

Theory: (Interferometric) Narrow-Angle Astrometry

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Outline



- Astrometry
- Utility of Narrow-Angle Astrometry
- Interferometric Astrometry
- Ground-Based Narrow-Angle Interferometric Astrometry
- Tolerances
- Summary



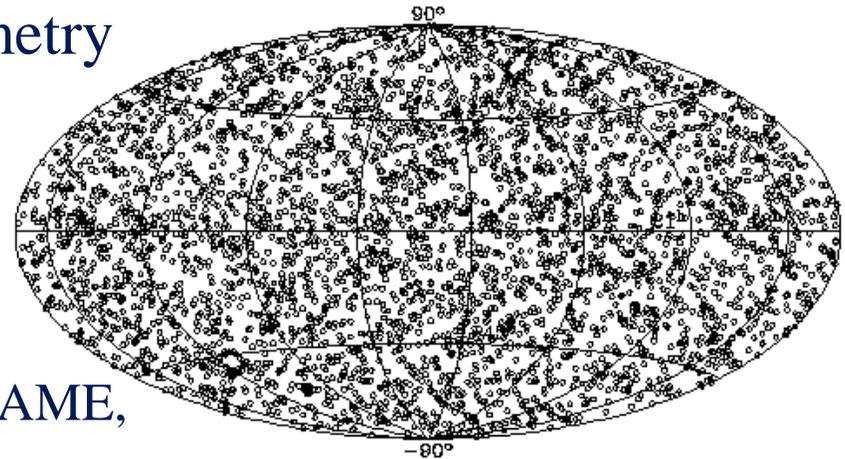
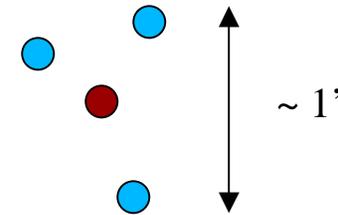
Astrometry

- A Few Working Descriptions...
- Astrometry: Measurement of the *Apparent* Positions and Motions of Astronomical Objects on the Celestial Sphere
 - Projection of 3-space positions and motions onto the apparent space of astronomical angles (such as α and δ)
 - To some approximation, inertial observers observe objects in linear motion
 - Because near-Earth observers move non-inertially (e.g. we orbit the sun), objects at finite distance exhibit non-linear motions. We call this parallax (π)...
 - Parallax was first observationally exploited by Bessel to measure the distance to 61 Cyg (well before electricity)
 - Canonical astrometric representation is two position angles (α, δ), two time rates of change (μ_α, μ_δ), and parallax (π)

Narrow-Angle Astrometry



- Narrow-Angle Astrometry: Relative astrometry over a *small* field
 - Operative Issue: small?
 - Ground-based: Tens of arcminutes to tens of arcseconds
 - Space-based (SIM): 1 -- 2 deg
- To be contrasted with Wide-Angle or Global Astrometry: Astrometry Over the 4π Celestial Sphere
 - Absolute parallax (vs relative parallax)
 - Quasi-static frame
 - E.G.: FK4, NPOI, Hipparcos, FAME, SIM, GAIA



Narrow-Angle Astrometry



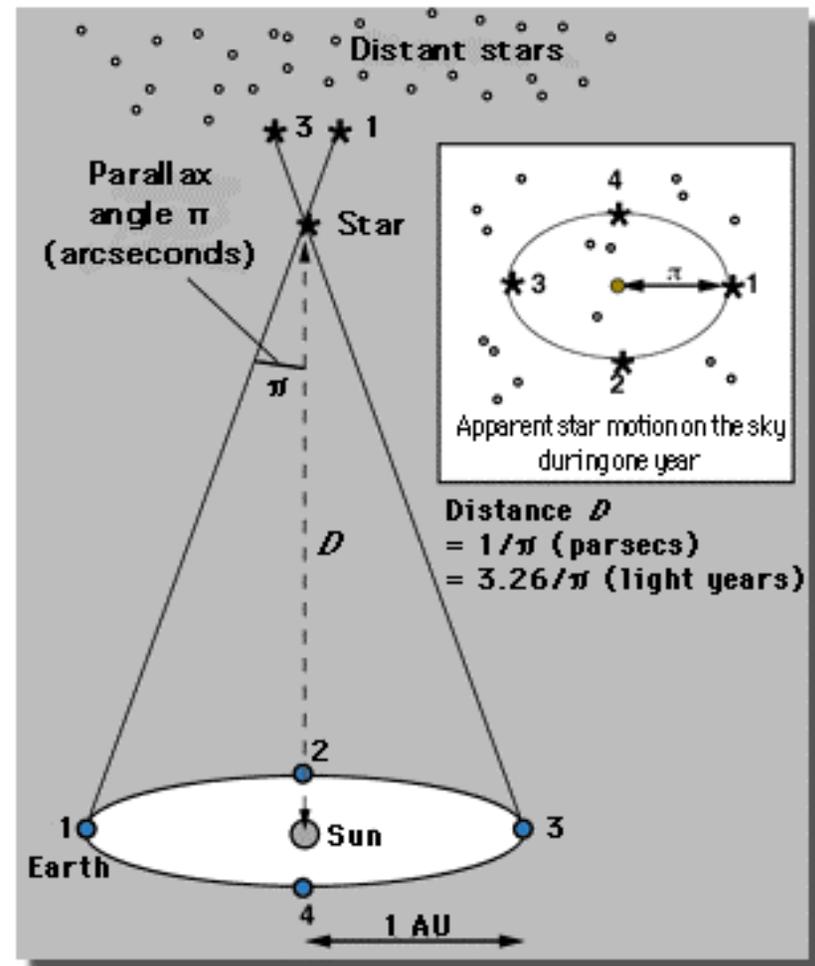
(cont)

- In the Narrow-Angle Regime:
 - Unless reference objects are thought to be distant (e.g. QSOs), narrow-angle reference frames are not static
 - Net proper motion in reference objects results in linear drift in the reference frame
 - Drift does not effect our ability to detect/measure relative *acceleration* (i.e. *non-linear* motions)

