Interferometric Narrow-Angle Astrometry: Data Analysis

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Overview

- Introduction
- Reminder: Measurement Tolerances
- (External) Baseline Knowledge
- Target Delay Measurement & Bias Calibration
- Separation Estimation
- What Next...?
- Summary

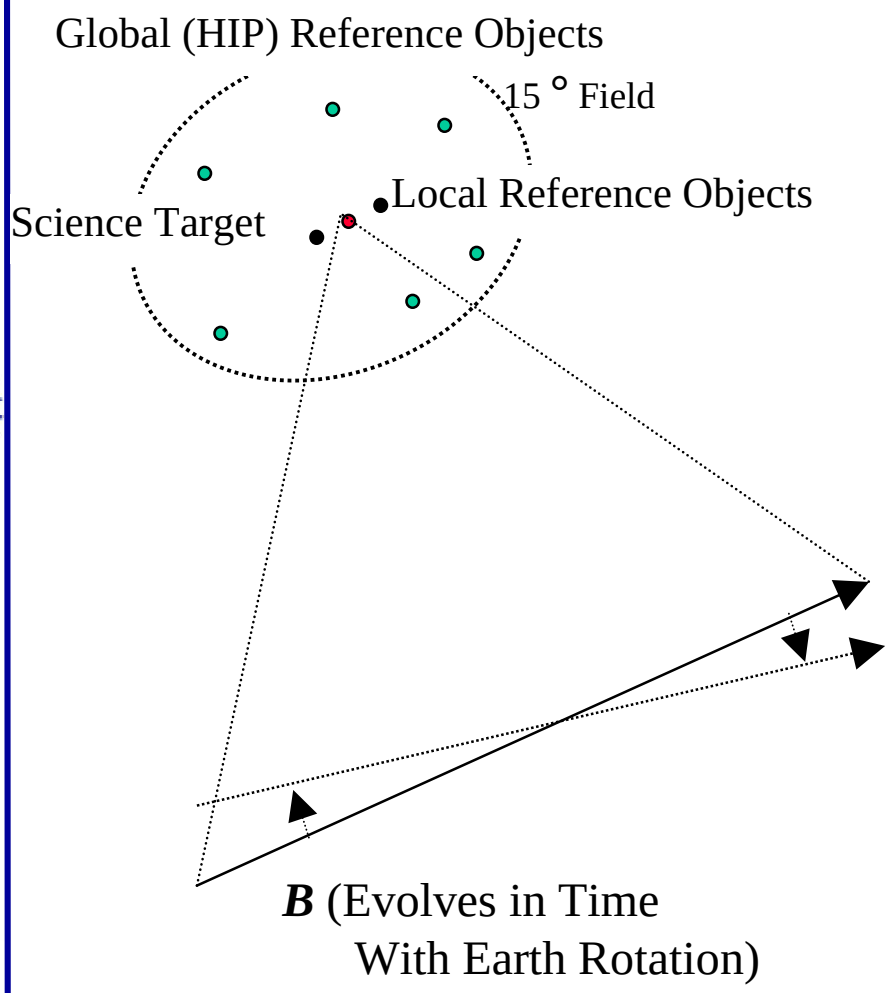
Interferometric Tolerances

$$\Delta s \approx \frac{\Delta d}{B}$$

$$\delta\Delta s \approx \frac{\delta\Delta d}{B} + \frac{\Delta d}{B^2}\delta B = \frac{\delta\Delta d}{B} + \Delta s\frac{\delta B}{B}$$

- $\sigma_d \sim \sigma_s * B$; $\sigma B / B \sim \sigma s / s$
- Take B ~ 100 m, Δs ~ 20” (10^{-4} rad)
- To Make a 10 μas ($5*10^{-11}$ rad) measurement:
  - Must measure Δd to ~ $2.5*10^{-9}$ m >> (2.5 nm) <<
  - Must measure B to 2.5 parts in 10^7 >> (25 μm) <<

Narrow-Angle Astrometric Sequence



- Observe baseline reference objects (~10 mas -- 15 deg field) to establish baseline 3-vector
- Differential delay measurement of target WRT two nearby (~20”) reference objects
- Separation vectors at epoch; establish relative frame parameters (pos, μ, π)
- Science target motions compared against local frame

(External) Baseline Knowledge

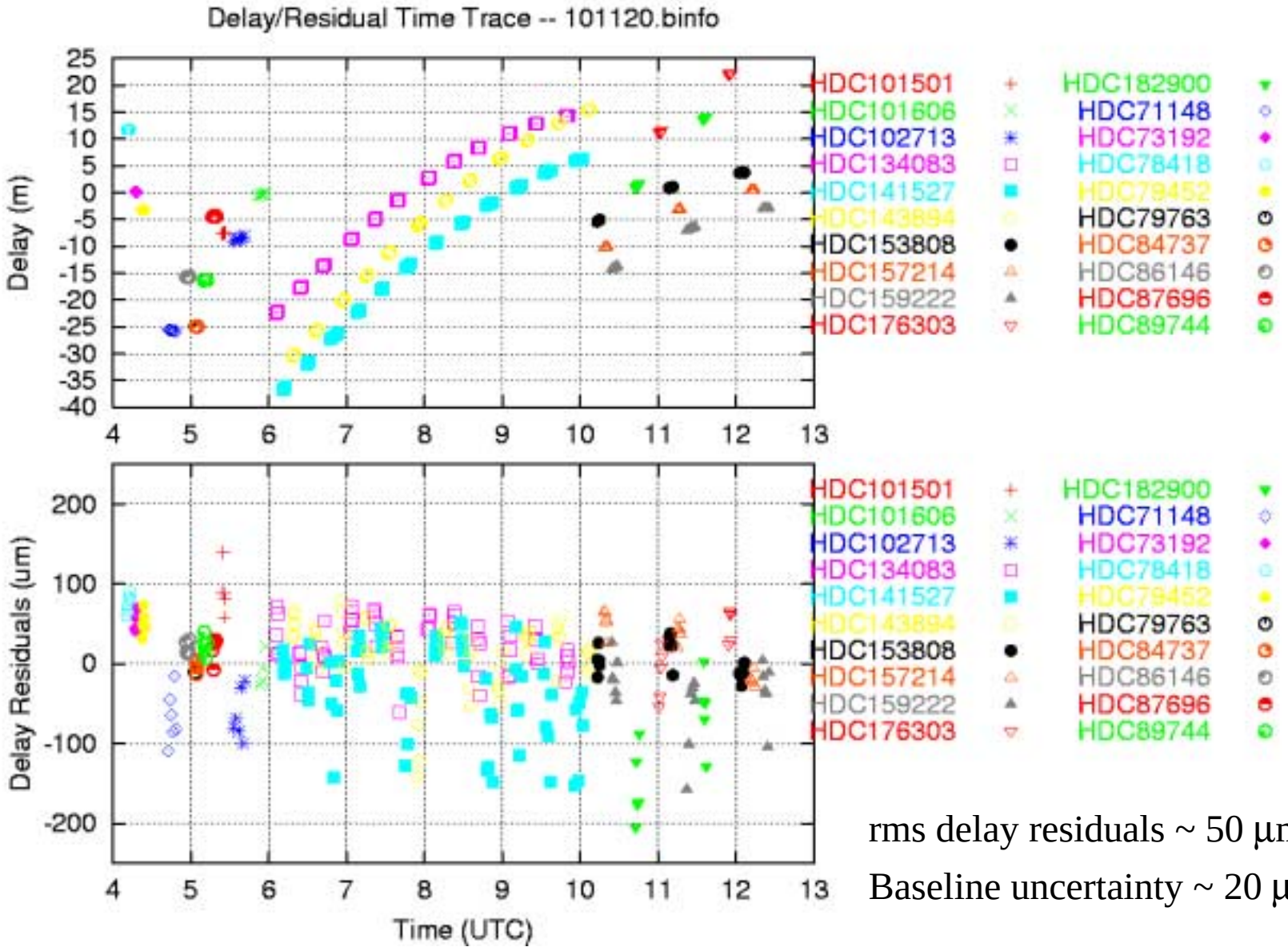
- O(25 μm) Baseline Knowledge
- Canonical Model: Observations of Known (e.g. Hipparcos) Stars Over a Wide (~ 1 rad) Angle
- Again Delay is the Observable:

$$d_i = \hat{s}_i(t) \bullet \underline{B} + C$$

$$\begin{pmatrix} d_i \\ . \\ . \\ d_n \end{pmatrix} = \begin{pmatrix} s_{i-x} s_{i-y} s_{i-z} 1 \\ . \\ . \\ s_{n-x} s_{n-y} s_{n-z} 1 \end{pmatrix} \bullet \begin{pmatrix} B_x \\ B_y \\ B_z \\ C \end{pmatrix}$$

- Problem is Linear if Star Positions are Known (Hipparcos)

Example PTI Baseline Fit



rms delay residuals ~ 50 μm

Baseline uncertainty ~ 20 μm

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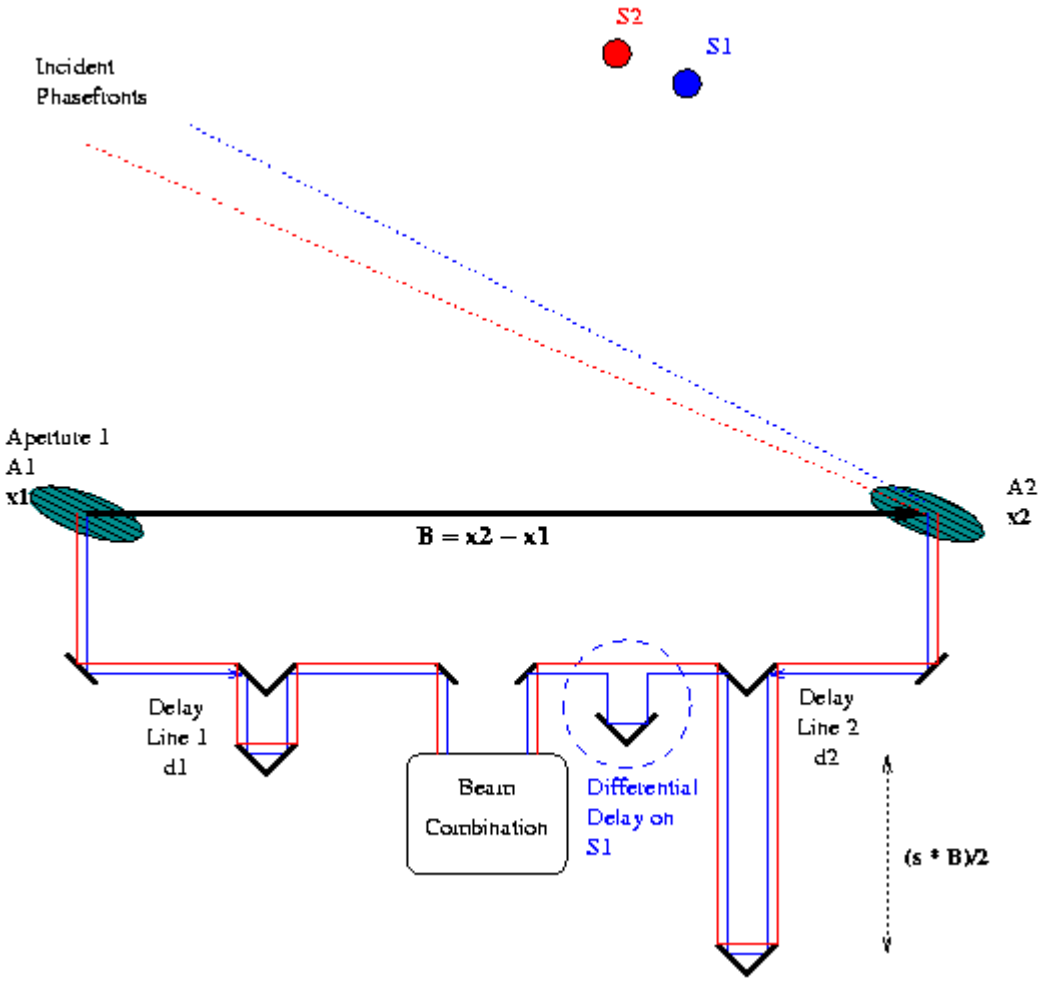
Differential Interferometric Astrometry



