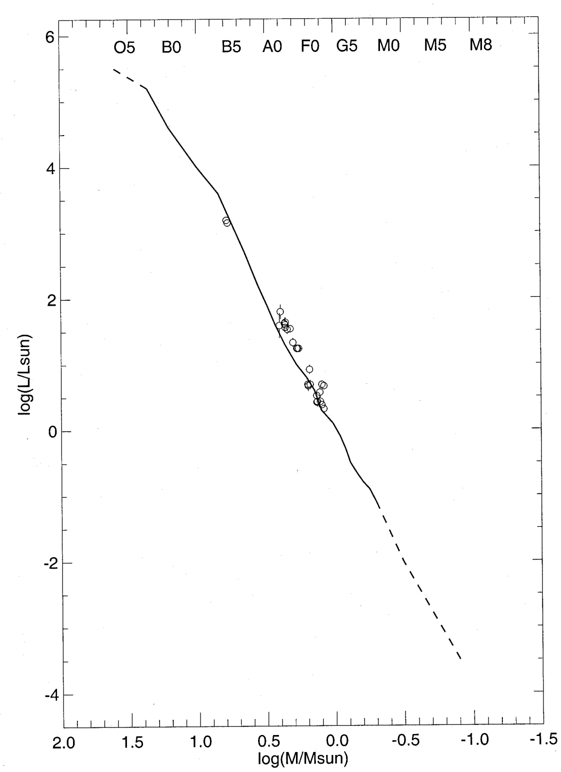


An overview of stellar masses

These diagrams (courtesy of W. Hartkopf) show all main-sequence stellar masses (right), and those with accuracies of better than 2% (bottom left) and 1% or better (bottom right).

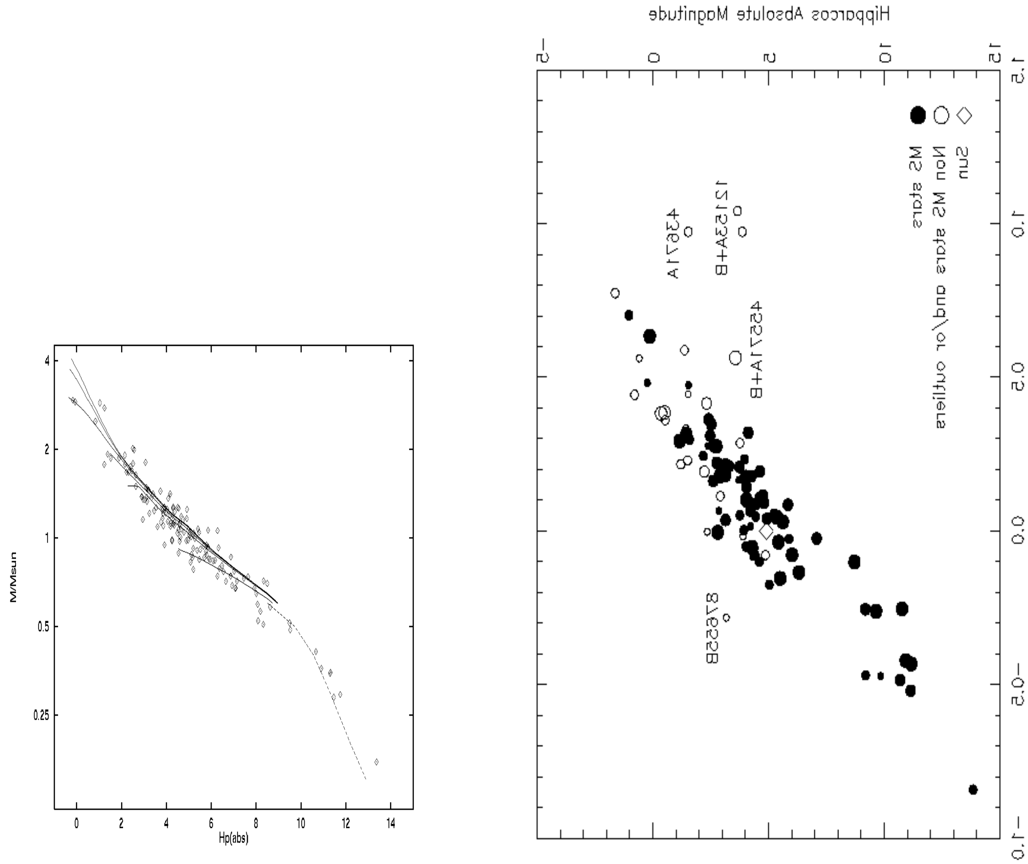






Inventory of precise masses and luminosities – Hipparcos

Astrometric binaries are the only class discussed here allowing the determination of stellar masses and luminosities without the requirement of spectroscopy. Parallax and absolute orbits of both components around the center of mass must be measured. For binaries near the resolution limit of Hipparcos the measured signal corresponds to the "hippacentre", a point between photocenter and center of mass. The reduction benefits a lot from relative ground based orbits of the binaries, but the resulting accuracy is only between 5% and 30%.