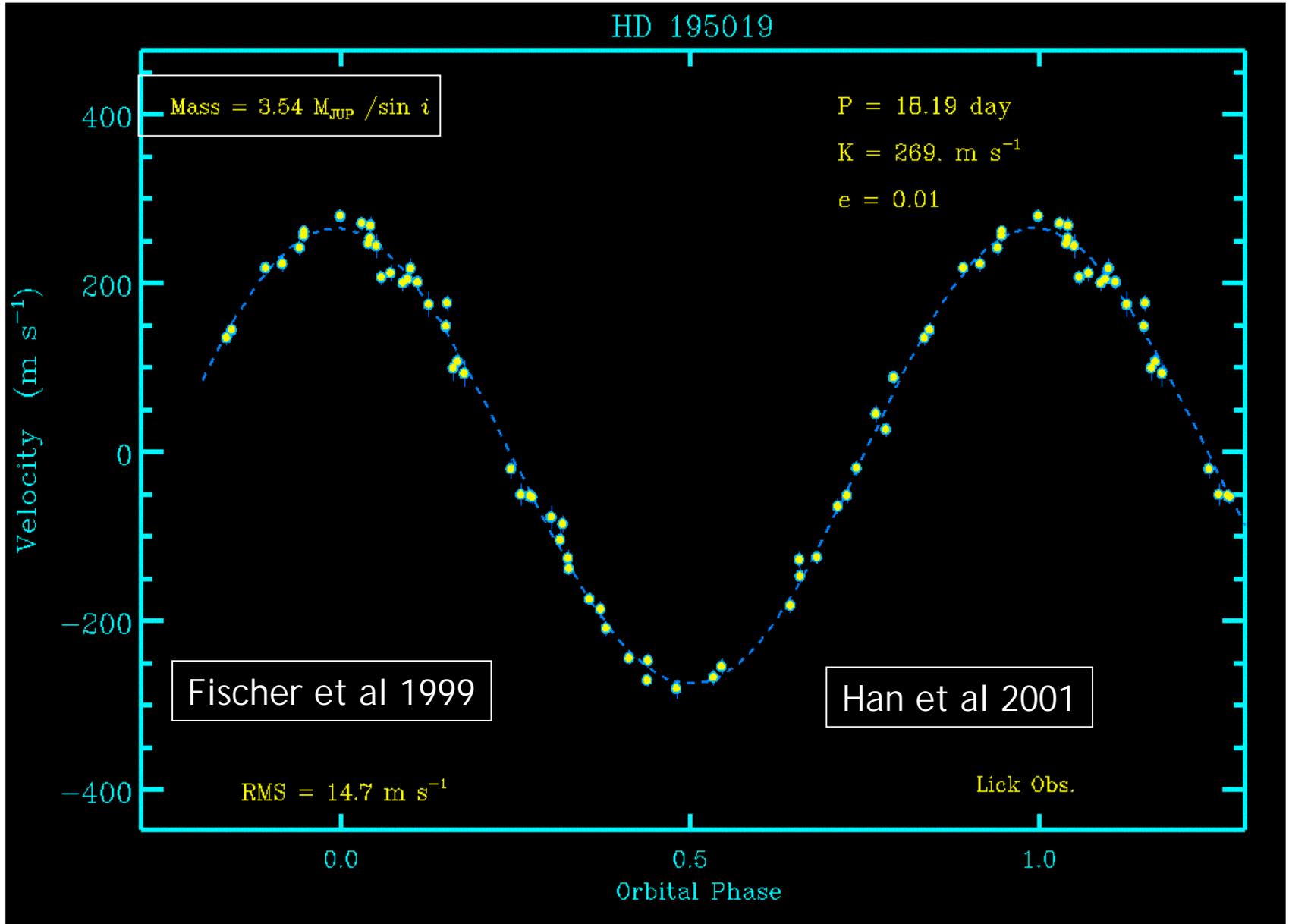
- So Can We Think of Something Interesting to do With Non-Linear (Relative) Motions of Objects?

Trigonometric Parallax

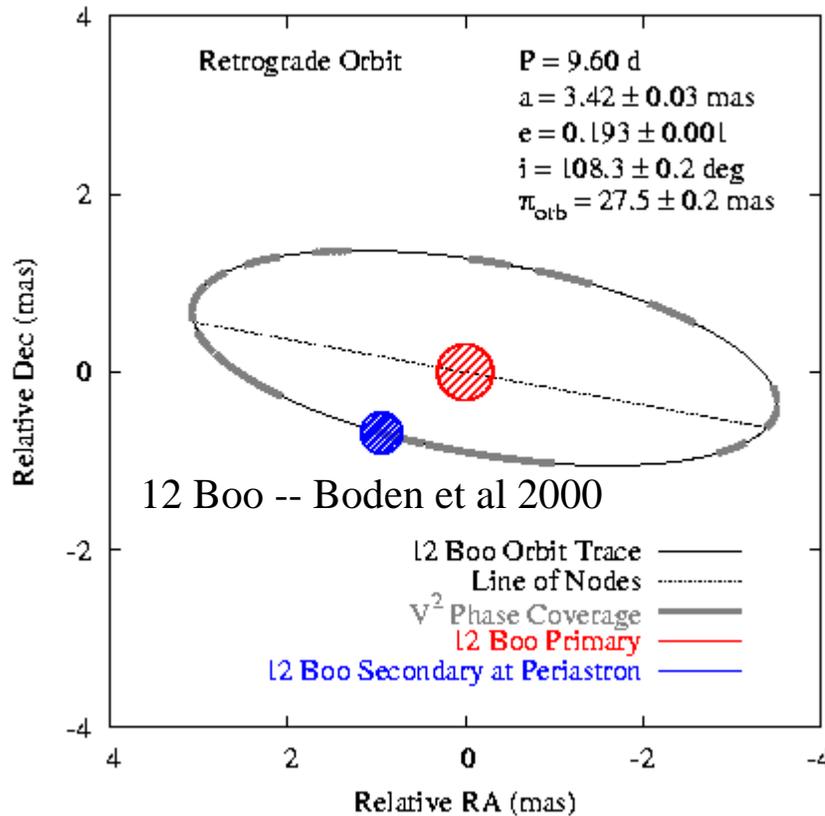
- Stars at Finite Distance Exhibit *Apparent* Elliptical Motions
- The Amplitude of These Apparent Motions is Related to the Distance to the Star
- Is the Distance to a Star Interesting?