- Two Simultaneous Beam Combiners, Independently Tracking Two Stellar Sources
- Differential Delay Mechanism
- Metrology In Each Interferometer Measuring the Relative Delay
- Relative Delay Difference is the Observable

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Differential Interferometric Astrometry (cont)

$$d_2 - d_1 = (\hat{s}_2 \bullet \underline{B} + C_2) - (\hat{s}_1 \bullet \underline{B} + C_1)$$

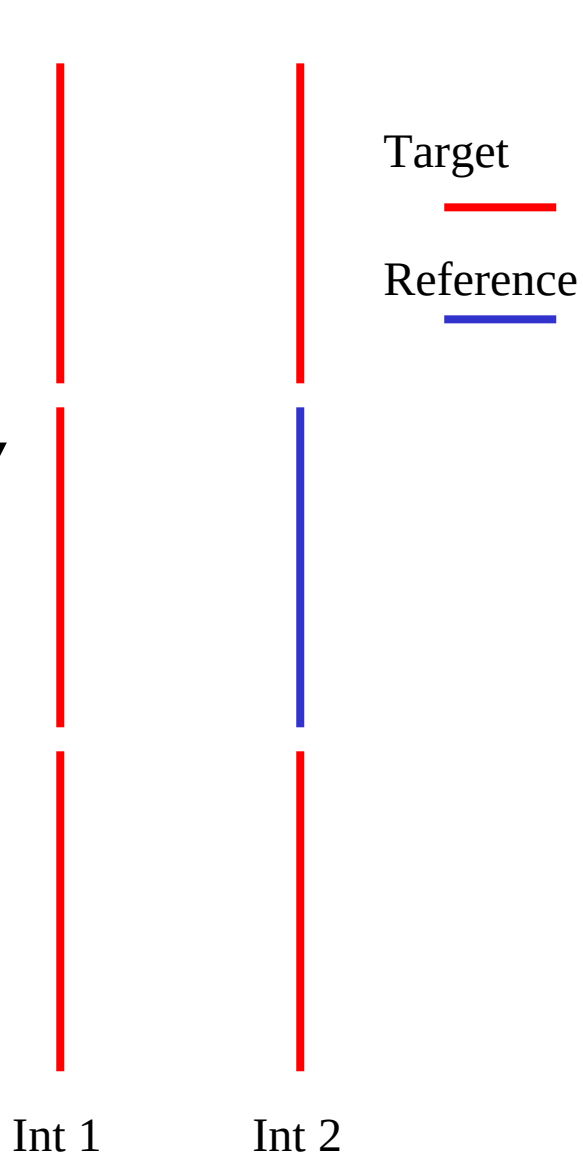
$$= (\hat{s}_2 - \hat{s}_1) \bullet \underline{B} + (C_2 - C_1)$$

$$\Delta d = \underline{\Delta s} \bullet \underline{B} + \Delta C$$

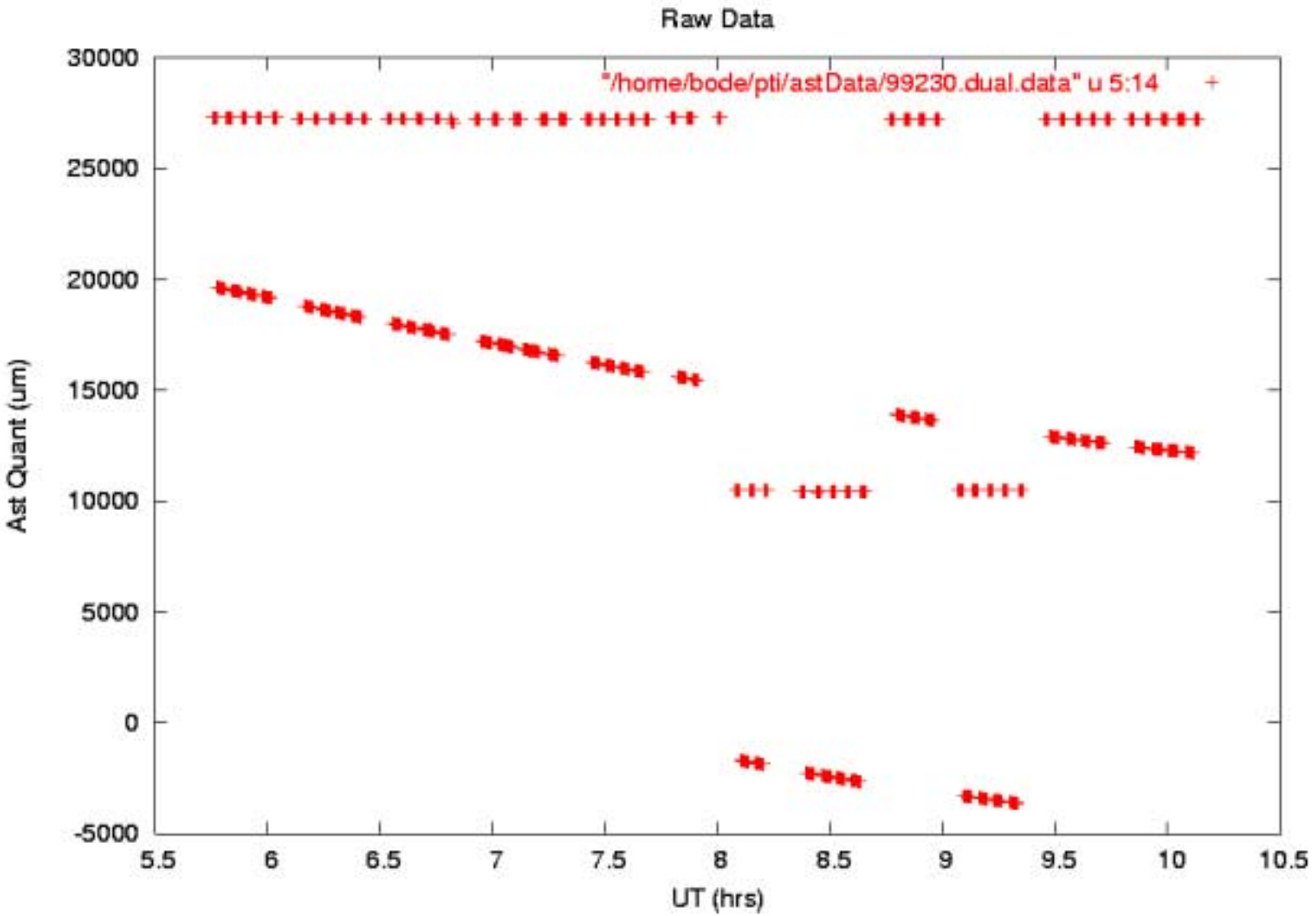
- Delay Difference (Δd) is Observable Proxy for Sky Separation ($\underline{\Delta s}$)
- Differential Delay Contains Instrumental Signature (Bias Term -- Δ*C*) That Must Be Calibrated

Bias Term Calibration

- Two Separate Interferometers >> Two Separate Bias Terms
- Simultaneous Measurement of the Same Star with Both Interferometers Yields Estimate of Relative Bias Difference



Bias Calibration -- Example



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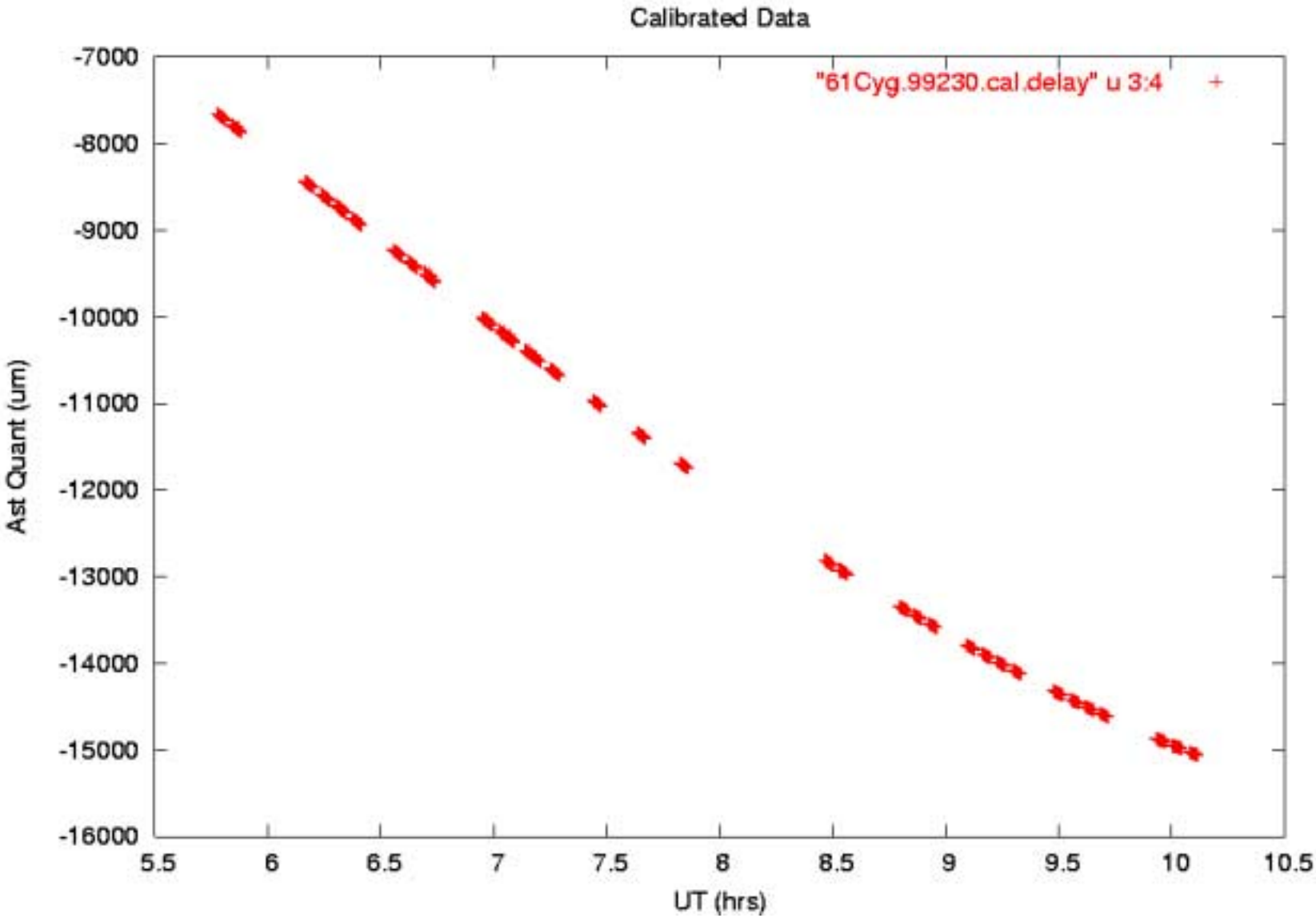
Bias Calibration -- Example