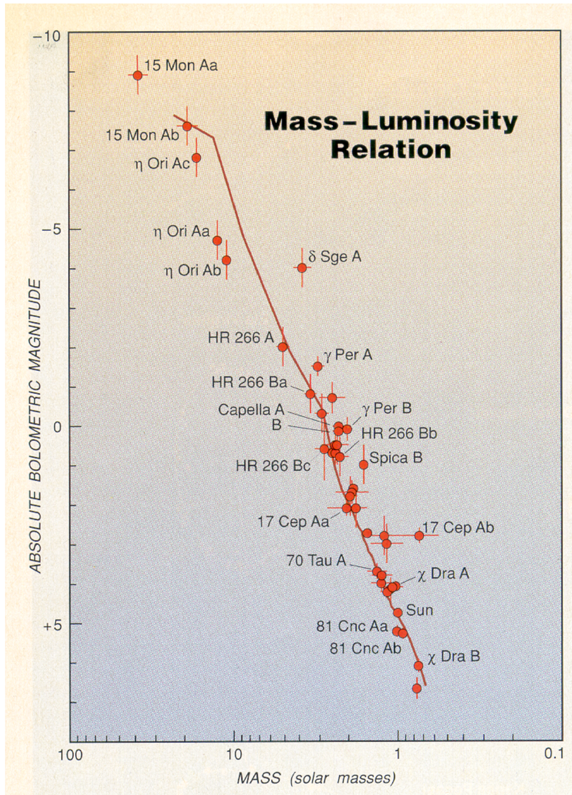


The figure shows mass vs absolute Hipparcos magnitude diagrams derived from two independent reductions of Hipparcos transit data done by Martin and Söderhjelm. (The figure on the left includes theoretical isochrones for a range of ages.) (The figure on the right was flipped twice to match the axis orientation of the other figure.)

Selected references

- Martin, C., et al. 1997, A&AS, 122, 571
- Martin, C., & Mignard, F. 1998, A&A, 330, 585
- Martin, C., et al. 1998, A&AS, 133, 149
- Söderhjelm, S. 1999, A&A, 341, 121

Inventory of precise masses and luminosities – Speckle



Mass uncertainties are typically in the range 3% to 10%, while luminosities are determined to about 10%. The speckle technique resolves visual binaries down to the diffraction limit of the telescope.

Principle sources

- CHARA, McAlister, Hartkopf, Mason, et al.
  "Binary star orbits from speckle interferometry. I – XII."

- USNO, Worley, Douglass, Mason, Hartkopf, et al.

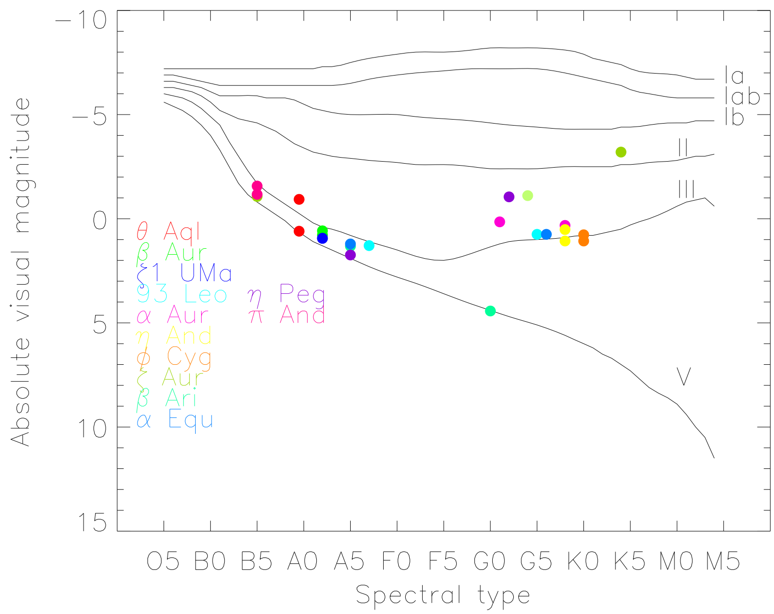
Selected references

- Torres, G., et al. 1997, ApJ, 474, 256

- Scarfe, C., et al. 1994, AJ, 107, 1529

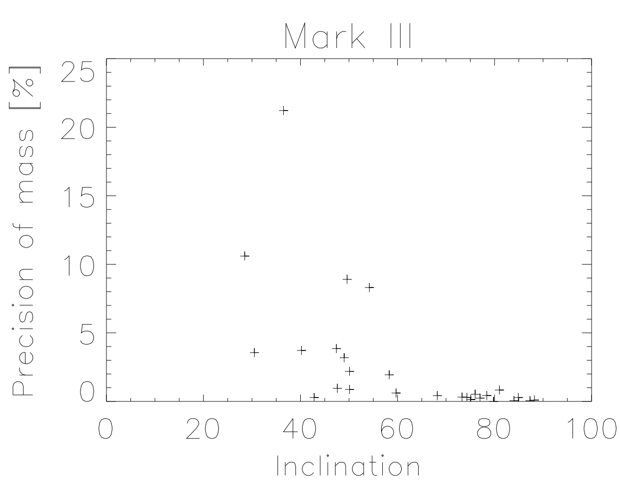
- McAlister, H. A. 1996, *Sky and Telescope*, 92, 5/28

Inventory of precise masses and luminosities – Mark III and NPOI observations of spectroscopic binaries