RV-Only Orbit



Binary (& Similar) Systems

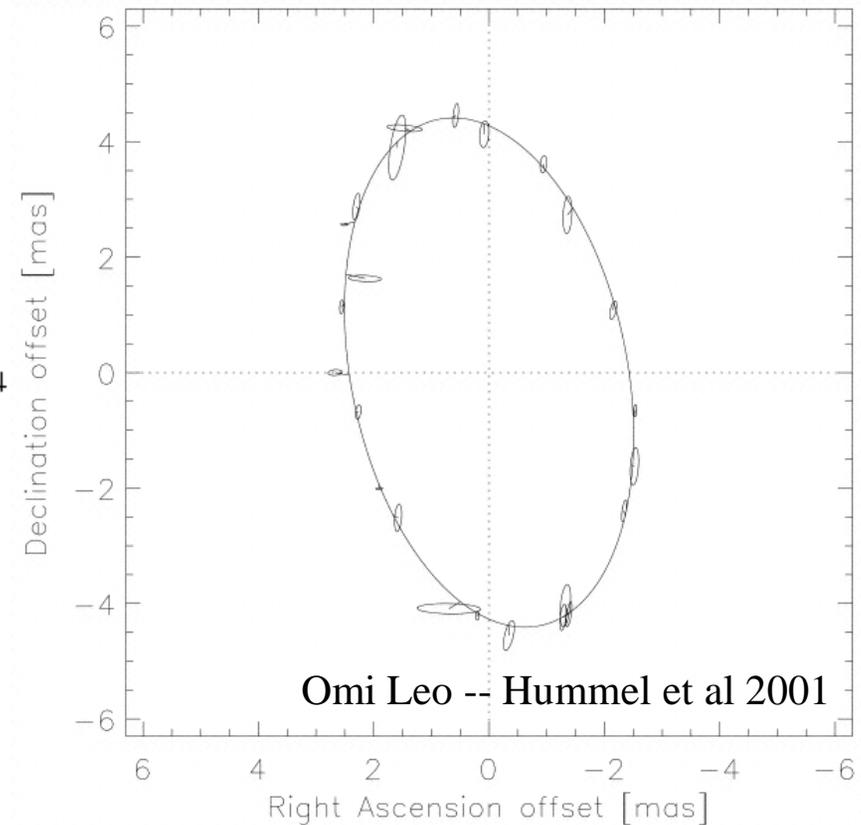


Keplerian Ellipse Projected on Sky is Still Ellipse

So How Does Relative Astrometry Assist in Determining Physical Parameters?

Motion in Time (Kepler Equation)
 Allows Unambiguous Euler Angle Determination

=> Masses and Distances

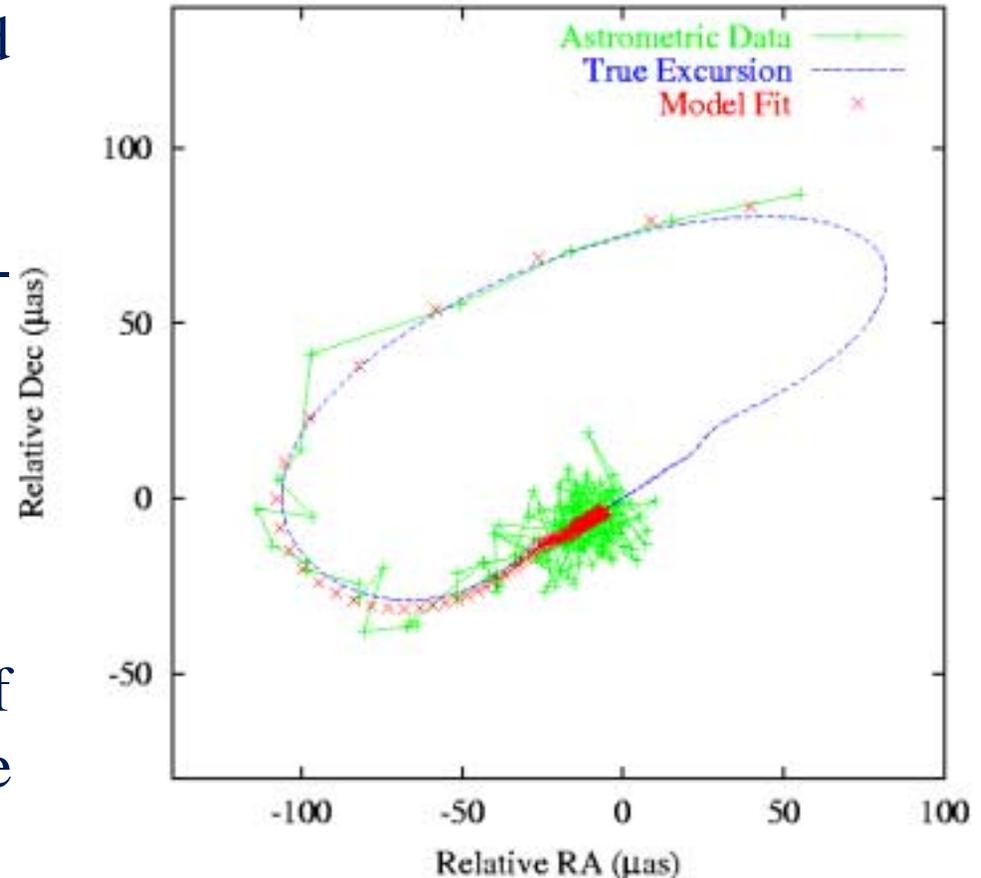


Astrometric Microlensing



- Microlensing (Unresolved Gravitational Lensing) Generates Astrometric Effects (as Well as Better-Known Photometric Effects)

- Differential Astrometry of Event Breaks Mass/Range Degeneracy, Allows Unambiguous Determination of Lens Mass



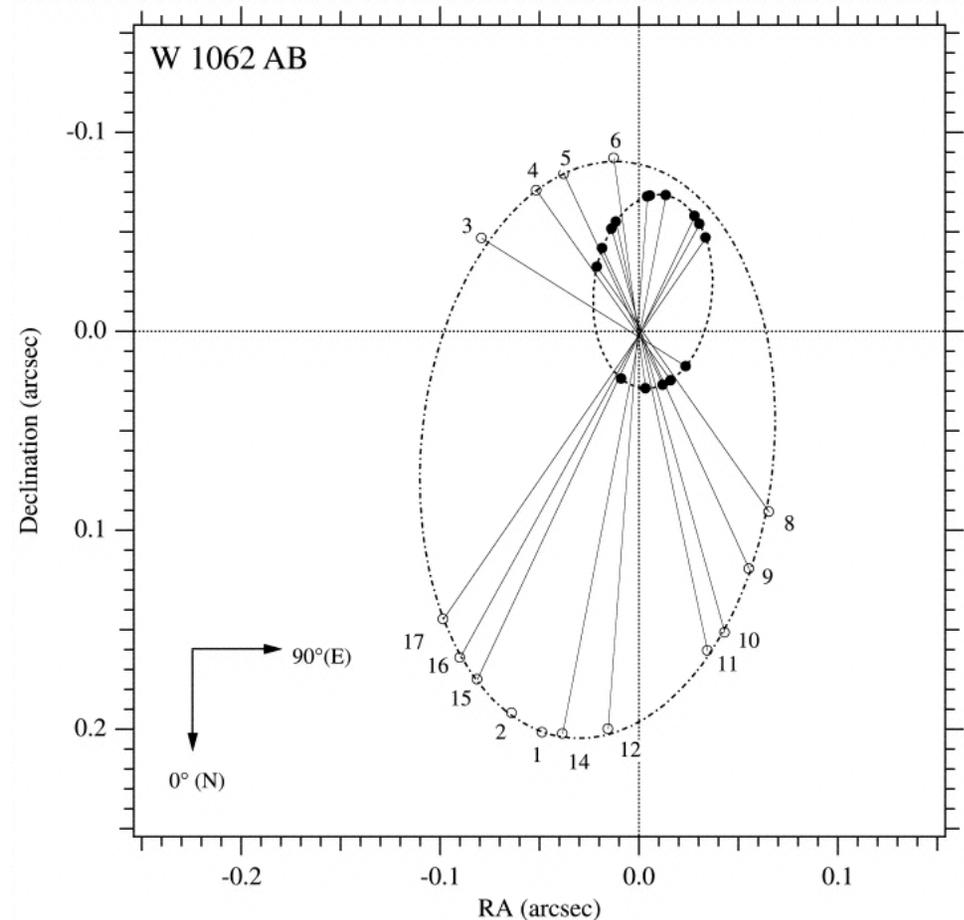
Boden, Shao, & Van Buren 1998
 Paczynski 1997
 Gould & Salim 1999
 Safizadeh, Dalal, & Griest 1999
 SIM KP (Gould PI)