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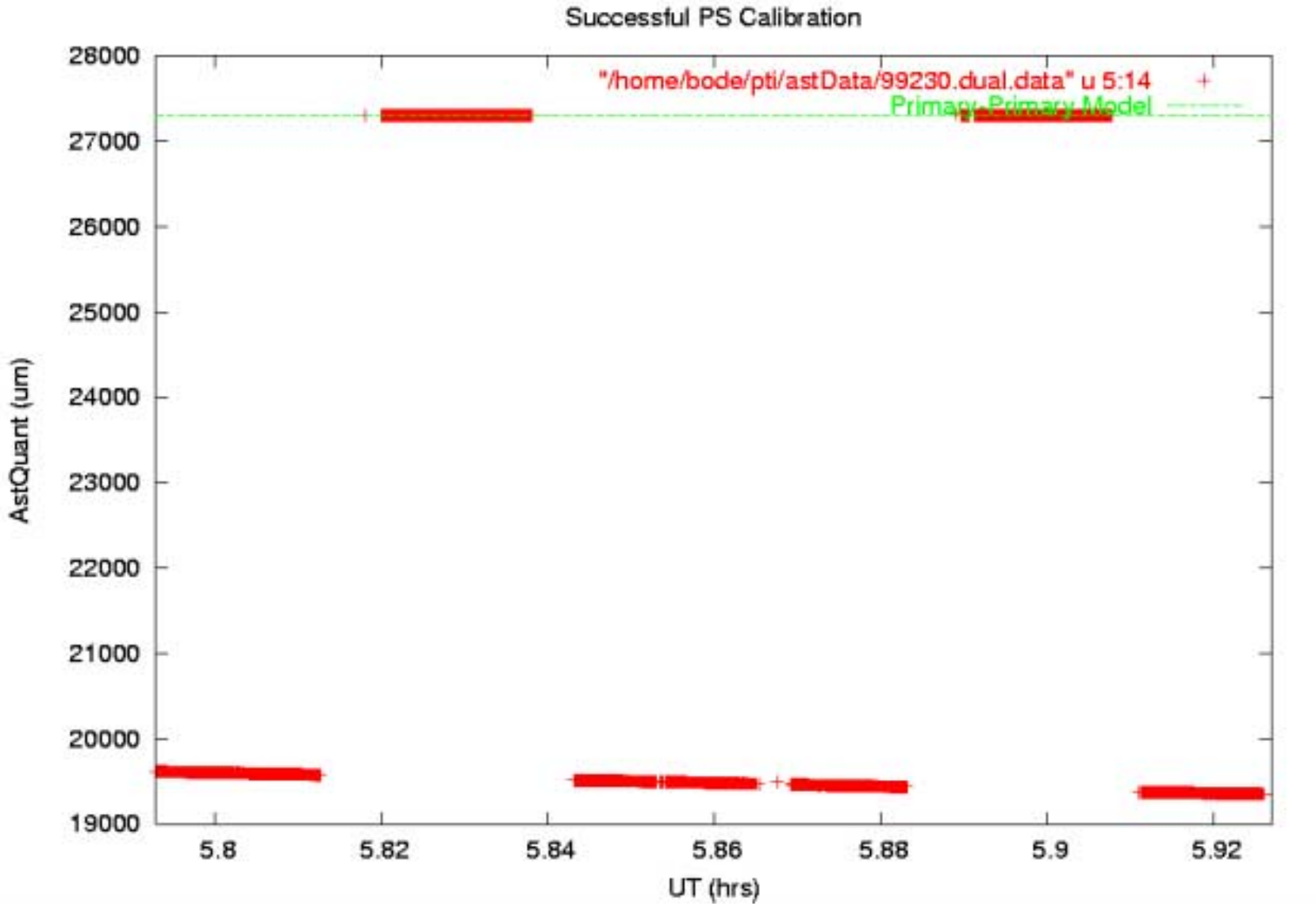
Bias Calibration -- Example



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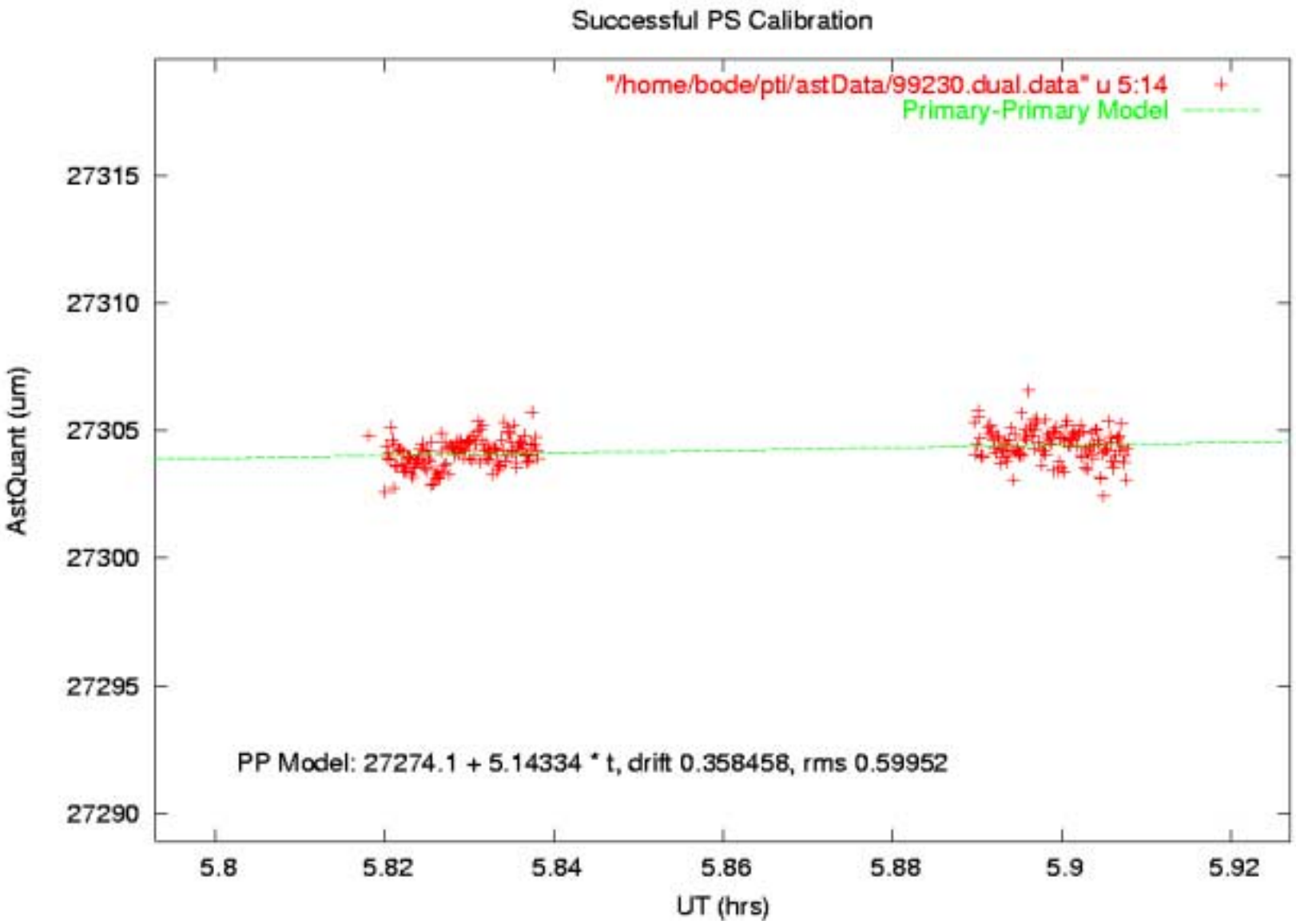
Bias Calibration -- Example



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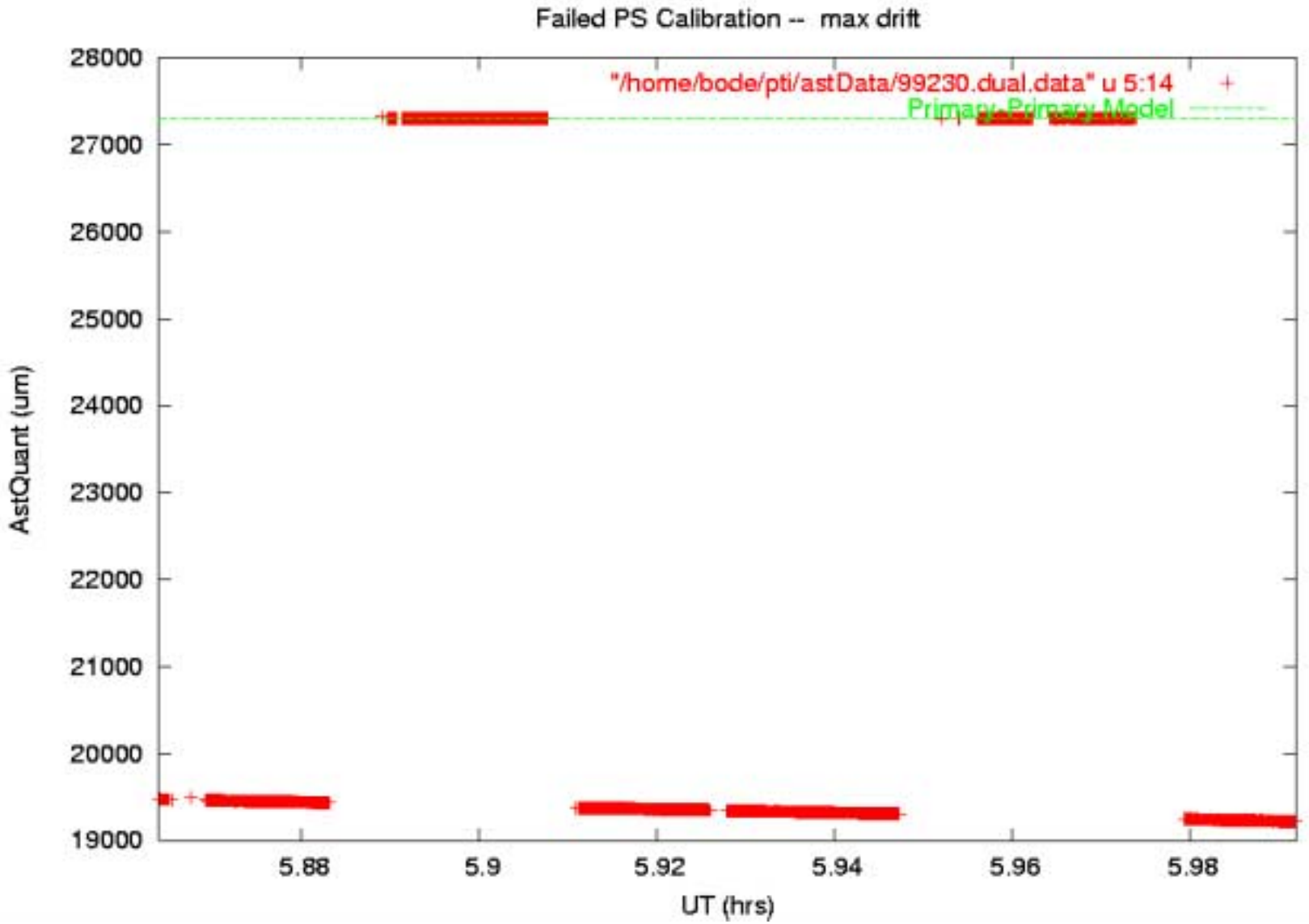
Bias Calibration -- Example