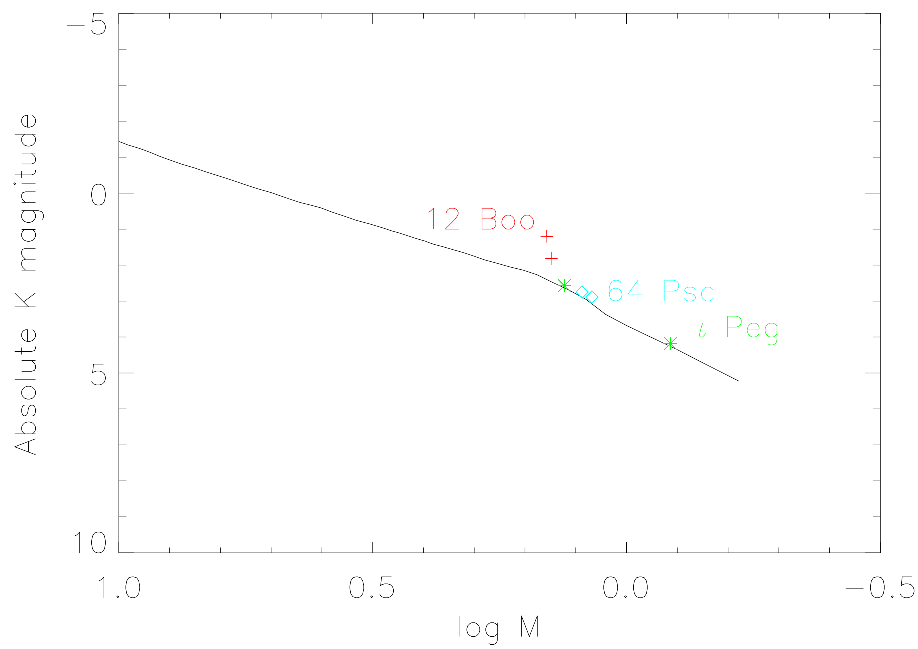
Selected references for high precision masses (% mass/ % luminosity σ)

- Armstrong J.T., et al. 1992, AJ, 104, 2217 (ϕ Cygni, 3.3%/12%)
- Hummel, C. A., et al. 1995, AJ, 110, 376 (β Aur, 1.2%/7%)
- Hummel, C. A., et al. 1998, AJ 116, 2536 (ζ 1 UMa, 2.8%/6.3%)
- Hummel, C. A., et al. 1994, AJ, 107, 1859 (α Aur, 1.6–2.2%/1.5–3.3%)



Theoretical mass uncertainties for binary orbits from the Mark III if there were negligible radial velocity errors.

Inventory of precise masses and luminosities – PTI



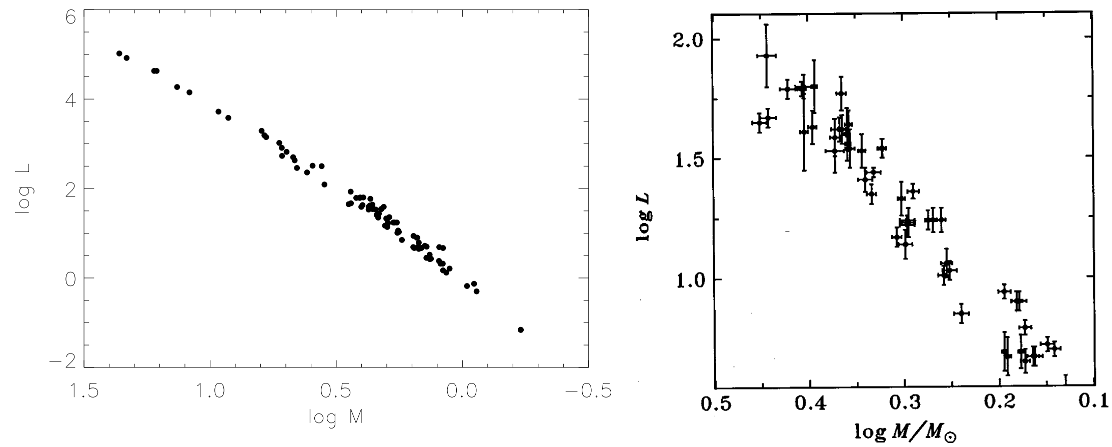
Selected references (% mass/ % luminosity σ)

- Boden, A. F., et al. 1999, ApJ, 515, 356 (ι Peg, 1.1%/2.5%)
- Boden, A. F., et al. 1999, ApJ, 527, 360 (64 Psc, 1.6%/9%)
- Boden, A. F., et al. 2000, ApJ, 536, 880 (12 Boo, 1.5%/7%)

Comparison of Hipparcos parallaxes with orbital parallaxes [mas]

Star	HIC	M3/PTI/NPOI	±	Hipparcos	±
θ Aql	99473	13	1	11.4	0.9
β Aur	28360	40	1	39.7	0.8
93 Leo	57565	13.8	0.5	14.4	0.9
η And	4463	13.1	0.3	13.4	0.7
β Ari	8903	53	2	54.7	0.8
β Per	14576	35.4	1.1	35.1	0.9
α Equ	104987	18.1	0.8	17.5	0.9
ζ Aur	23453	3.8	0.1	4.1	0.8
ϕ Cyg	96683	12.4	0.3	13.0	0.6
θ^2 Tau	20894	21.2	0.8	21.9	0.8
α Aur	24608	75.1	0.5	77.3	0.9
Mizar A	65378	39.4	0.3	41.7	0.6
64 Psc	3810	43.3	0.5	41.8	0.8
ι Peg	109176	86.9	1.0	85.1	0.7
12 Boo	69226	27.1	0.4	27.3	0.8
o Leo	47508	24.2	0.1	24.1	1.0

Inventory of precise masses and luminosities – EB



The mass-luminosity relationship after Andersen, 1991, based on observations of eclipsing binaries. On the right is a zoom-in on a section of the diagram, showing that the scatter considerably exceeds the observational errors.