(Partial) List of NA



Techniques

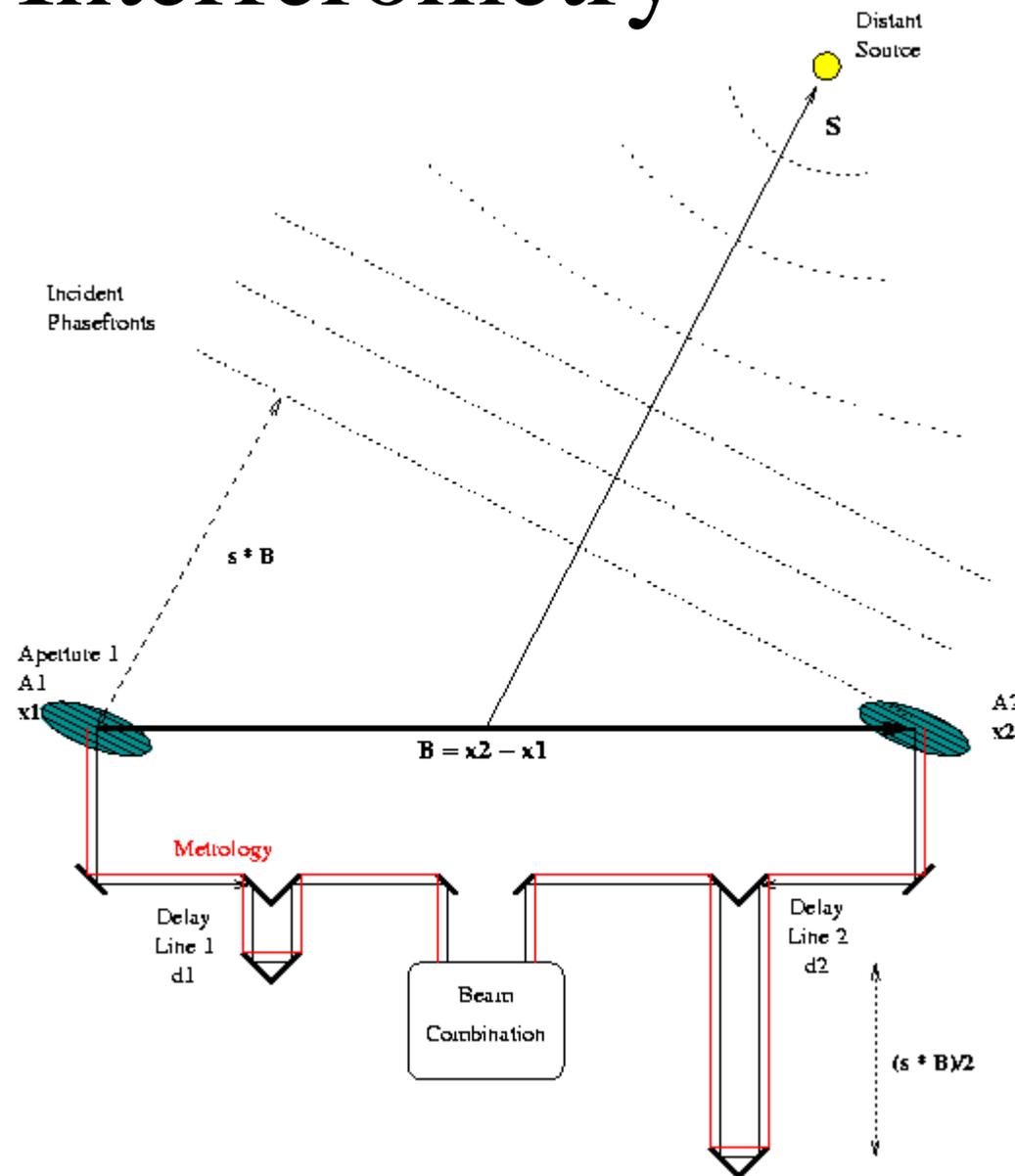
- CCD Astrometry
 - Monet et al
 - Pravdo & Shaklin
- Speckle Astrometry
 - McAlister, Hartkopf, & Mason
 - Ghez, Leinert, etc.
- HST FGS
 - Benedict, Franz, Nelan, Henry, McAruthur, et al.
- L-B Interferometric Visibility
 - Hummel, Armstrong, Pan, Boden