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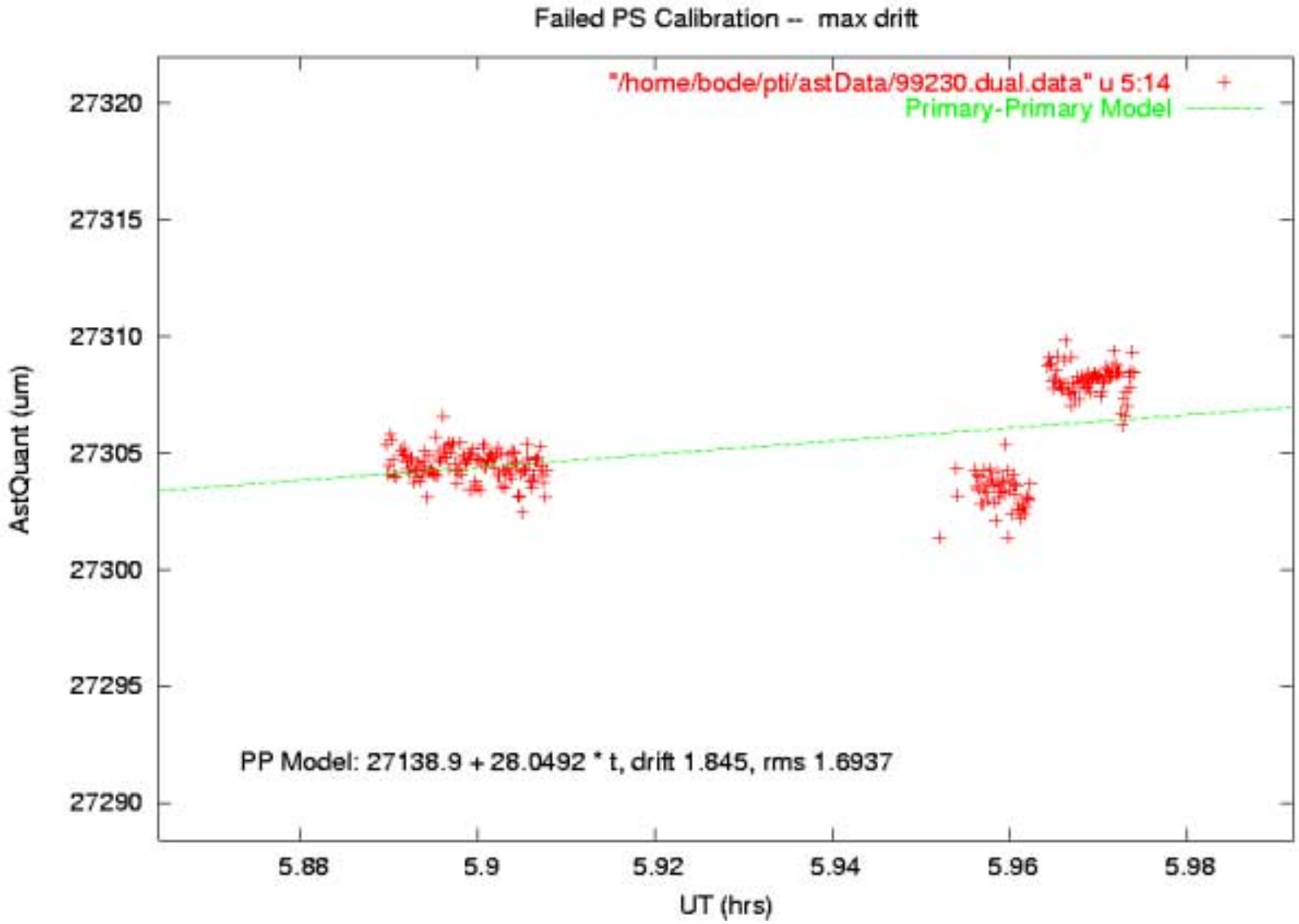
Bias Calibration -- Example



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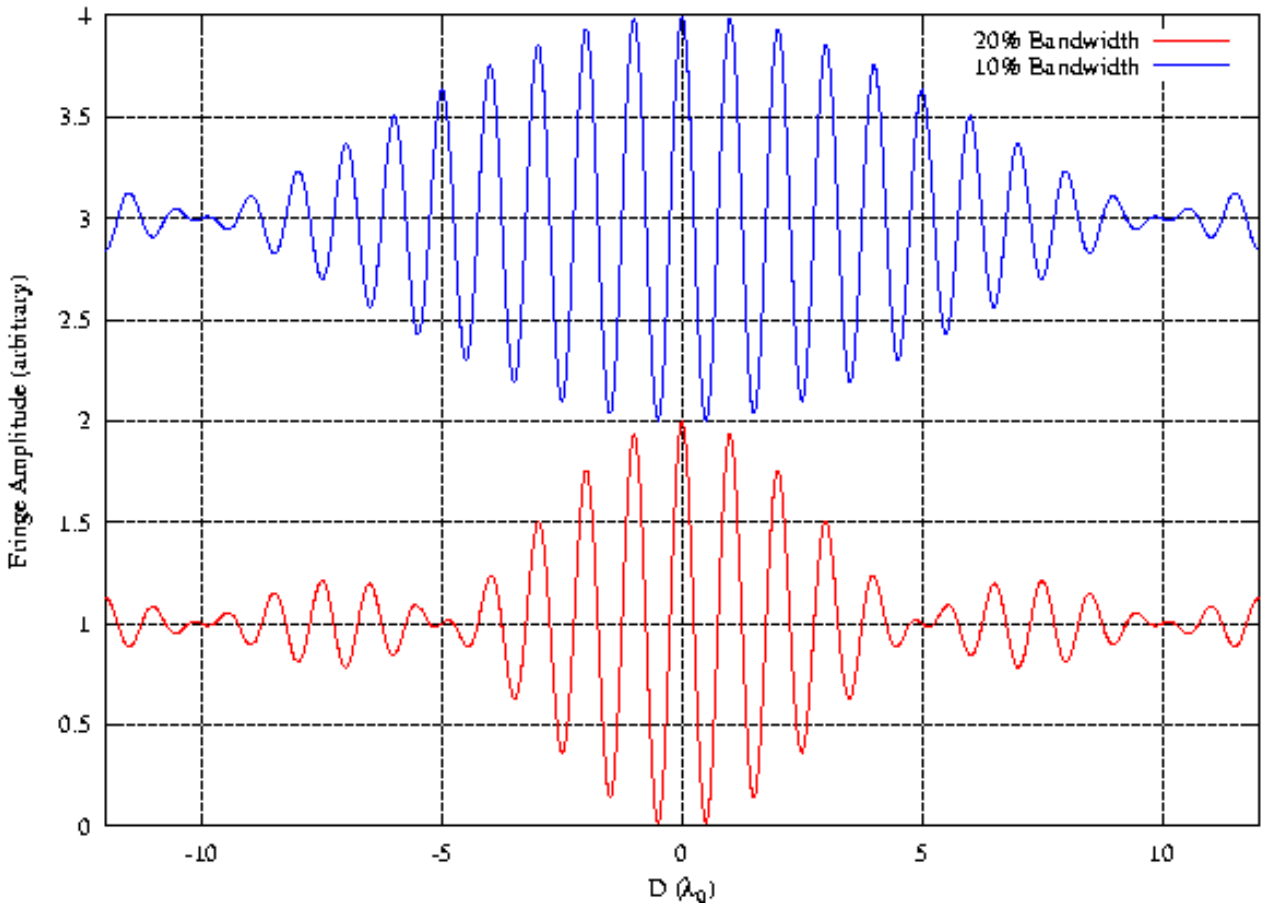
Delay is not Just Delay...



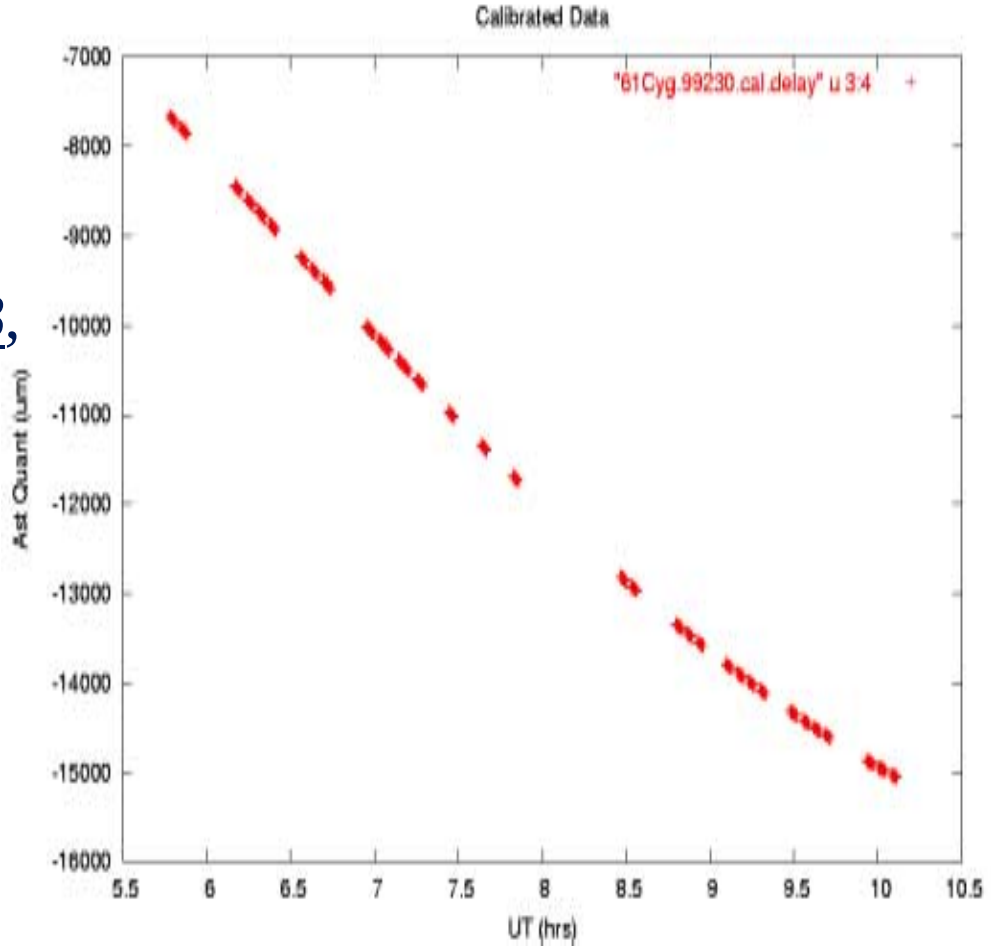
- Astrometric Delay Quantity -- Composite of Metrology Delay and Fringe "Group Delay"

Separation Estimation

- Having Calibrated and Removed the Differential Bias Term, One is Left With:

$$\Delta d = \underline{\Delta s} \bullet \underline{B}$$

- Given that we Know $\underline{B}$, Estimating $\underline{\Delta s}$ is Straightforward...