Typical properties of a well studied binary system (after Andersen 1995, IAU 166)

- Mass: 1%
- Radius: 1%
- Luminosity: 10%
- Distance: 3%

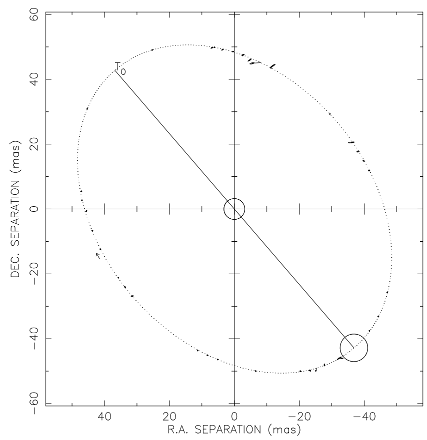
Selected references

- Andersen, J. 1991, *Astron. Astrophys. Rev.*, 3, 91
  "Absolute dimensions of eclipsing binaries. I — XXIII."
  A collaboration with Nordström, Clausen, Torres, Stefanik, Latham et al.
- Andersen, J., et al. 1989, A&A, 211, 346
- Casey, W., et al. 1998, AJ, 115, 1617
- Torres, G., et al. 2000, AJ, 119, 1942

State of the art – visual 1

Observational parameters:

- Spectroscopy: $K_{1,2} \pm 0.1$ km/s,
  $M_{1,2}/M_\odot = 1.0385\,10^{-7}(1-e^2)^{3/2}(K_1+K_2)^2 K_{2,1} P$, with $K_{1,2}$ in km/s and P in days.
- Interferometry: $i \pm 0.1°$, $a \pm 0.1$ mas, $\Delta m_\lambda \pm 0.05$ mag, $D_{1,2}$ $\pm$ 0.1 mas
- Photometry: $m_{1,2,\lambda} \pm 0.02$ mag



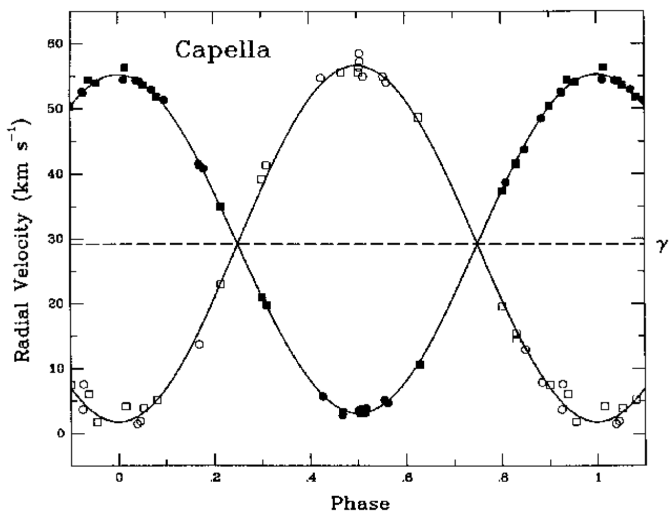


TABLE 1. Orbital elements and component parameters from the Mark III interferometer.

Parameter	value	uncertainty
a/mas	56.47	0.05
e	0.0000	0.0002
i	$137\overset{\circ}{.}18$	$0\overset{\circ}{.}05$
Ω (2000.0)	$40\overset{\circ}{.}8$	$0\overset{\circ}{.}1$
T_0	JD2447528.45	0.02
P/days	104.022	0.002
D_1/mas	6.4	0.3
D_2/mas	8.5	0.1
$\Delta m_{800\mathrm{nm}}$	$-0\overset{\mathrm{m}}{.}05$	$0\overset{\mathrm{m}}{.}05$
$\Delta m_{550\mathrm{nm}}$	$+0\overset{\mathrm{m}}{.}15$	$0\overset{\mathrm{m}}{.}05$
$\Delta m_{450\mathrm{nm}}$	$+0\overset{\mathrm{m}}{.}28$	$0\overset{\mathrm{m}}{.}10$
Note:	$\Delta m \equiv m_{\mathrm{Aa}} - m_{\mathrm{Ab}}$	

TABLE 3. Orbital elements from spectroscopy and speckle interferometry (Barlow *et al.* 1993).

a	55.7±0.3 mas
i	$136\overset{\circ}{.}7 \pm 0\overset{\circ}{.}4$
Ω	$40\overset{\circ}{.}9 \pm 0\overset{\circ}{.}3$
T_0	JD 2442119.249 ± 0.077 days
P	104.0233 ± 0.0008 days
K_1	26.05±0.10 km/s
K_2	27.40±0.30 km/s
Note:	52 orbital periods added to T_0 give T_0 = 2447528.461.