Benedict et al 2001



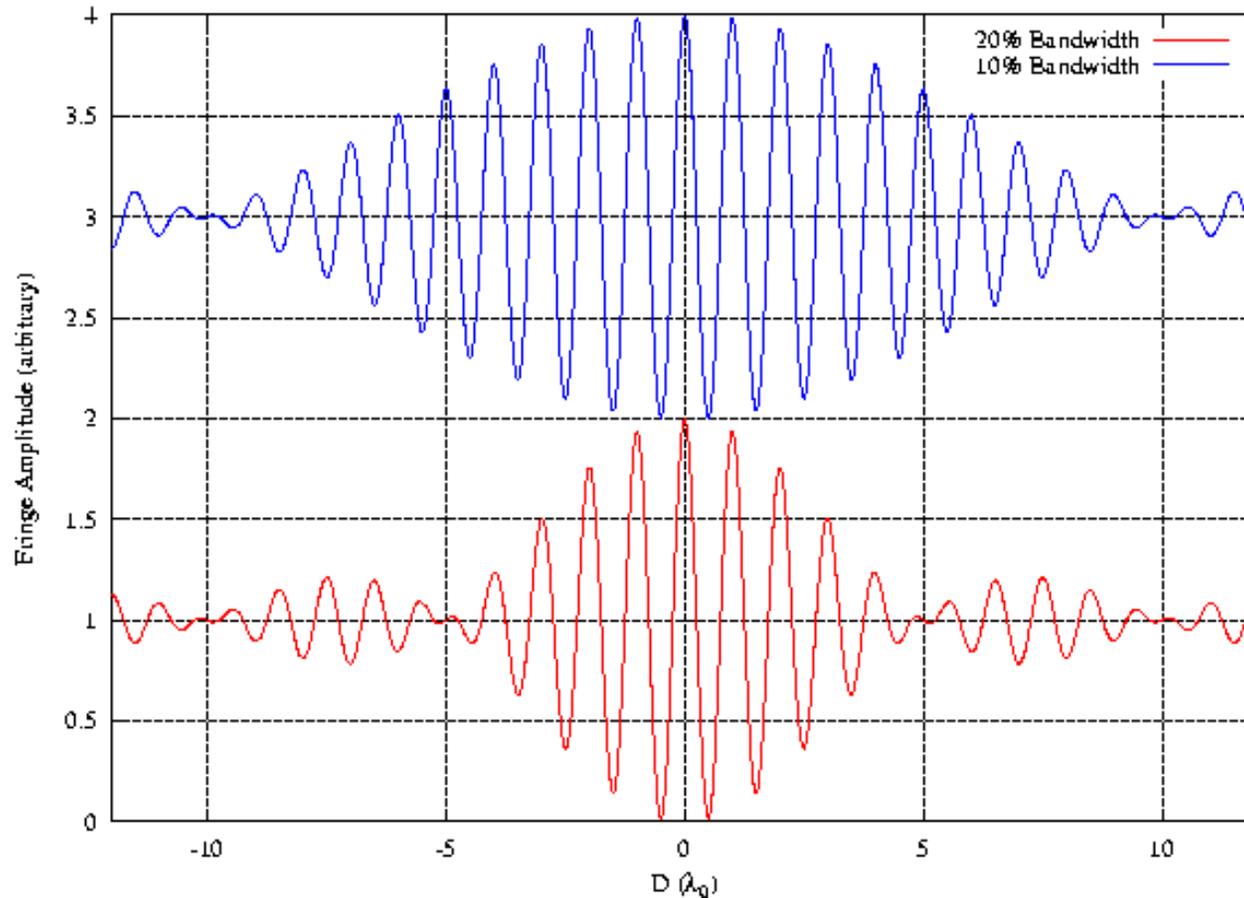
Consider Long-Baseline Interferometry



Features:

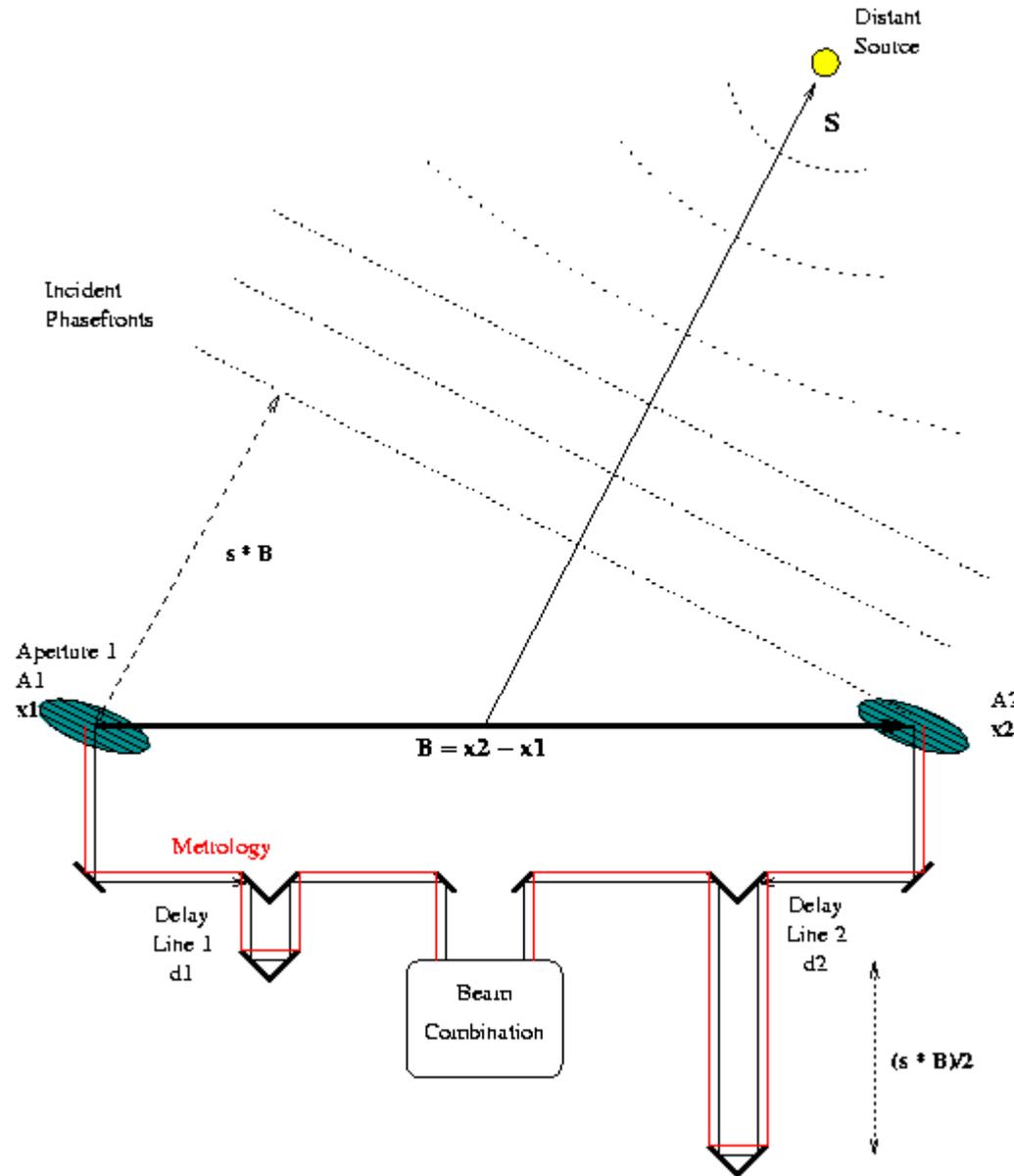
- High Angular Resolution
- Small Field of Regard
- Impossible to Build

Polychromatic Fringe Position



- Polychromatic Fringes Where Delays are Equilibrated
- Proxy for Object Position in Space of Relative Delay

Interferometric Astrometry



➤ Interferometers Do not Measure Angles, They Measure *Delays* as *Proxy* for Angles

➤ Astrometric Equation

$$d = \hat{s} \bullet \underline{B} + C$$

➤ Bias *C* Accounts for Measurement Imperfections

The Paper That Started It



A11

Astron. As

“One is that a long-baseline interferometer can find ...reference stars very near an arbitrary target star, so ...atmospherically-induced motions of both stars are highly correlated.”