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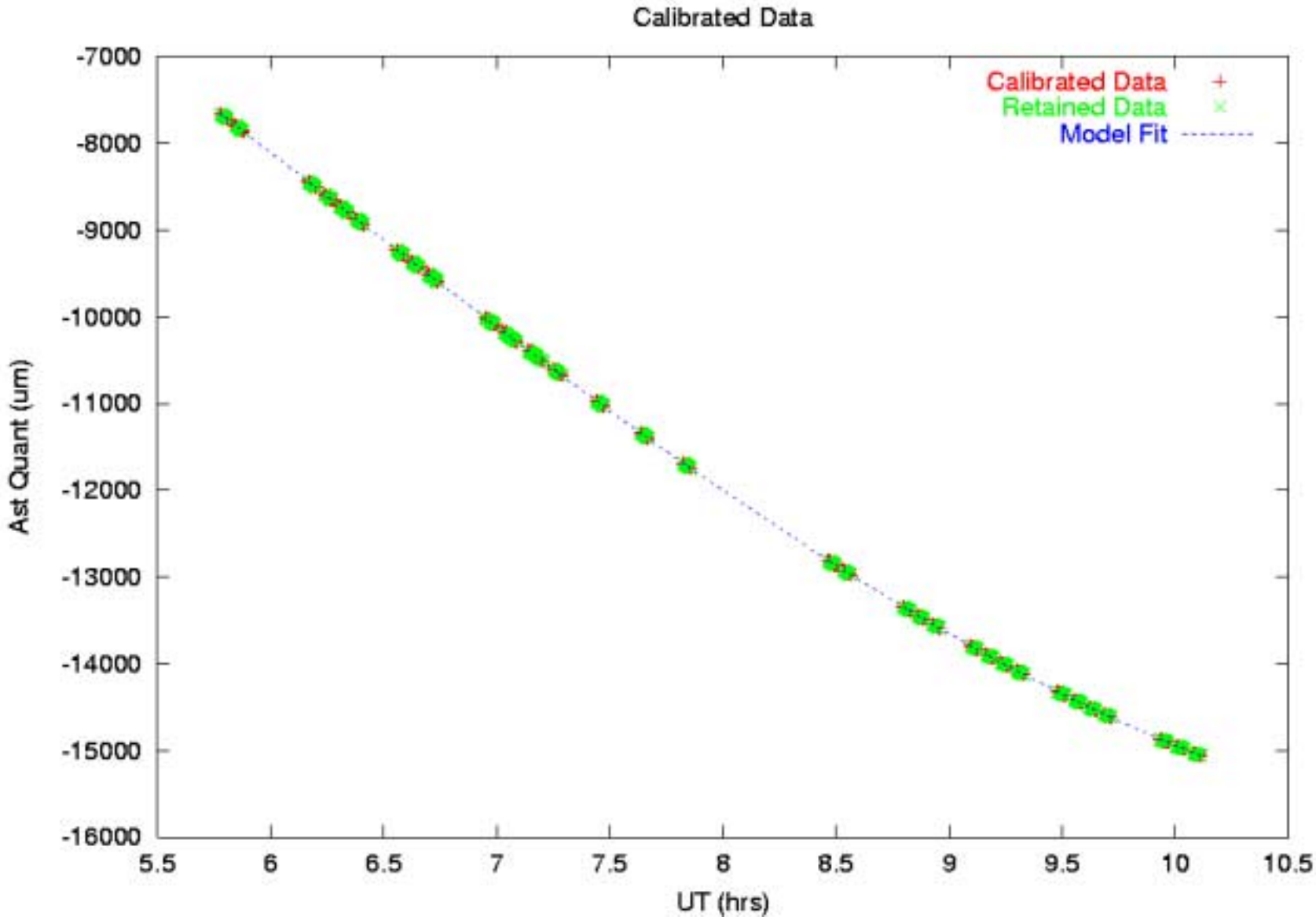
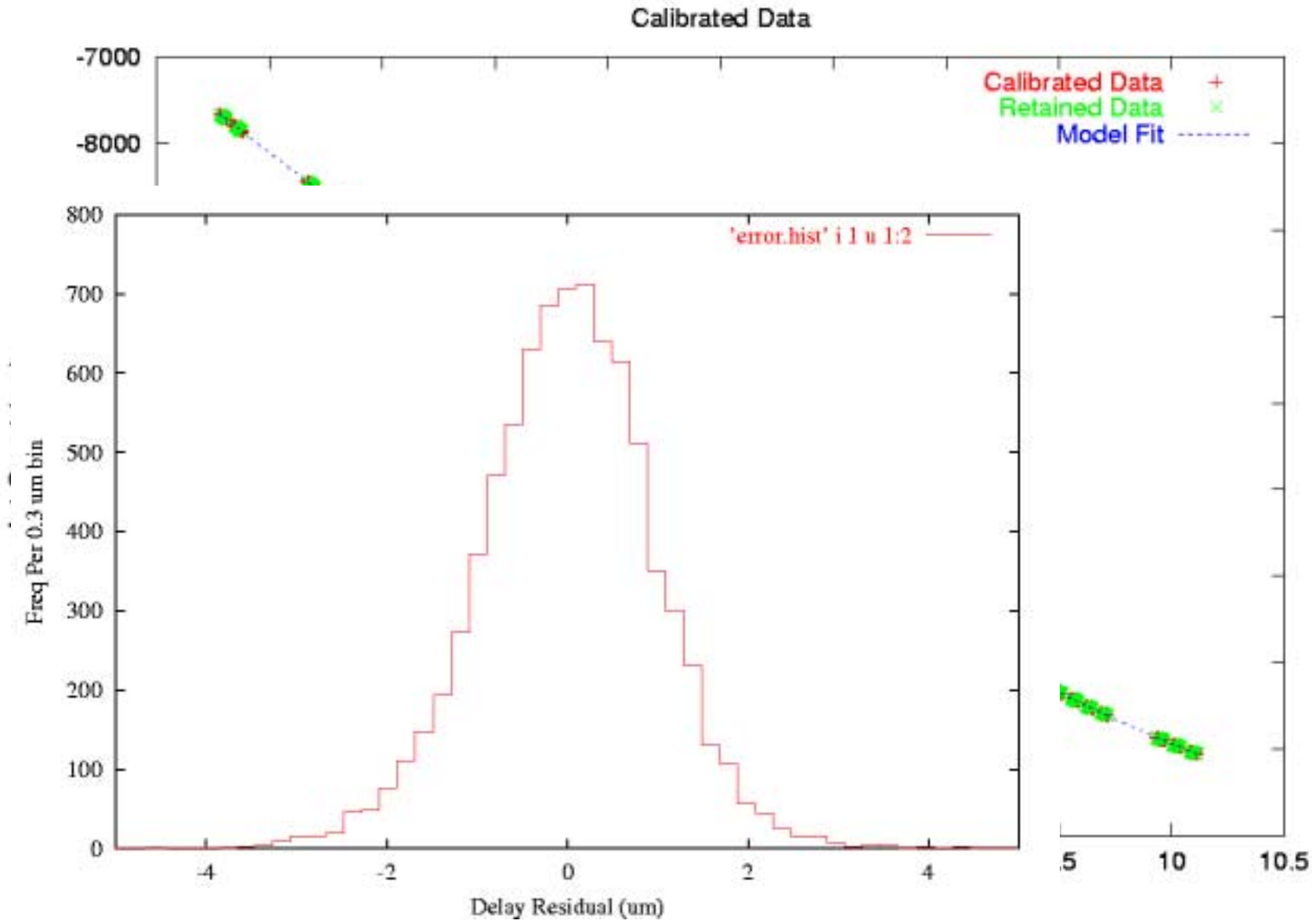
Separation Estimation -- Example