Examples from:

Hummel, C., et al. 1994, AJ, 107, 1859
Barlow, D., et al. 1993, PASP, 105, 476

State of the art – visual 2

Steps in the analysis:

- derive masses: ± 2%
- derive distance (here 13.3 ± 0.1 pc), absolute magnitudes, and colors
- derive linear radius, $R/R_{\odot} = 215(D/2)/\pi$, where π is the orbital parallax
- derive T from absolute bolometric flux and diameters...or
- derive T from abolute magnitudes, bolometric correction and diameters
- derive total luminosity from integrated flux measurements and distance (here $L_{\rm Aa+Ab}/L_{\odot} = 153$)
- compare total luminosity to luminosity based on T and diameters
- compare T and colors to calibrations

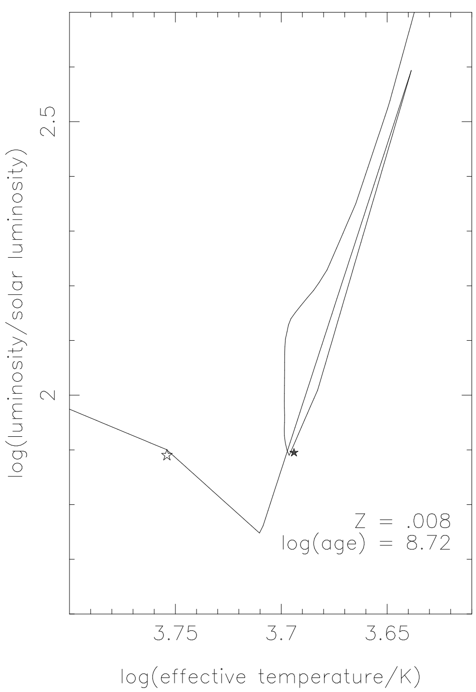


TABLE 4. Limb-darkening coefficients A and magnitudes.

	Band, effective wavelength [nm]		
	$I_{\rm C}$	V	B
	800	550	450
$A^{a}_{\rm Aa}$	0.540	0.749	0.885
$A^{a}_{\rm Ab}$	0.476	0.681	0.804
$M_{\rm Aa}$	$-0^{\rm m}.62 \pm 0^{\rm m}.04$	$0^{\rm m}.29 \pm 0^{\rm m}.04$	$1^{\rm m}.16 \pm 0^{\rm m}.05$
$M_{\rm Ab}$	$-0^{\rm m}.57 \pm 0^{\rm m}.04$	$0^{\rm m}.14 \pm 0^{\rm m}.04$	$0^{\rm m}.88 \pm 0^{\rm m}.05$
$m^{b}_{\rm Aa+Ab}$	$-0^{\rm m}.73$	$0^{\rm m}.08$	$0^{\rm m}.88$

[a] Limb-darkening coefficients (adopted, see Sec. 3)
[b] Johnson *et al.* (1966)

TABLE 5. Physical parameters of the Capella stars.

Parameter	Ab	Aa
Mass/$\mathcal{M}_{\odot}$	2.56 ± 0.04	2.69 ± 0.06
Luminosity/$L_{\odot}$	77.6 ± 2.6	78.5 ± 1.2
$T_{\rm eff}$/K	5700 ± 100	4940 ± 50
$(B-V)$	$0^{\rm m}.74 \pm 0^{\rm m}.07$	$0^{\rm m}.87 \pm 0^{\rm m}.08$
$(V-I)_{\rm C}$	$0^{\rm m}.71 \pm 0^{\rm m}.05$	$0^{\rm m}.91 \pm 0^{\rm m}.06$
Radius/$R_{\odot}$	9.2 ± 0.4	12.2 ± 0.2

Examples from:

Hummel, C., et al. 1994, AJ, 107, 1859
Barlow, D., et al. 1993, PASP, 105, 476

State of the art – eclipsing binaries 1

Observational data:

- Photometry: i, $R/a \pm 1\%$, $\Delta m_\lambda \pm 0.05$ mag, $m_{1,2} \pm 0.02$ mag
- Spectroscopy: $M_{1,2}(\sin i)^3$, $K_{1,2} \pm 0.1$ km/s

i	88°.45 5		r_A	0.0380 5
$e\cos\omega$	-0.0634 3		k	1.613 10
$e\sin\omega$	0.178 10		r_B	0.0613 10
e	0.189 10			
ω	109°.6 1.0			

	U	B	V	R	I
J_B/J_A	0.184	0.296	0.408	0.484	0.556
L_B/L_A	0.433	0.736	1.021	1.224	1.414

	u	v	b	y
J_B/J_A	0.198	0.252	0.356	0.418
L_B/L_A	0.472	0.621	0.872	1.038

K_A (km s^{-1})	50.90±0.08	50.95±0.08
K_B (km s^{-1})	49.24±0.07	49.20±0.08
γ_A (km s^{-1})	-1.76±0.06	-1.76±0.06
γ_B (km s^{-1})	-1.92±0.06	-1.92±0.06
e_A	0.188±0.0020	0.1855±0.0016
e_B	e_B=e_A	0.1895±0.0015
ω_A	109°.9±0°.6	111°.0±0°.5
ω_B	ω_B=ω_A+180°	289°.6±0°.4
σ_A (km s^{-1})	0.43	0.40
σ_B (km s^{-1})	0.37	0.37
M_B/M_A	1.034±0.002	
asini ($R_\odot$)	47.78±0.05	
$M_A \sin^3 i$ ($M_\odot$)	1.194±0.004	
$M_B \sin^3 i$ ($M_\odot$)	1.234±0.005	

Example from: Andersen, J., et al. 1988, A&A, 196, 128
"AI Phoenicis: a case study in stellar evolution"

State of the art – eclipsing binaries 2

Steps in the analysis:

- derive masses ($\pm 1\%$)
- obtain effective temperature from multi-color photometry
- derive luminosities ($\pm 10\%$) from effective temperature and radius.
- derive distance luminosity, bolometric correction, and apparent magnitude