ONOMY
AND
PHYSICS

Potential of long-baseline infrared interferometry for narrow-angle astrometry

Shao & Colavita 92

M. Shao and M.M. Colavita

Jet Propulsion Laboratory,

Received February 10, accep

Abstract. Narrow-angle astrometry has many astrophysical applications, from the measurement of stellar parallaxes to the search for planets around nearby stars. Long-focus telescopes with photoelectric detectors have achieved accuracies of a few milliarcsec in 1 h. This accuracy is limited primarily by atmospheric turbulence, and is consistent with models of atmospheric turbulence. However, applying turbulence models to observations with long-baseline interferometers yields astrometric errors that are far smaller than can be achieved with long-focus telescopes. The predictions for the ultimate accuracy of ground-based narrow-angle astrometry using long-baseline infrared (2.2 μm) stellar interferometers are promising. With the construction of a telescope like Mauna Kea, the astrometric limit for a 1 h astrometric measurement is expected to be of the order of 10 microarcsec for a 1 h of integration. This two-order-of-magnitude improvement over conventional measurements is due to two effects. One is that a long-baseline infrared interferometer can find useful reference stars very near an arbitrary target star, so that the atmospherically-induced motions of both stars are highly correlated. The second is that the baseline length can be much larger than the separation of the stellar beams in the turbulent atmosphere, resulting in a reduction in astrometric error with increasing baseline length.

“The second is that the baseline length can be much larger than the separation of the stellar beams in the turbulent atmosphere, resulting in a reduction in astrometric error with increasing baseline length.”

In narrow-angle astrometry, the position of a target star is measured relative to a number of reference stars in a wide field of view, typically $\sim 0.5^\circ$. A detailed analysis using conventional models of turbulence (Lindgren 1980), as well as empirical data (Han 1988), shows that the turbulence-limited accuracy is proportional to the star separation to the 1/3 power, and to first order, is independent of the size of the telescope. As a result, for many years the interferometry community did not perceive narrow-angle astrometry as an area where long-baseline interferometry would offer a significant advantage over conventional techniques.

The differential motion of stars attributable to turbulence occurs because the beams of light from each star follow different paths through the atmosphere, as shown schematically in Fig. 1. If one traces the light from two stars $0.25''$ apart from the telescope up through the atmosphere, at the top of the turbulent atmosphere, ~ 10 km, the two beams are separated by ~ 50 m. In

Long-Baseline Astrometry

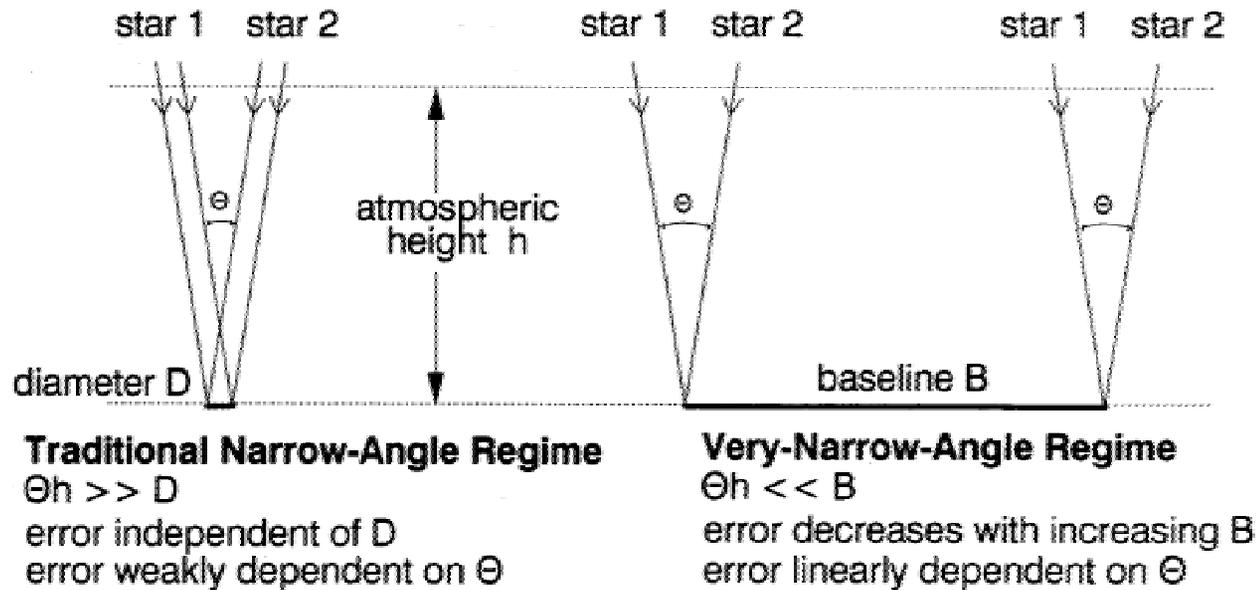


Fig. 1. Schematic of the atmospheric paths for narrow-angle astrometry with short and long baselines

- Atmospheric Noise Dominated By Low-Frequency Component
- Longer-Baselines Increase the *Correlation* of Atmospheric Turbulence
- This Correlation is What We Exploit Differentially