Separation Estimation -- Example



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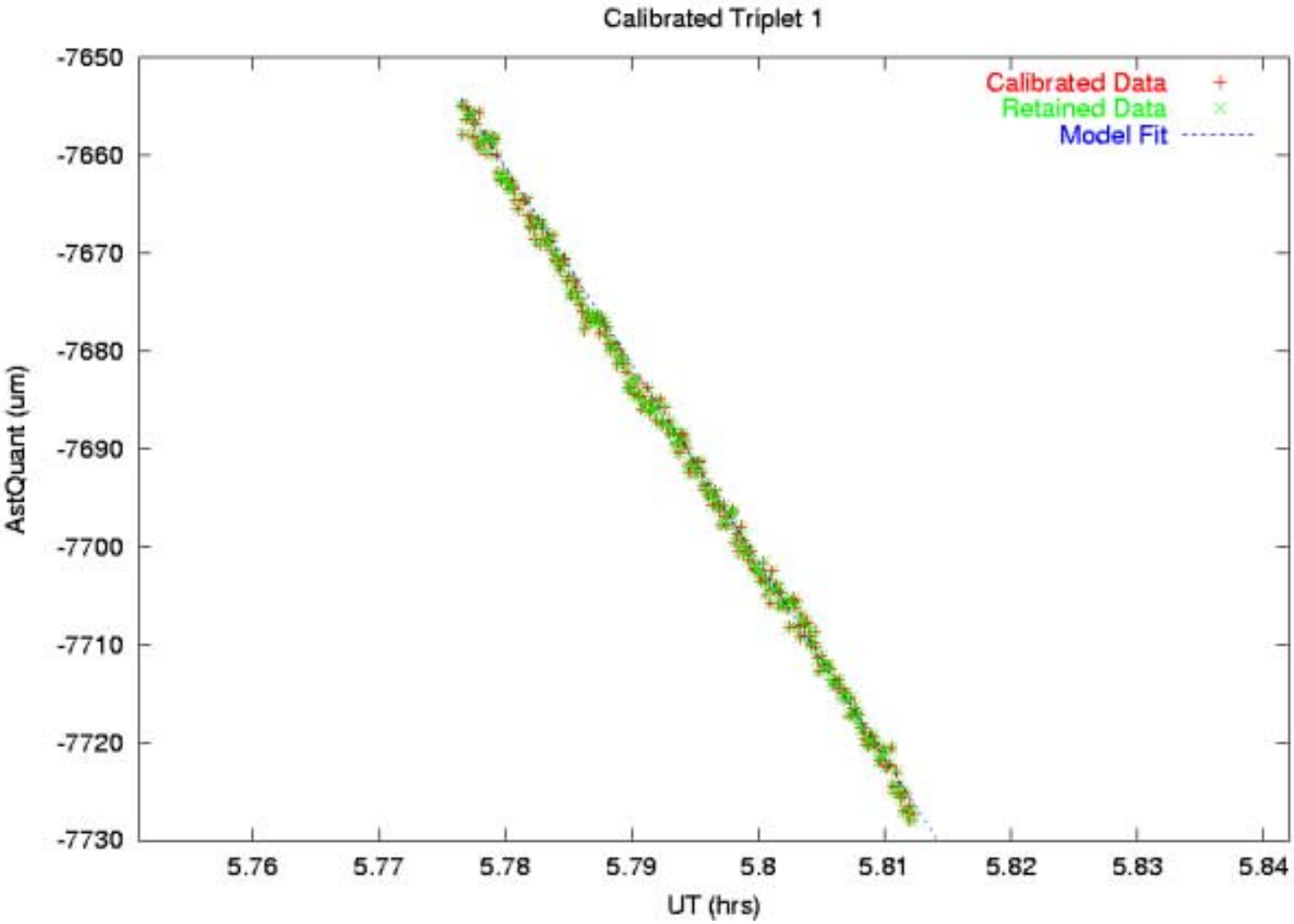
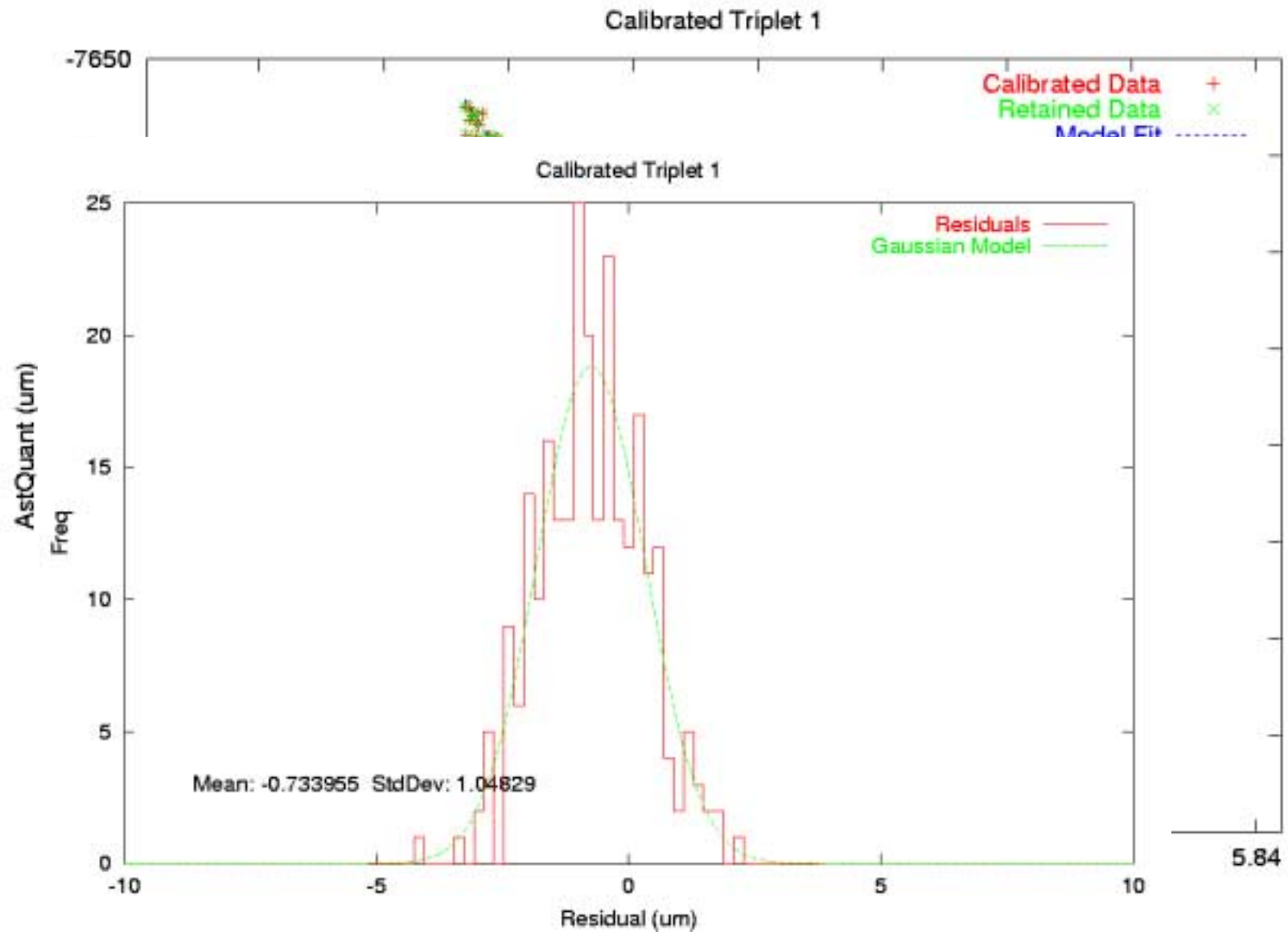
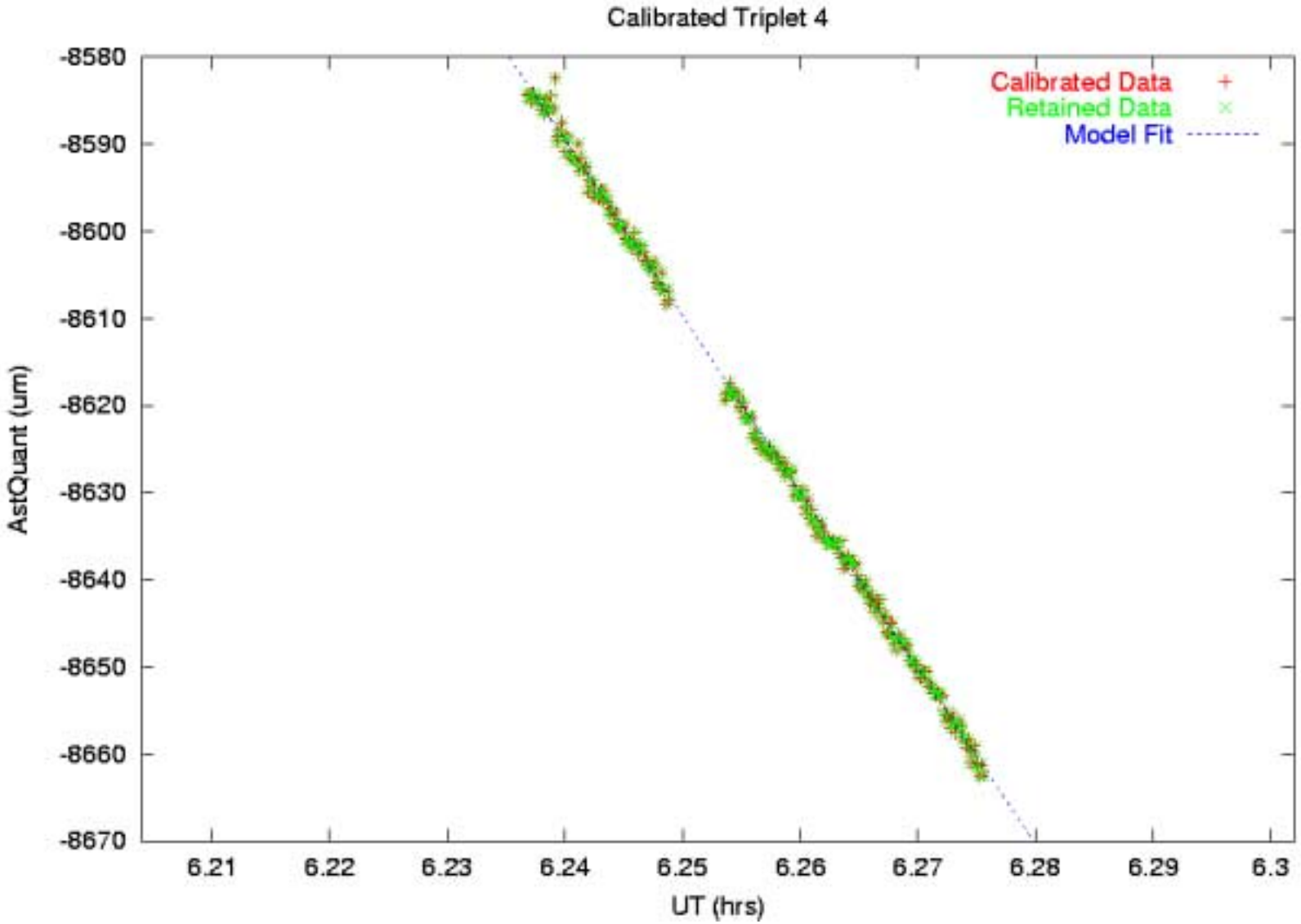
Separation Estimation -- Example