Absolute dimensions:		
$M/M_\odot$	1.1954±0.0041	1.2357±0.0045
$R/R_\odot$	1.816±0.024	2.930±0.048
log g (cgs)	3.997±0.012	3.596±0.014
$v\sin i$ (km s^{-1})	4±1	6±1
[Fe/H]	-0.14±0.1	
Photometric data:		
T_e (K)[a]	6310±150	5010±120
M_{bol}[b]	3.06±0.11	3.03±0.11
log $L/L_\odot$	0.67±0.04	0.69±0.04
$B.C.$[b]	-0.06	-0.26
M_V	3.12±0.11	3.29±0.11
Distance (pc)	162±6 (no reddening)	
(U,V,W)[c] (km s^{-1})	(+30,-23,+5)	

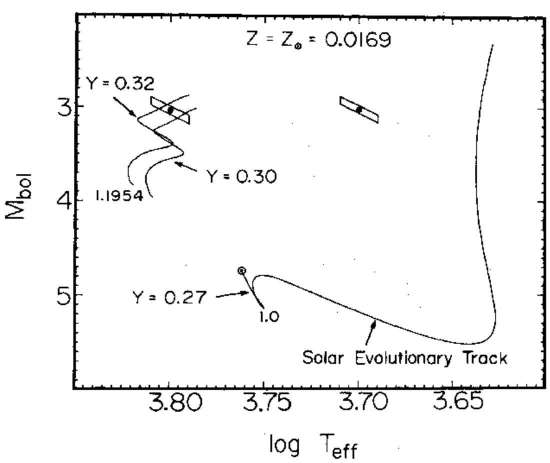
[a] VandenBerg and Hrivnak (1985)
[b] Popper (1980, Table 1), assuming $M_{bol,\odot}$ = 4.75
[c] proper motions from SAO

Example from: Andersen, J., et al. 1988, A&A, 196, 128
"AI Phoenicis: a case study in stellar evolution"

State of the art – eclipsing binaries 3

Steps in the comparison with evolutionary models

- Determine metal abundance from spectroscopic analysis, here (AI Phe) $Z = 0.012 \pm 0.003$
- Adjust helium abundance, a free parameter, by matching the luminosity, here $Y = 0.27 \pm 0.02$
- Compute track for less evolved component for stellar masses and determine age
- Compute evolutionary state for more evolved component and compare with observations. Note that there are no free parameters.



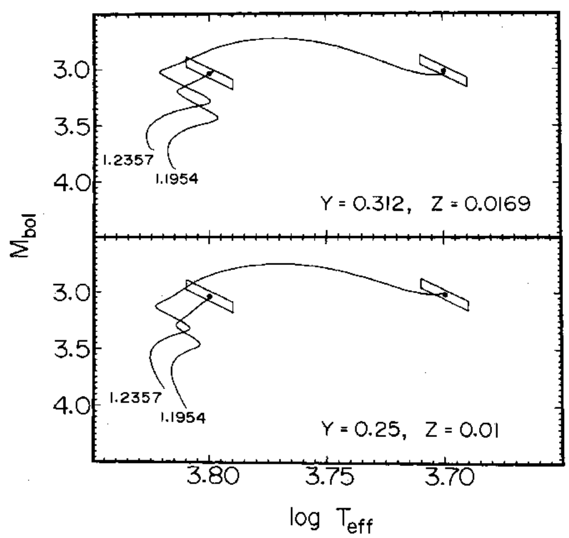


Table 8. Estimated ages (10^9 yr) for the components of AI Phe

	Z	Y	Star A	Star B
Models:	0.0169	0.312	3.70±0.06	3.80±0.01
	0.0100	0.250	4.65±0.06	4.65±0.02
Observed:	0.0122 ± 26	0.27 ± 2	4.1 ±4	

Example from: Andersen, J., et al. 1988, A&A, 196, 128
"AI Phoenicis: a case study in stellar evolution"

Some useful calibrations

Bolometric correction

Gubochkin, A. N. & Miroshnichenko, A. S. 1991, Kin. and Phys. of Cel. Bodies, 7, 59

$$BC = \begin{cases} -0.0508T_3^2 + 0.762T_3 - 2.831 & (4.7 < T_3 < 10,\ T_3 \equiv T/1000) \\ 0.0032T_3^2 - 0.260T_3 + 1.978 & (10 < T_3) \end{cases}$$

Flower, P. J. 1996, ApJ, 469, 355 (corrected)

$$BC = \begin{cases} -19053.73 + 15514.49\log T - 4212.788(\log T)^2 + 381.4763(\log T)^3 & (\log T < 3.70) \\ -37051.02 + 38567.26\log T - 15065.149(\log T)^2 + 2617.2464(\log T)^3 & \\ \qquad -170.62381(\log T)^4 & (3.70 < \log T < 3.90) \\ -118115.45 + 137145.97\log T - 63623.381(\log T)^2 + 14741.2924(\log T)^3 & \\ \qquad -1705.87278(\log T)^4 + 78.873172(\log T)^5 & (3.90 < \log T) \end{cases}$$

Interstellar extinction

Schmidt-Kaler, Th. 1982, Landolt-Börnstein series VI/2b

$$\frac{A_V}{E_{B-V}} = 3.30 + 0.28(B-V)_0 + 0.04E_{B-V},\ A_V = V - V_0,\ E_{B-V} = (B-V) - (B-V)_0$$

Effective temperature

Flower, P. J. 1996, ApJ, 469, 355 (corrected)

$$\log T = \begin{cases} 3.9791 - 0.6550(B-V) + 1.7407(B-V)^2 - 4.6088(B-V)^3 & \\ \qquad +6.7926(B-V)^4 - 5.3969(B-V)^5 + 2.1930(B-V)^6 & \\ \qquad -0.3595(B-V)^7 & \text{MS, Subgiants, Giants} \\ 4.0126 - 1.0550(B-V) + 2.1333(B-V)^2 - 2.4598(B-V)^3 & \\ \qquad +1.3494(B-V)^4 - 0.2839(B-V)^5 & \text{Supergiants} \end{cases}$$