Experimental Verification

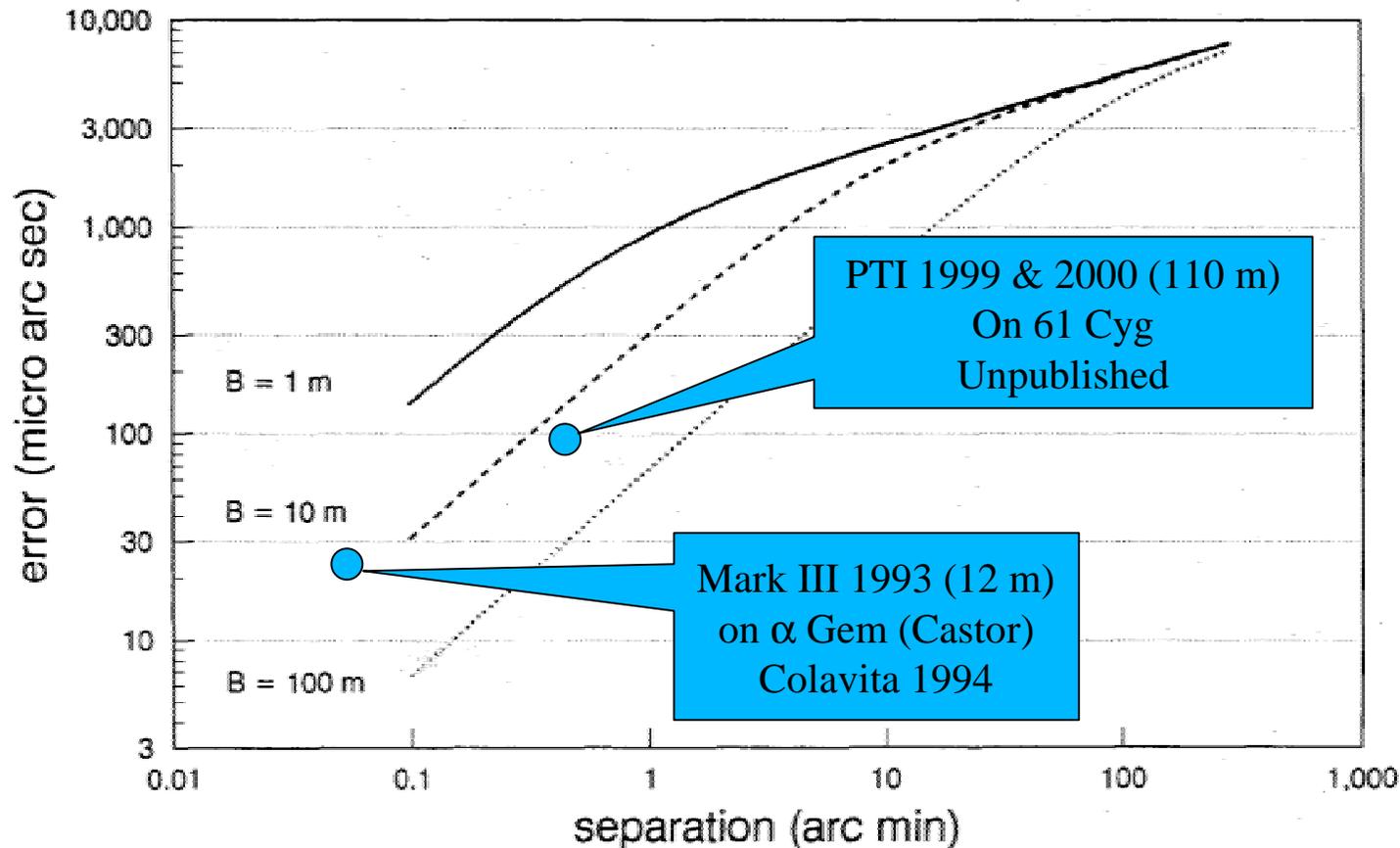
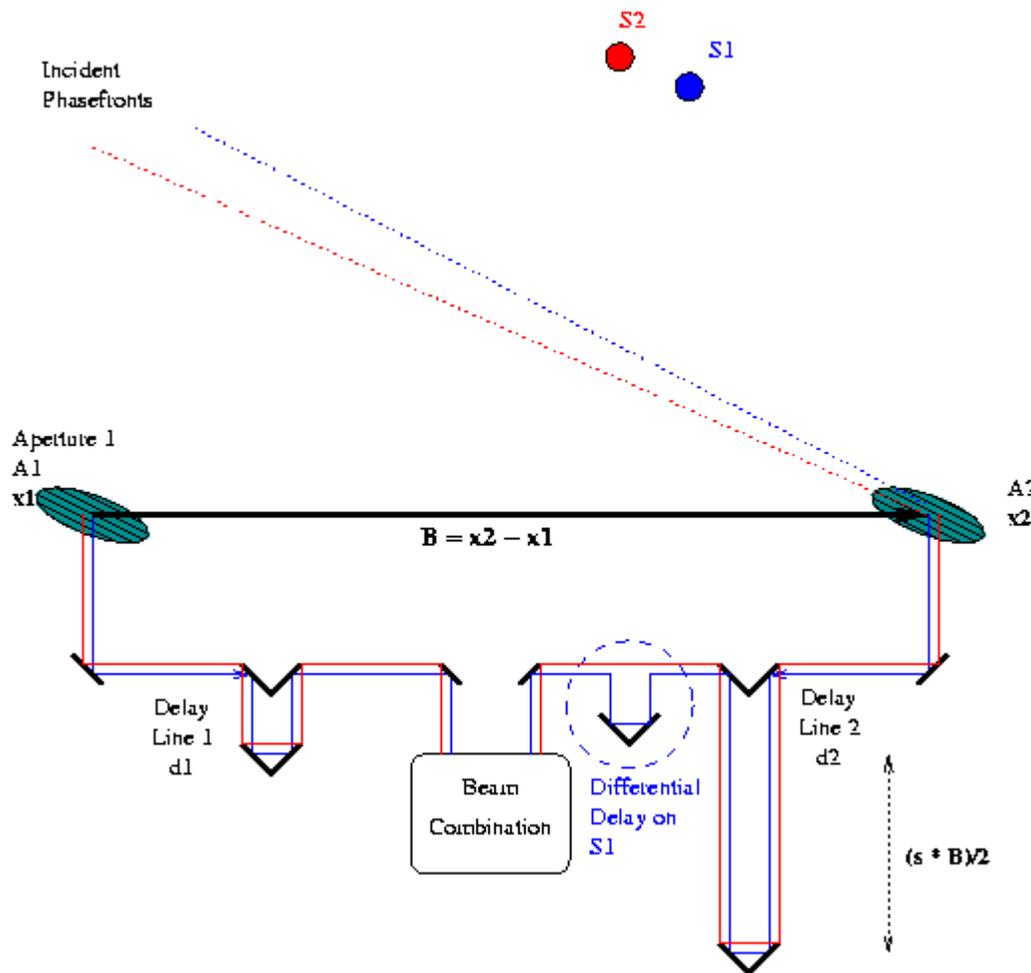


Fig. 2. Narrow- and very-narrow-angle astrometric error for several baseline lengths using measured Mauna Kea turbulence profiles and an integration time of 1 h

Differential Interferometric Astrometry



- Two Simultaneous Beam Combiners, Independently Tracking Two Stellar Sources
- Differential Delay Mechanism
- Metrology In Each Interferometer Measuring the Relative Delay
- Relative Delay Difference is the Observable

Differential Interferometric Astrometry (cont)



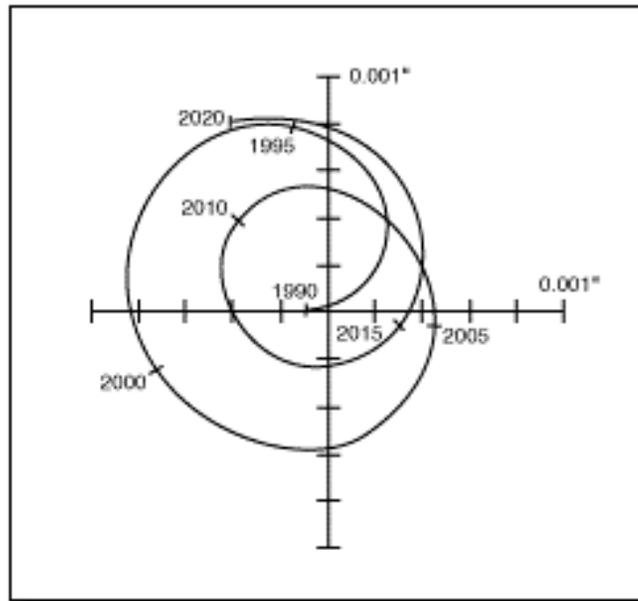
$$\begin{aligned}
 d_2 - d_1 &= (\hat{s}_2 \bullet \underline{B} + C_2) - (\hat{s}_1 \bullet \underline{B} + C_1) \\
 &= (\hat{s}_2 - \hat{s}_1) \bullet \underline{B} + (C_2 - C_1) \\
 \Delta d &= \underline{\Delta s} \bullet \underline{B} + \Delta C
 \end{aligned}$$