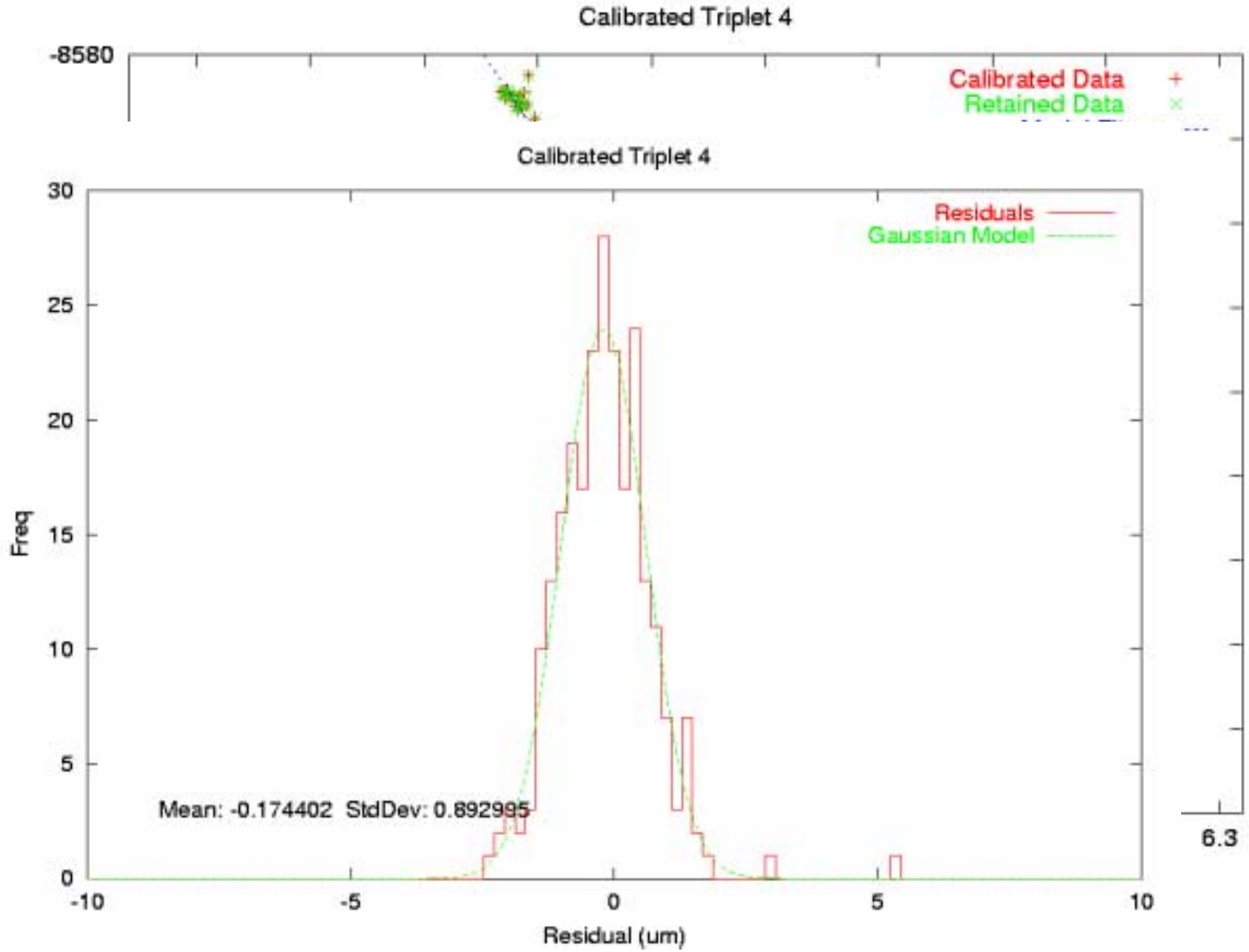
Separation Estimation -- Example



Separation Estimation -- Example



Separation Estimation -- Example



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The Result -- A Relative Separation

- The Output From a Future ISC Astrometry Application...
- In a Form Only a Unix Geek Could Love...

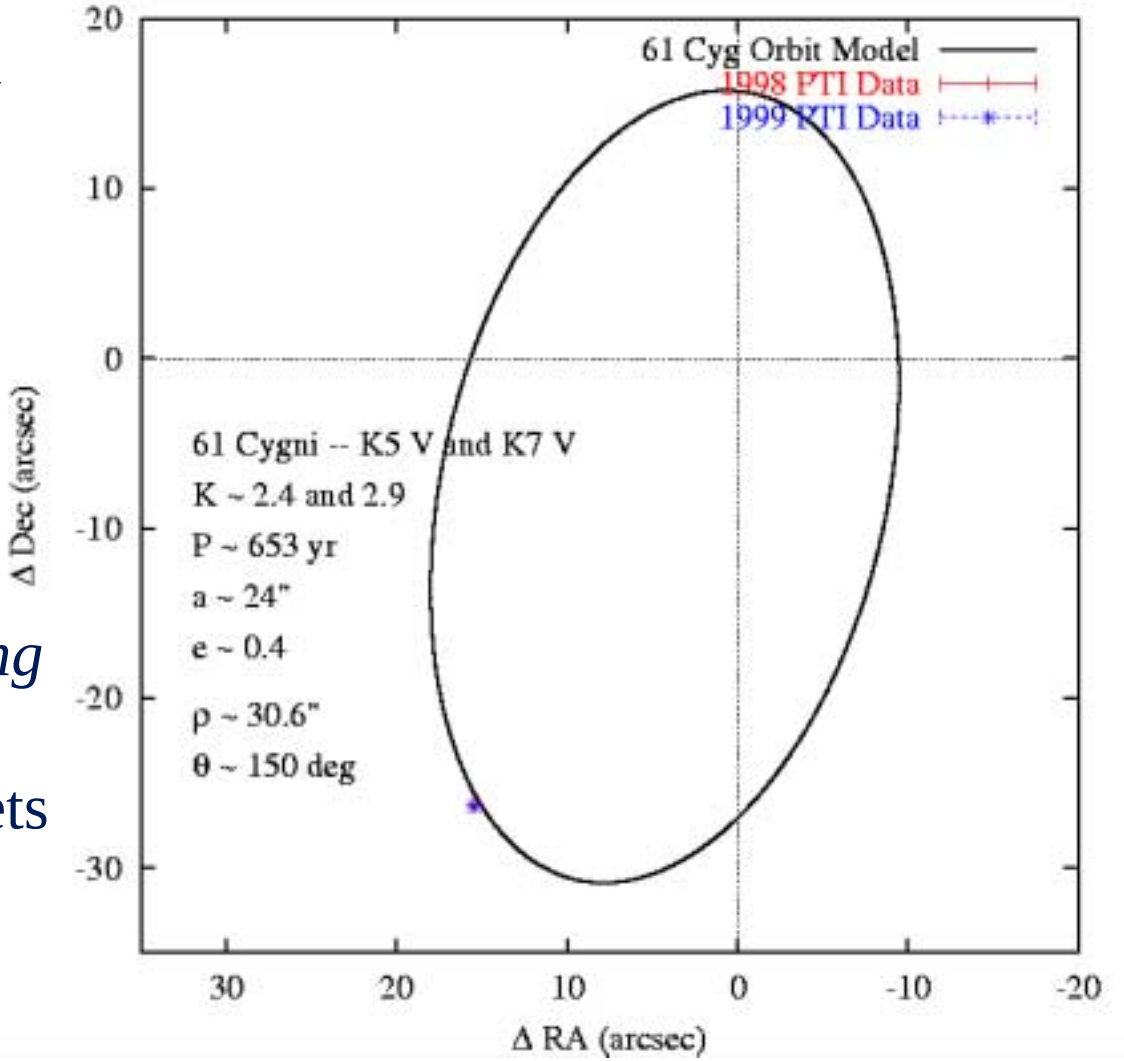
bode@gnomad: /home/bode/eXtras/gnuplot-3.7.1

File Edit Settings Help

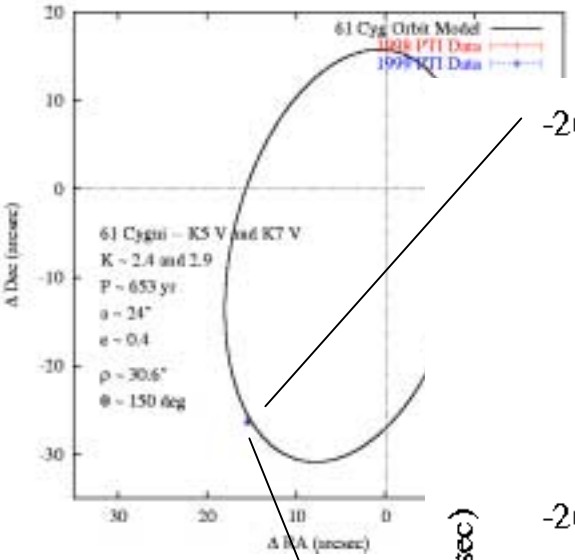
```
# executing: sepFit /home/bode/pti/sepFit/99130.bline /home/bode/pti/astrometry/
61Cyg/61cyg.init /home/bode/pti/astrometry/61Cyg/61Cyg.99230.cal.delay -boost -n
oPre
# option boostCorrect
# option noPrecession
# 2 command line arguments processed
# Simbad Search HD 201091: Type: Variable of BY Dra type  K5V V=5.21 * 61 Cyg A
   V* V1803 Cyg    ** STF 2758AB
# Data taken on: 8/18/1999 UT
# Avg epoch of input delay observations (MJD) 51408.334766086
# Input model separation: 30.646501 (arcsec)   149.681 (deg)
# Fit terminated after 7 iterations
# Fit chi-squared per DOF 1.72048
# RMS/Peak residual delay (m): 1.31167e-06/6.10319e-06
# 8149 out of 8171 delays used (0.997308)
# Output model separation: 30.601261 (arcsec)   149.661 (deg)
# separation error est (arcsec, deg): 7.00865e-05  0.000135219
# delta RA/Dec separation (arcsec):   15.45732   -26.41038      9.45086e-05
3.45819e-05     -0.611462
8/18/1999       230     51408.334766086    15.45732      -26.41038      9.45086e
-05     3.45819e-05     -0.611462

--More--(76%)
```

The Target -- 61 Cygni

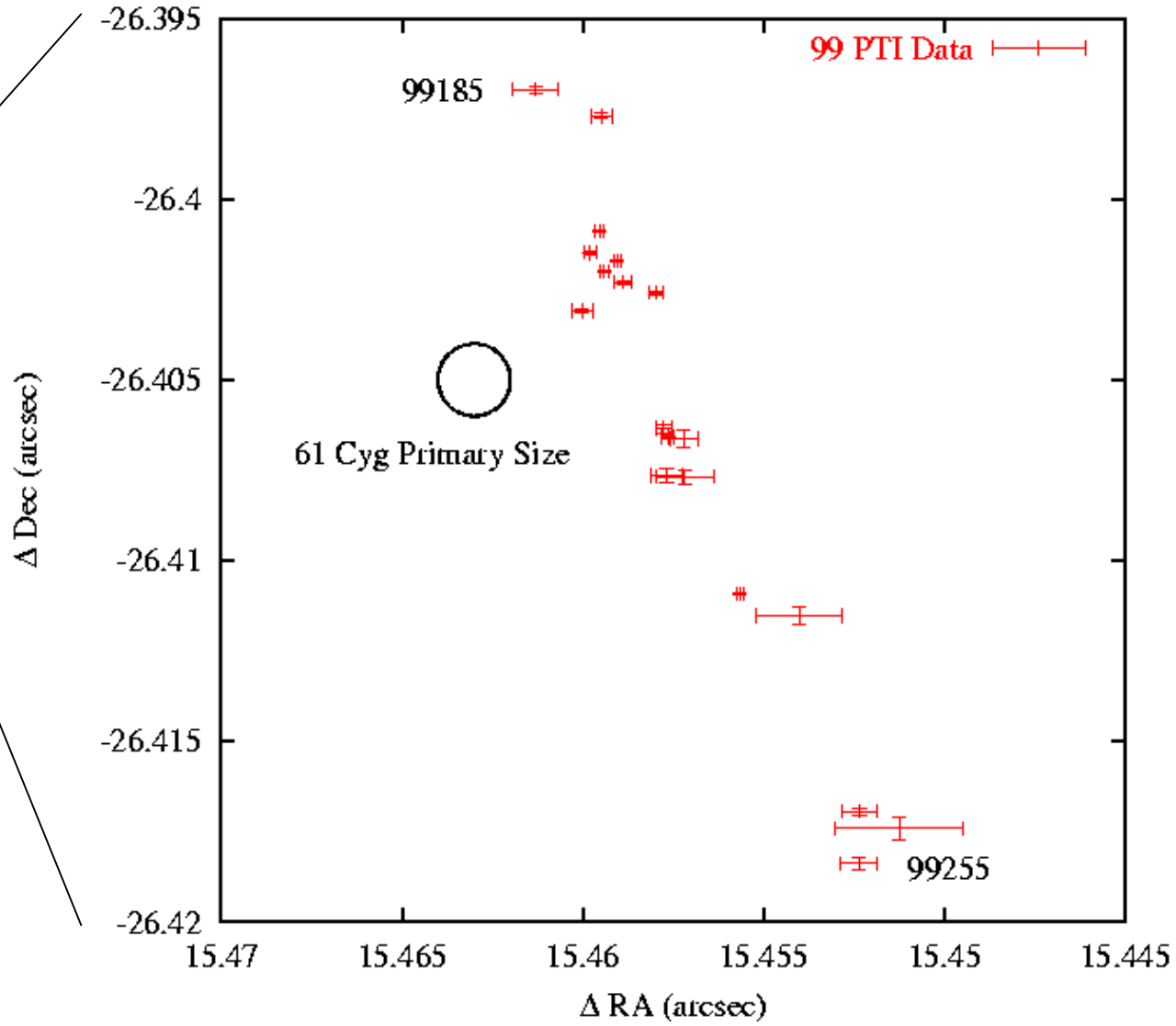
- 61 Cygni -- Nearby K-dwarf Visual Binary (*K*~2.5)
- ~ 650 yr Period Eccentric Orbit
- Have it on Good Authority (Marcy) that There is *Nothing* Going on in This System WRT Planets