Zero-magnitude bolometric flux outside the Earth's atmosphere

$(m_{\rm bol} = 0)$ star $\equiv 2.48\,10^{-5}$ erg cm^{-2} s^{-1}

Zero-magnitude fluxes for the Johnson system

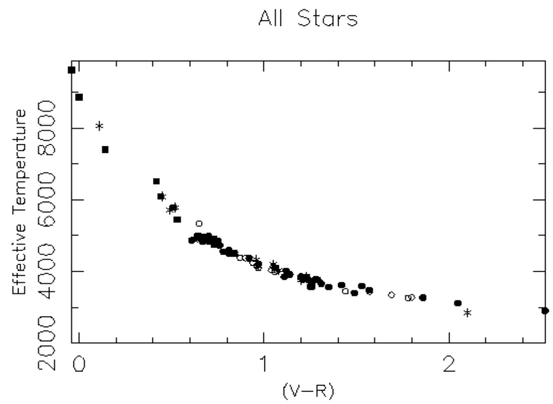
Filter	$\lambda_{\rm eff}$ [μm]	$F_{\nu,0}$ [W/(m^{2}Hz)]	$F_{\lambda,0}$ [erg/(cm^{2}sÅ)]
U	0.36	$1.88\,10^{-23}$	$4.22\,10^{-9}$
B	0.44	$4.65\,10^{-23}$	$6.40\,10^{-9}$
V	0.55	$3.95\,10^{-23}$	$3.75\,10^{-9}$
R	0.70	$2.87\,10^{-23}$	$1.75\,10^{-9}$
I	0.90	$2.24\,10^{-23}$	$8.4\,10^{-10}$

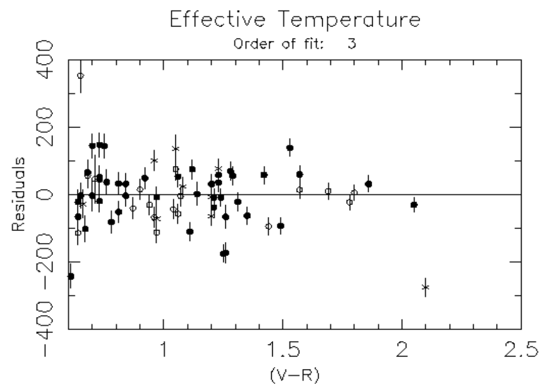
Stellar diameter, effective temperature, and surface brightness calibrations

Effective temperatures are derived from angular diameters and bolometric fluxes.

$$F_{\rm bol} = \sigma(\theta/2)^2 T_{\rm eff}^4,$$

where F is the flux measured above the atmosphere, σ is the Stefan-Boltzmann constant ($\sigma = 5.67\,10^{-5}$ erg cm^{-2} s^{-1} K^{-4}), θ is the limb-darkened angular diameter in radians.



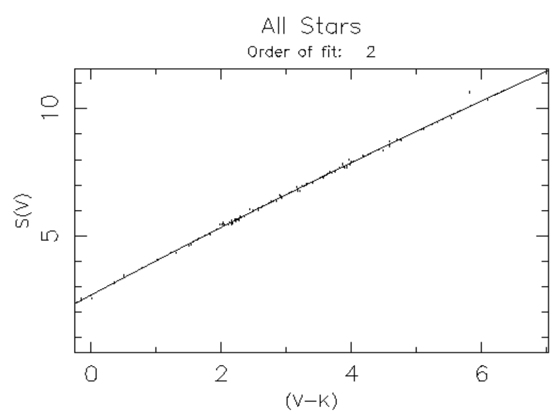


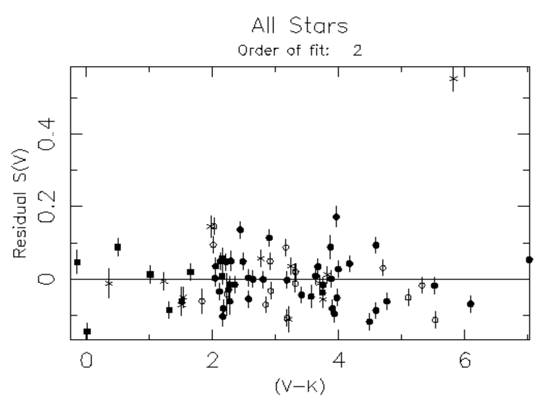
(Mozurkewich, D., et al. *in prep*, based on data of the Mark III stellar interferometer.)

The stellar surface brightness is defined through

$$S_V = V + 5\log(\theta),$$

where θ is measured in milliarcseconds.