- Delay Difference (Δd) is Observable Proxy for Sky Separation ($\underline{\Delta s}$)
- Differential Delay Contains Instrumental Signature (ΔC) That Must Be Calibrated

Detection Space for Exo-Planets

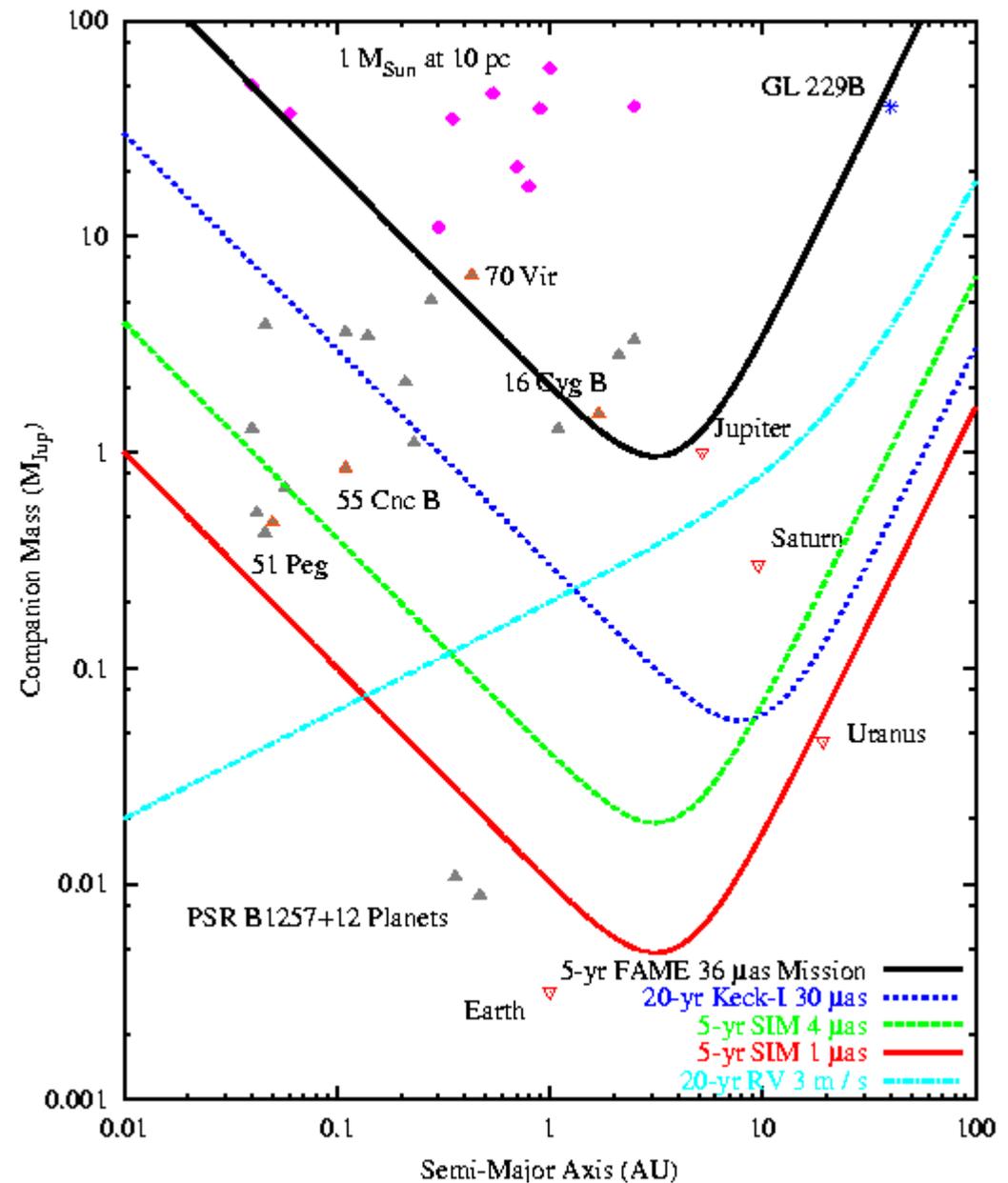


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Astrometric displacement of the Sun due to Jupiter as seen from 10 parsecs.

Scale: 1 Jupiter in a Jupiter orbit produces an 1 mas p-p apparent motion



Astrometric Implications for Exo-Planetary Studies



- Selection Effects for Astrometry Quasi-Orthogonal to RV Studies
 - Architectural studies of Planetary Systems probably only possible by combining the two techniques
- Co-Planarity in Multiple Planet Systems
- Currently Six Known Multiple Planet Systems
 - υ Andromeda -- three planets, two eccentric ($e \sim 0.3$) and likely interacting
 - ✧ Laughlin & Adams 1999 predict “stability”: non-coplanarity, non-stationarity in υ And system
 - Gl 876 -- two planets is near 2:1 mean motion resonance
 - ✧ Maybe coplanar, maybe not...
- Astrometry is (Probably) the Only Empirical Technique With the Potential of Directly Accessing the Issue of Coplanarity in Exo-Planet Systems

Interferometric Tolerances



$$\Delta s \approx \frac{\Delta d}{B}$$

$$\delta \Delta s \approx \frac{\delta \Delta d}{B} + \frac{\Delta d}{B^2} \delta B = \frac{\delta \Delta d}{B} + \Delta s \frac{\delta B}{B}$$

- $\sigma_d \sim \sigma_s * B$; $\sigma_B / B \sim \sigma_s / s$
- Take $B \sim 100 \text{ m}$, $\Delta s \sim 20''$ (10^{-4} rad)
- To Make a $10 \mu\text{as}$ ($5 * 10^{-11} \text{ rad}$) measurement:
 - Must measure Δd to $\sim 2.5 * 10^{-9} \text{ m}$ (2.5 nm)
 - Must measure B to 2.5 parts in 10^7 (25 μm)

Summary



- **Narrow-Angle Astrometry Allows Detection of (Apparent) Non-Inertial Motions of Target**
 - Parallax
 - Dynamical (e.g. Keplerian) motions
- **Atmosphere is Nice to Breathe, but Atmospheric Turbulence Limits Ground-Based Measurements**
 - Go to space (\$\$\$) -- Hipparcos, FAME, SIM, GAIA
 - Try to devise strategies around the atmosphere
- **Dual-Star, Long-Baseline Interferometric Astrometry**
 - Shao & Colavita 92
- **Tolerances (For Exo-Planetary Work)**
 - nm-class measurements of relative delay
 - μm -class measurements (knowledge) of interferometric baseline