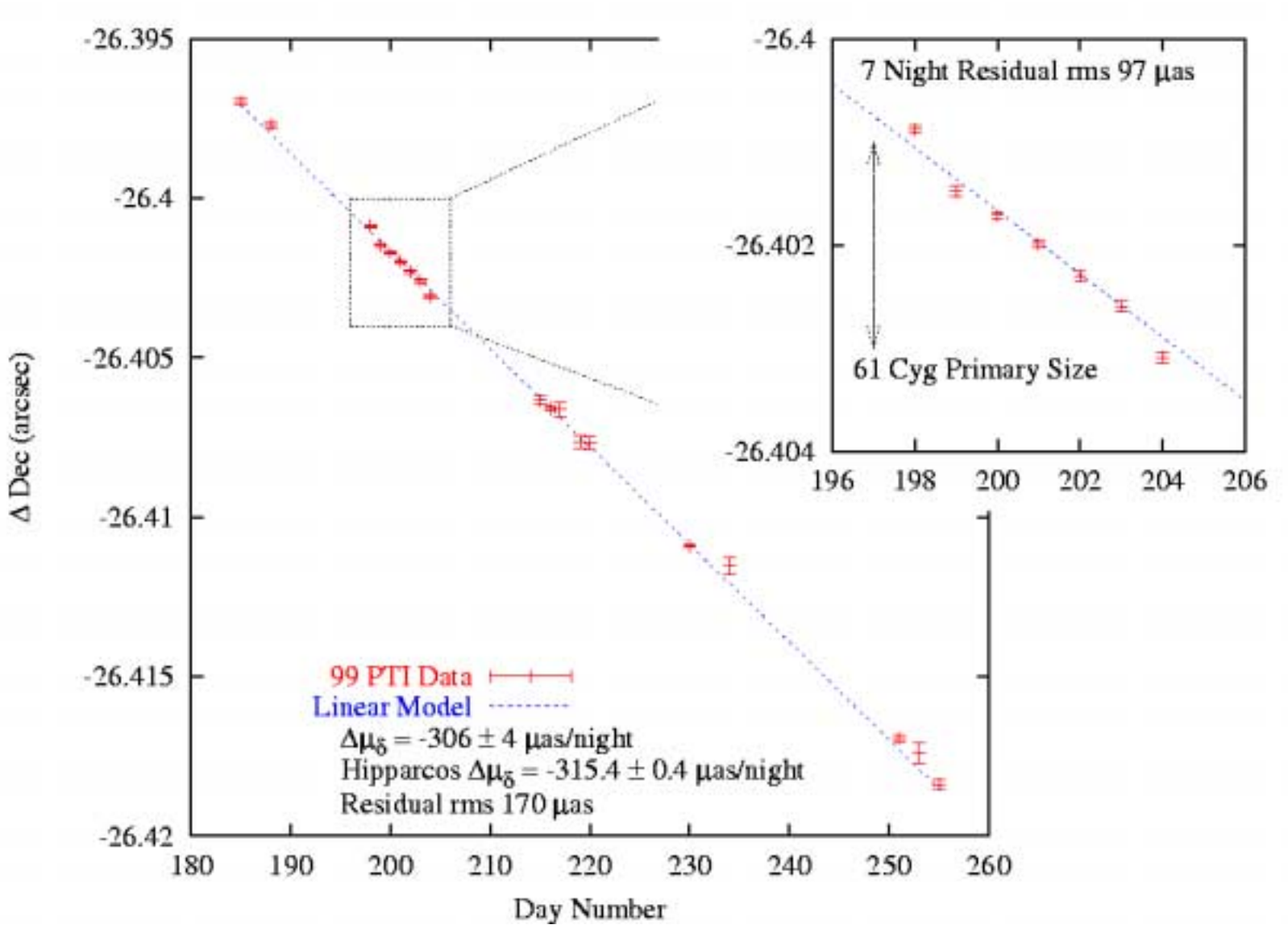
PTI Astrometry on 61 Cygni



2000x



1999 Declination-Only Data



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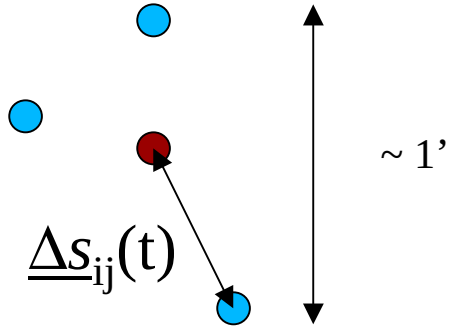
Separation Vectors -- What Next?

- Now That I Have These Separation Vectors, What Do I Do With Them?
- In General, The Target Star and its References Have Relative Motions:
  - Relative Proper Motion
  - Differential Parallax
  - Possible Gravitational Companions
- Necessary to Solve for Relative Frame
  - Relative $\underline{\mu}$ and π
  - Perturbations about the nominal relative values represent dynamical signature



What is it that keeps the relative frame non-rotational?

Wide-angle reference stars used to determine the baseline

Harmonic Detection

- The Common Detection Algorithm for Harmonic Signals is the Lomb-Scargle Periodogram -- Least Squares Fit of sin Function
- Performs Well for Data That Spans More Than One Period

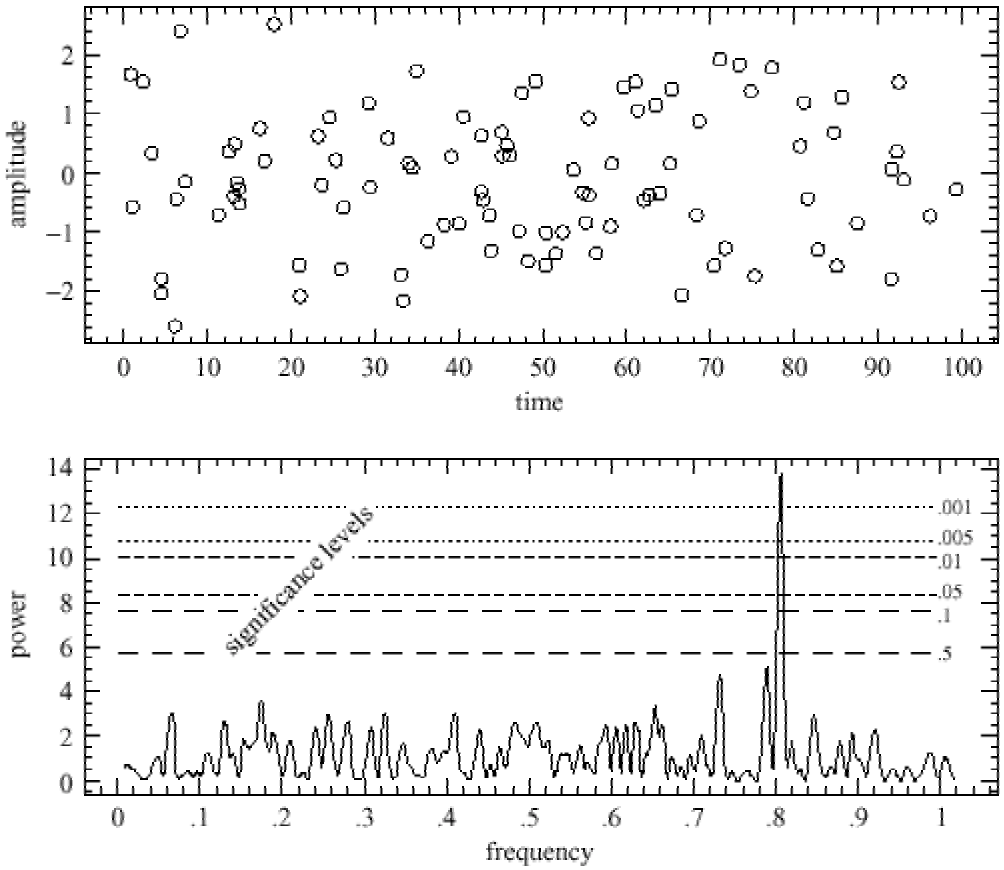
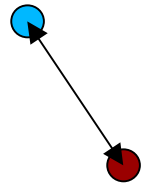


Figure 13.8.1. Example of the Lomb algorithm in action. The 100 data points (upper figure) are at random times between 0 and 100. Their sinusoidal component is readily uncovered (lower figure) by the algorithm, at a significance level better than 0.001. If the 100 data points had been evenly spaced at unit interval, the Nyquist critical frequency would have been 0.5. Note that, for these unevenly spaced points, there is no visible aliasing into the Nyquist range.

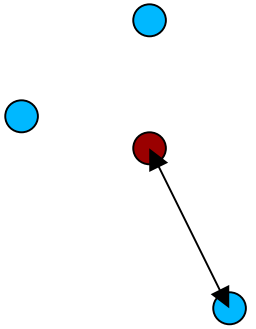
Numerical Recipes, 2nd Ed

Reference Stars

- If Target Exhibits Periodic Signal Against One Reference Star -- Unable to Determine Who is Doing the Wobbling



- With Two (Or More) Reference Objects, Target Motion Shows Up as Common Periodicity in Multiple Target-Reference Periodograms



Summary

- Canonical Model:
  - Baseline calibration from wide-angle reference stars
    - Calibrates the baseline 3-vector
  - Separation estimation
    - Relative bias term calibration
    - Separation estimation from calibrated differential delay
  - Separation dataset
    - Relative pos, μ, and π
    - Harmonic signal in residuals to relative frame fit