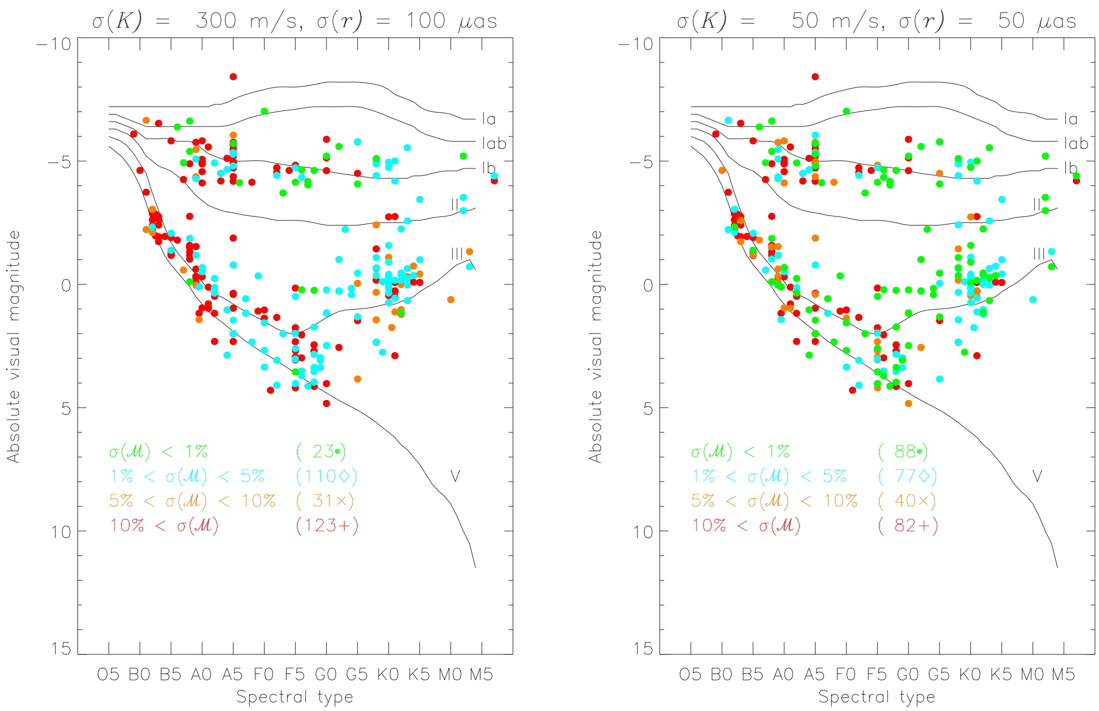


These calibrations can be used to determine limb-darkened diameters of stars of any luminosity class.

$$S_V = \left\{ \begin{array}{l} 2.646 + 4.235(V-R) - 0.022(V-R)^2 - 0.102(V-R)^3 \\ 2.680 + 1.361(V-K) - 0.015(V-K)^2 \end{array} \right.$$

An outlook for interferometric observations of spectroscopic binaries

- Input catalog: Eighth Catalog of the Orbital Elements of Spectroscopic Binary Systems (Batten et al. 1989)
- Select systems with $V < 6$
- Estimate a and $K_{1,2}$ based on spectroscopic elements and spectral types (assume components are identical), select systems with $a > 1$ mas
- Set uncertainties $\sigma(K)$ and $\sigma(\rho)$
- Estimate accuracy of masses obtainable



An outlook for astrometric observations of binaries – FAME

- Measure absolute orbits of binary components
- Measure orbital inclinations
- Measure parallaxes and derive luminosities and total masses

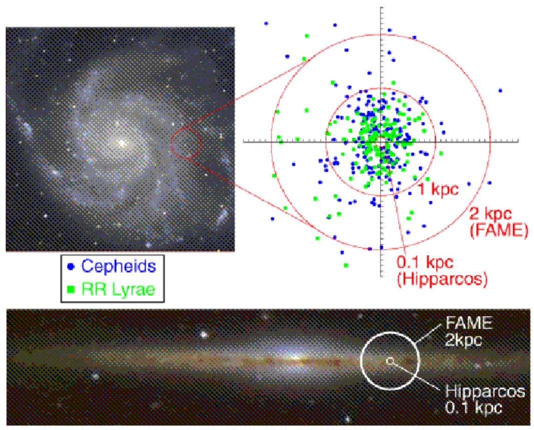


Figure 2-1. Hippacros and FAME
Observation Coverage in the Milky Way

Mission specifics

- Parallaxes and positions errors: 50μas at $V = 9$.
- Parallaxes to 10% or better out to 2.5 kpc distance from the Sun
- Magnitude limit: $V = 15$.

Example of single-lined spectroscopic binary η Pegasi

Parameter	Mark III, NPOI, RV	Hipparcos	FAME
K_1/(km/s)	14.20 ± 0.19		
a/mas	45.02 ± 0.06		
a_{PC}/mas^b		13.60 ± 0.88	
$i/°$	68.28 ± 0.05	70.6 ± 3.1	
$\Omega/°$ (J2000.0)	20.90 ± 0.04	23.6 ± 3.5^c	
T (JD−244E4)	7140.3 ± 0.4	7170 ± 9	
e	0.1677 ± 0.0009	0.155 ± 0.016^d	
$\omega/°$	-5.5 ± 0.1	5.6 ± 5.5^d	
P/days	817.41 ± 0.04	818 ± 2.2^d	
$\pi('')$		15.18 ± 0.56	15.18 ± 0.05
$\mathcal{M}_1/\mathcal{M}_\odot$		3.2 ± 0.4	3.20 ± 0.04
$\mathcal{M}_2/\mathcal{M}_\odot$		2.0 ± 0.2	1.98 ± 0.03

^{a}Systematic error 0.1 mas, ^{b}Photo center, ^{c}Orbit of secondary
^{d}Adopted from spectroscopy, ^{e}Adopted from photometry